Discrete Mathematical Modeling of Powder Bed 3D Printing Process
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3D Printers & Applications

**Powder bed** printers are machines in which a heat source melts the 2D cross section of the object into the bed of powdered material. Then another layer of powder is swept over and sintered onto the previous layer.

**3D printing** is a cutting edge technology with multiple applications:
- Cooling channels which conform to the contours of the object for improved thermal dissipation.
- Building parts with complex geometry which would be difficult and expensive to manufacture with traditional methods.
- The option to build and test prototypes during the development phase.

**Heat Source Model**

- **Heat flux from laser:** \( q_{\text{laser}} = \frac{Q - r_j^3}{r_{\text{laser}}} \) where \( Q \) is the total power of the laser, \( r_j \) is the radius of the particle, \( r_{\text{laser}} \) is the radius of the laser.
- **Heat flux between two particles:** \( q_{ij} = k_i (T_i - T_j) \) where \( k_i \) is the heat transfer coefficient, and \( T_i, T_j \) are the temperatures of particle \( i \) and \( j \).
- **Total heat flux:** \( q_i = q_{\text{laser}} + \sum_{j=1}^N q_{ij} \)
- **Temperature update** for particle \( i \):
  \[
  T_i^{\text{new}} = T_i^i + \frac{q_i}{m_i C_p} \Delta t
  \]
  where \( T_i^i \) is the initial temperature, \( q_i \) is the total initial energy flux, \( m_i \) is the mass, and \( C_p \) is the specific heat capacity of particle \( i \).
- If particle \( i \) and \( j \) are in contact and both above the sintering temperature a bond is formed between them.

**The Powder Bed Model**

**Our Algorithm:**
- Every new sphere is randomly generated from chosen distribution.
- If two or more particles are in contact they can spawn a new sphere.
- The location of the new sphere is determined by solving a system of equations relating to the location and radii of the parent particles.

The distribution of powder particle size was determined to be a **Weibull distribution**:
\[
 f(x; \lambda, k) = \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-(x/\lambda)^k}
\]

\( x \) is the diameter of the particle, \( k = 31.4 \) \( \mu \text{m} \) is the scale parameter, \( \lambda = 3.55 \) \( \mu \text{m} \) is the shape parameter.

**Packing of Random Spheres**

The distribution of spheres closely matches the probability density function from which the radii were randomly drawn.

**Building the Powder Bed**

Once the cube is filled with spheres the bed is created by exploiting the symmetry of the cube. By reflecting the cube about its face it is possible to determine the contacts between two joined cubes.

**Simulated Print of a Square**

**Conclusions**

**Summary:** An algorithm was developed to fill a cube with spheres of random radii. Symmetry of the cube was used to build a simulated powder bed. A discrete model of the 3D printing process was developed to study the affect of varying printing parameters.

**Conclusions:** The packing of particles will affect the final object. The path the laser takes will affect the internal bonding of the object. Residual heat from previous laser passes will affect the building process and must be taken into consideration.

**References**
