Techniques for Finding Derivatives

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If \( f \) and \( g \) are differentiable at \( x \), then
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If $f$ and $g$ are differentiable at $x$, then

4. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$. 
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These rules are sufficient for the differentiation of all polynomials.

**Example 1:** \( \frac{d}{dx}[8x^{10}] = \)
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Example 1: $\frac{d}{dx} [8x^{10}] = 8 \frac{d}{dx} [x^{10}] = \ldots$
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The Product & Quotient Rules
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The derivative of the product of two non-constant functions is **never** equal to the product of their derivatives.
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or

\[ (f(x)g(x))' = f(x)g'(x) + f'(x)g(x) \]
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Notice that \( f'(x)g'(x) = 1(1) = 1 \), which is quite different, and incorrect.

Example 5: Find the derivative of \((2x^2 + 3)(4x + 5)\).
Example 4: Let $f(x) = x$, and $g(x) = x$, so that $f(x)g(x) = x \cdot x = x^2$. Then the Product Rule gives us $(x \cdot x)' = x(x)' + (x)'x = x(1) + (1)x = 2x$, which agrees with $(x^2)' = 2x$.

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Example 5: Find the derivative of $(2x^2 + 3)(4x + 5)$.

Let $f(x) = 2x^2 + 3$, and $g(x) = 4x + 5$. 
Example 4: Let \( f(x) = x \), and \( g(x) = x \), so that \( f(x)g(x) = x \cdot x = x^2 \). Then the Product Rule gives us
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Example 4: Let $f(x) = x$, and $g(x) = x$, so that $f(x)g(x) = x \cdot x = x^2$. Then the Product Rule gives us

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\]
The Quotient Rule

The derivative of the quotient of two non-constant functions requires an even more complicated formula:

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

or, in function notation,

$$\left( \frac{f}{g} \right)' = \frac{g f' - f g'}{g^2}$$

or

$$\left( \frac{f(x)}{g(x)} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

**Example 7:** Let $f(x) = x + 1$, and $g(x) = x - 1$, so that $\frac{f(x)}{g(x)} = \frac{x + 1}{x - 1}$. Then the Quotient Rule gives us:

$$\left( \frac{x + 1}{x - 1} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} =$$

$$\frac{(x - 1)(x + 1)' - (x + 1)(x - 1)'}{(x - 1)^2} =$$
The Quotient Rule

The derivative of the quotient of two non-constant functions requires an even more complicated formula:

\[
\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}
\]

or, in function notation,

\[
\left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2}
\]

or

\[
\left( \frac{f(x)}{g(x)} \right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}
\]

Example 7: Let \( f(x) = x + 1 \), and \( g(x) = x - 1 \), so that \( \frac{f(x)}{g(x)} = \frac{x + 1}{x - 1} \). Then the Quotient Rule gives us:

\[
\left( \frac{x + 1}{x - 1} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}
\]

\[
(x - 1)(x + 1)' - (x + 1)(x - 1)' =
\]

\[
(x - 1)(1) - (x + 1)(1) =
\]

\[
(x - 1) - (x + 1) =
\]
The Quotient Rule

The derivative of the quotient of two non-constant functions requires an even more complicated formula:

\[
\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}
\]

or, in function notation,

\[
\left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2}
\]

or

\[
\left( \frac{f(x)}{g(x)} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}
\]

Example 7: Let \( f(x) = x + 1 \), and \( g(x) = x - 1 \), so that \( \frac{f(x)}{g(x)} = \frac{x + 1}{x - 1} \). Then the Quotient Rule gives us:

\[
\left( \frac{x + 1}{x - 1} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{(x - 1)(x + 1)' - (x + 1)(x - 1)'}{(x - 1)^2}
\]

\[
\frac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2} = \frac{(x - 1) - (x + 1)}{(x - 1)^2} =
\]
\[
\frac{x - 1 - x - 1}{(x - 1)^2} =
\]
\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \]

\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}.
\]

Notice that \(\frac{f'(x)}{g'(x)} = \)
Techniques for Finding Derivatives-6

\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}.
\]

Notice that \( \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \).
\[
\frac{x - 1 - x - 1}{(x - 1)^2} = -\frac{2}{(x - 1)^2}.
\]

Notice that \(\frac{f'(x)}{g'(x)} = \frac{1}{1} = 1\), which is quite different, and incorrect.
\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}.
\]

Notice that \( \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \), which is quite different, and incorrect.

**Example 8:** Differentiate \( \frac{x^2 - 4x + 2}{x + 3} \)
\[-\frac{x-1 - x - 1}{(x-1)^2} = \frac{-2}{(x-1)^2}.
\]

Notice that \(\frac{f'(x)}{g'(x)} = \frac{1}{1} = 1\), which is quite different, and incorrect.

---

**Example 8:** Differentiate \(\frac{x^2 - 4x + 2}{x + 3}\)

**Solution:** \(\left(\frac{x^2 - 4x + 2}{x + 3}\right)' = \)
\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}.
\]

Notice that \( \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \), which is quite different, and incorrect.

**Example 8:** Differentiate \( \frac{x^2 - 4x + 2}{x + 3} \)

**Solution:**

\[
\left( \frac{x^2 - 4x + 2}{x + 3} \right)' = \frac{(x + 3)(x^2 - 4x + 2)' - (x^2 - 4x + 2)(x + 3)'}{(x + 3)^2}
\]
$$\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}. \quad \text{Notice that } \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1, \text{ which is quite different, and incorrect.}$$

**Example 8:** Differentiate \( \frac{x^2 - 4x + 2}{x + 3} \)

**Solution:** 

\[
\left( \frac{x^2 - 4x + 2}{x + 3} \right)' = 
\frac{(x + 3)(x^2 - 4x + 2)' - (x^2 - 4x + 2)(x + 3)'}{(x + 3)^2} = 
\frac{(x + 3)(2x - 4) - (x^2 - 4x + 2)(1)}{(x + 3)^2}
\]
$$\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}.$$  

Notice that \( \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \), which is quite different, and incorrect.

**Example 8:** Differentiate \( \frac{x^2 - 4x + 2}{x + 3} \)

**Solution:**  
\[
\left( \frac{x^2 - 4x + 2}{x + 3} \right)' =
\]
\[
\frac{(x + 3)(x^2 - 4x + 2)' - (x^2 - 4x + 2)(x + 3)'}{(x + 3)^2} =
\]
\[
\frac{(x + 3)(2x - 4) - (x^2 - 4x + 2)(1)}{(x + 3)^2} =
\]
\[
\frac{2x^2 + 2x - 12 - x^2 + 4x - 2}{(x + 3)^2} =
\]
\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}
\]

Notice that \( \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \), which is quite different, and incorrect.

**Example 8:** Differentiate \( \frac{x^2 - 4x + 2}{x + 3} \)

**Solution:**

\[
\left(\frac{x^2 - 4x + 2}{x + 3}\right)' = \frac{(x + 3)(x^2 - 4x + 2)' - (x^2 - 4x + 2)(x + 3)'}{(x + 3)^2}
\]

\[
= \frac{(x + 3)(2x - 4) - (x^2 - 4x + 2)(1)}{(x + 3)^2}
\]

\[
= \frac{2x^2 + 2x - 12 - x^2 + 4x - 2}{(x + 3)^2}
\]
\[
\frac{x - 1 - x - 1}{(x - 1)^2} = \frac{-2}{(x - 1)^2}.
\]

Notice that \( \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \), which is quite different, and incorrect.

**Example 8:** Differentiate \( \frac{x^2 - 4x + 2}{x + 3} \)

**Solution:**

\[
\left( \frac{x^2 - 4x + 2}{x + 3} \right)' = \frac{(x + 3)(x^2 - 4x + 2)' - (x^2 - 4x + 2)(x + 3)'}{(x + 3)^2} = \frac{(x + 3)(2x - 4) - (x^2 - 4x + 2)(1)}{(x + 3)^2} = \frac{2x^2 + 2x - 12 - x^2 + 4x - 2}{(x + 3)^2} = \frac{x^2 + 6x - 14}{(x + 3)^2}.
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution: \( \left( \frac{\sqrt{t}}{2t + 3} \right)' = \)
Example 9: Differentiate $\frac{\sqrt{t}}{2t + 3}$

Solution: \[
\left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)'
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution: \[
\left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{\frac{1}{2} t^{-\frac{1}{2}}}{2t + 3} \right)'
\]

\[
\frac{(2t + 3) \left( t^{-\frac{1}{2}} \right)' - \left( t^{-\frac{1}{2}} \right) (2t + 3)'}{(2t + 3)^2} =
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution:

\[
\left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)'
\]

\[
\frac{(2t + 3) \left( t^{\frac{1}{2}} \right)' - \left( t^{\frac{1}{2}} \right) (2t + 3)'}{(2t + 3)^2} = \frac{(2t + 3) \left( \frac{1}{2} t^{\frac{-1}{2}} \right) - t^{\frac{1}{2}} (2)}{(2t + 3)^2} =
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution:
\[
\left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)
\]
\[
= \frac{(2t + 3) \left( t^{\frac{1}{2}} \right)' - (2t + 3) t^{\frac{1}{2}} (2t + 3)'}{(2t + 3)^2}
\]
\[
= \frac{(2t + 3) \left( \frac{1}{2} t^{-\frac{1}{2}} \right) - t^{\frac{1}{2}} (2)}{(2t + 3)^2} = \frac{(2t + 3) \frac{1}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2}
\]
\[
= \frac{t}{\sqrt{t}} + \frac{3}{2\sqrt{t}} - 2\sqrt{t}
\]
Example 9: Differentiate $\frac{\sqrt{t}}{2t + 3}$

Solution:

\[
(\sqrt{t} \div (2t + 3))' = \left(\frac{t^{\frac{1}{2}}}{2t + 3}\right)'
\]

\[
\frac{(2t + 3) \left( t^{\frac{1}{2}} \right)' - (t^{\frac{1}{2}}) (2t + 3)'}{(2t + 3)^2} = 
\]

\[
\frac{(2t + 3) \left( \frac{1}{2} \cdot t^{-\frac{1}{2}} \right) - t^{\frac{1}{2}} (2)}{(2t + 3)^2} = \frac{(2t + 3) \frac{1}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2} = 
\]

\[
\frac{t}{\sqrt{t}} + \frac{3}{2\sqrt{t}} - 2\sqrt{t} = \frac{\sqrt{t} + \frac{3}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2} = 
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution:

\[
\left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)'
\]

\[
\frac{(2t + 3) \left( t^{\frac{1}{2}} \right)' - (t^{\frac{1}{2}}) (2t + 3)'}{(2t + 3)^2}
\]

\[
\frac{(2t + 3) \left( \frac{1}{2} t^{-\frac{1}{2}} \right) - \frac{1}{2} t^{-\frac{1}{2}} (2)}{(2t + 3)^2}
\]

\[
= \frac{(2t + 3) \frac{1}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2}
\]

\[
= \frac{\frac{t}{\sqrt{t}} + \frac{3}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2}
\]

\[
= \frac{\frac{3}{2\sqrt{t}} - \sqrt{t}}{(2t + 3)^2}
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution:

\[
 \left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)'
\]

\[
\frac{(2t + 3) \left( t^{\frac{1}{2}} \right)' - \left( t^{\frac{1}{2}} \right)(2t + 3)'}{(2t + 3)^2} = \frac{(2t + 3) \left( \frac{1}{2} t^{-\frac{1}{2}} \right) - t^{\frac{1}{2}} (2)}{(2t + 3)^2} = \frac{(2t + 3) \frac{1}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2} = \]

\[
\frac{\sqrt{t} + \frac{3}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2} = \frac{\sqrt{t} + \frac{3}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2} = \frac{3}{2\sqrt{t}} - \sqrt{t} = \frac{3}{2\sqrt{t}} - \sqrt{t} (2 \sqrt{t}) = \]

\[
\frac{3}{2\sqrt{t}} - \sqrt{t} \left( \frac{2\sqrt{t}}{2\sqrt{t}} \right) = \]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution: \( \left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)' \)

\[
\frac{(2t + 3) \left( t^{\frac{1}{2}} \right)' - (t^{\frac{1}{2}}) (2t + 3)'}{(2t + 3)^2} = (2t + 3) \frac{1}{2\sqrt{t}} - 2\sqrt{t}
\]

\[
\frac{t}{\sqrt{t}} + \frac{3}{2\sqrt{t}} - 2\sqrt{t} = \frac{\sqrt{t} + \frac{3}{2\sqrt{t}} - 2\sqrt{t}}{(2t + 3)^2} = \frac{\frac{3}{2\sqrt{t}} - \sqrt{t}}{(2t + 3)^2} = \frac{3}{2\sqrt{t}} - \sqrt{t}
\]

\[
\frac{3}{2\sqrt{t}} - \sqrt{t} \left( \frac{2\sqrt{t}}{2\sqrt{t}} \right) = \frac{3}{2\sqrt{t}} - \sqrt{t}
\]
Example 9: Differentiate \( \frac{\sqrt{t}}{2t + 3} \)

Solution:

\[
\left( \frac{\sqrt{t}}{2t + 3} \right)' = \left( \frac{t^{\frac{1}{2}}}{2t + 3} \right)'
\]

\[
(2t + 3) \left( t^{\frac{1}{2}} \right)' - (t^{\frac{1}{2}}) (2t + 3)' \\
(2t + 3)^2
\]

\[
(2t + 3) \left( \frac{1}{2} t^{-\frac{1}{2}} \right) - t^{\frac{1}{2}} (2) \\
(2t + 3)^2
\]

\[
\frac{t}{\sqrt{t}} + \frac{3}{2\sqrt{t}} - 2\sqrt{t} \\
(2t + 3)^2
\]

\[
\frac{3}{2\sqrt{t}} - \sqrt{t} \\
(2t + 3)^2
\]

\[
\frac{3}{2\sqrt{t}} - \sqrt{t} \left( \frac{2\sqrt{t}}{2\sqrt{t}} \right) = \frac{3 - 2t}{2\sqrt{t}(2t + 3)^2}
\]