Conservation laws of magnetohydrodynamic equations and their transformational properties

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Outline

- Magnetohydrodynamic model definition
- Methodology of research and problem statement
- 4 submodels:
  - viscous resistive plasma, dynamical case
  - viscous resistive plasma, stationary case
  - ideal plasma, dynamical case
  - ideal plasma, stationary case
- Conclusion

Magnetohydrodynamic model definition

Magnetohydrodynamic unitless PDE system:

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\left(p + \frac{1}{2}\mathbf{B} \cdot \mathbf{B}\right) + (\mathbf{B} \cdot \nabla)\mathbf{B} + a\, \Delta\mathbf{v}$$
$$\partial_t \mathbf{B} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} + b\, \Delta\mathbf{B}$$
$$\nabla \cdot \mathbf{v} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$

where $\mathbf{v}, \mathbf{B}, p$ are dependent variables; a, b are some parameters.

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$$\partial_t \mathbf{B} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} + b\,\Delta \mathbf{B}$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

where $\mathbf{v}, \mathbf{B}, p$ are dependent variables; a, b are some parameters.

Assume constants:

- density
- dynamic viscosity
- magnetic viscosity

Assume negligibly small:

- heat flows
- density gradients
- gravitation force

Methodology of research I

Local conservation law:

$$(D_t T + D_{\mathbf{x}} \cdot \mathbf{\Phi})|_{R[\mathbf{v},\mathbf{B},p]=0} = 0$$

Trivial conservation law:

$$T|_{R[\mathbf{v},\mathbf{B},p]=0} = -D_{\mathbf{x}} \cdot \mathbf{\Psi}[\mathbf{v},\mathbf{B},p]$$
$$\mathbf{\Phi}|_{R[\mathbf{v},\mathbf{B},p]=0} = D_t \mathbf{\Psi}[\mathbf{v},\mathbf{B},p] + D_{\mathbf{x}} \times \mathbf{\Gamma}[\mathbf{v},\mathbf{B},p]$$

Point symmetry generators are searched in the canonical form:

$$\mathrm{X} = \tau \partial/\partial t + \boldsymbol{\xi} \cdot \partial/\partial \mathbf{x} + \boldsymbol{\eta}^{\mathbf{v}} \cdot \partial/\partial \mathbf{v} + \boldsymbol{\eta}^{\mathbf{B}} \cdot \partial/\partial \mathbf{B} + \eta^p \partial/\partial p$$

Action of a point symmetry generator on a conservation law conserved density and flux vector:

$$\tilde{T} = \mathrm{X}^{(1)} T - (\mathbf{\Phi} \cdot D_{\mathbf{x}})\tau + T(D_{\mathbf{x}} \cdot \boldsymbol{\xi})$$
$$\tilde{\mathbf{\Phi}} = \mathrm{X}^{(1)} \mathbf{\Phi} - T(D_{\mathbf{x}} \cdot \boldsymbol{\xi}) - (\mathbf{\Phi} \cdot D_{\mathbf{x}})\boldsymbol{\xi} + \mathbf{\Phi} D_t \tau + \mathbf{\Phi}(D_{\mathbf{x}} \cdot \boldsymbol{\xi})$$

Methodology of research II

Determining equations for point symmetry generators:

$$\mathrm{X}^{(2)} R[\mathbf{v}, \mathbf{B}, p]|_{R[\mathbf{v}, \mathbf{B}, p]=0} = 0$$

Determining equations for conservation laws:

$$D_t T + D_{\mathbf{x}} \cdot \mathbf{\Phi} = \sum A[\mathbf{v}, \mathbf{B}, p](R[\mathbf{v}, \mathbf{B}, p])$$

Integrating by part we are getting

$$D_t T + D_{\mathbf{x}} \cdot \mathbf{\Phi} = \sum \Lambda[\mathbf{v}, \mathbf{B}, p] R[\mathbf{v}, \mathbf{B}, p]$$

where $\Lambda[\mathbf{v}, \mathbf{B}, p] = (\mathbf{\Lambda}^{fl}, \mathbf{\Lambda}^{mag}, \Lambda^{inc}, \Lambda^{sol})$.

Methodology of research II

Determining equations for point symmetry generators:

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Determining equations for conservation laws:

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Integrating by part we are getting

$$D_t T + D_{\mathbf{x}} \cdot \mathbf{\Phi} = \sum \Lambda[\mathbf{v}, \mathbf{B}, p] R[\mathbf{v}, \mathbf{B}, p]$$

where $\Lambda[\mathbf{v}, \mathbf{B}, p] = (\mathbf{\Lambda}^{fl}, \mathbf{\Lambda}^{mag}, \Lambda^{inc}, \Lambda^{sol})$.

Trivial multipliers: $\Lambda[\mathbf{v}, \mathbf{B}, p] = (\mathbf{0}, \nabla f(t, \mathbf{x}), 0, \partial_t f(t, \mathbf{x}))$

A minimal generating set of conservation laws is defined to be a minimal subset of non-trivial conservation laws whose image under point symmetry group is entire set of non-trivial conservation laws up to equivalence.

Conservation laws of the submodel 1 $(a^2 + b^2 \neq 0)$

angular momenta

$$T[\alpha_i] = \alpha_i \epsilon^i{}_{km} x^k v^m$$
$$\Phi^j[\alpha_i] = \alpha_i \epsilon^i{}_{km} \left(x^k v^m v^j - x^k B^m B^j + \delta^{mj} x^k \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) - a\ \delta^{jn} \nabla_n \left(x^k v^m \right) \right)$$

generalized momenta

$$T[\beta_i(t)] = \beta_i v^i$$
$$\Phi^j[\beta_i(t)] = \beta_i \left(v^i v^j - B^i B^j + \delta^{ij} \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) - a\ \delta^{jn} \nabla_n v^i \right) - v^j x^i \partial_t \beta_i$$

generalized magnetic fluxes

$$T[f(t, \mathbf{x})] = B^k \nabla_k f$$
$$\Phi^j[f(t, \mathbf{x})] = \left(v^j B^m - B^j v^m \right) \nabla_m f - B^j \partial_t f - b\ f \Delta B^j$$

incompressibility condition

$$T[h(t)] = 0$$
$$\Phi^j[h(t)] = h v^j$$

Point symmetries, submodel 1 ($a^2 + b^2 \neq 0$)

time translation

$$\mathrm{X} = \partial/\partial t$$

rotations

$$\mathrm{X}[a_i] = a_i \epsilon_k^{\ im} (x^k \partial/\partial x^m + v^k \partial/\partial v^m + B^k \partial/\partial B^m)$$

generalized Galilean boosts

$$\mathrm{X}[g_i(t)] = g^i \partial/\partial x^i + (\partial_t g^i)\partial/\partial v^i - (\partial_t^2 g_i) x^i \partial/\partial p$$

pressure shift

$$\mathrm{X}[w(t)] = w\ \partial/\partial p$$

scaling

$$\mathrm{X} = 2t\partial/\partial t + x^i \partial/\partial x^i - v^i \partial/\partial v^i - B^i \partial/\partial B^i - 2p\partial/\partial p$$

Action of symmetries, submodel 1 $(a^2 + b^2 \neq 0)$

	ang. mom. α_j	gen. mom. $\beta_j(t)$	incomp. cond. $h(t)$
time translation	0	gen. mom. $\tilde{\beta}_j(t) = \partial_t \beta_j(t)$	incomp. cond. $\tilde{h}(t) = \partial_t h(t)$
rotations, a_i	ang. mom. $\tilde{\alpha}_j = \epsilon_j^{\ im} \alpha_i a_m$	gen. mom. $\tilde{\beta}_j(t) = \epsilon_j^{\ im} \beta_i(t) a_m$	0
generalized Galilean boosts $g_i(t)$	gen. mom. $\tilde{\beta}_j(t) = \epsilon_j^{\ im} \alpha_i g_m(t)$	incomp. cond. $\tilde{h}(t) = (g_k(t)\partial_t \beta_j(t)$ $-\beta_j(t)\partial_t g_k(t))\delta^{jk}$	0
pressure shift $w(t)$	0	0	0
scaling	ang. mom. $\tilde{\alpha}_j = 3\alpha_j$	gen. mom. $\tilde{\beta}_j(t) = 2\partial_t(t\beta_j(t))$	incomp. cond. $\tilde{h}(t) = 3h(t)$ $+2t\partial_t h(t)$

	gen. mag. fluxes $f(t, \mathbf{x})$
time translation	gen. mag. fluxes $\tilde{f}(t, \mathbf{x}) = \partial_t f(t, \mathbf{x})$
rotations, a_i	gen. mag. fluxes $\tilde{f}(t, \mathbf{x}) = \epsilon_k^{\ im} x^k \nabla_i f(t, \mathbf{x}) a_m$
generalized Galilean boosts $g_i(t)$	gen. mag. fluxes $\tilde{f}(t, \mathbf{x}) = g^i(t) \nabla_i f(t, \mathbf{x})$
pressure shift $w(t)$	0
scaling	gen. mag. fluxes $\tilde{f}(t, \mathbf{x}) = f(t, \mathbf{x})$ $+2t\partial_t f(t, \mathbf{x}) + x^k \nabla_k f(t, \mathbf{x})$

Minimal generating set of conservation laws, submodel 1 $(a^2 + b^2 \neq 0)$

- projection of angular momentum on arbitrary direction
- infinite family of generalized magnetic fluxes

Conservation laws of the submodel 2 ($a^2 + b^2 \neq 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

angular momenta

$$\Phi^j[\alpha_i] = \alpha_i \epsilon^i_{km} \left(x^k v^m v^j - x^k B^m B^j + \delta^{mj} x^k \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) - a\ \delta^{jn} \nabla_n \left(x^k v^m \right) \right)$$

generalized momenta

$$\Phi^j[\beta_i] = \beta_i \left(v^i v^j - B^i B^j + \delta^{ij} \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) - a\ \delta^{jn} \nabla_n v^i \right)$$

generalized magnetic fluxes

$$\Phi^j[f(\mathbf{x})] = \left(v^j B^m - B^j v^m \right) \nabla_m f - b\ f \Delta B^j$$

incompressibility condition

$$\Phi^j = v^j$$

solenoidal condition

$$\Phi^j = B^j$$

Point symmetries, submodel 2 ($a^2 + b^2 \neq 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

rotations

$$\mathrm{X}[a_i] = a_i \epsilon_k{}^{im} (x^k \partial/\partial x^m + v^k \partial/\partial v^m + B^k \partial/\partial B^m)$$

space shifts

$$\mathrm{X}[c_i] = c^i \partial/\partial x^i$$

pressure shift

$$\mathrm{X} = \partial/\partial p$$

scaling

$$\mathrm{X} = x^i \partial/\partial x^i - v^i \partial/\partial v^i - B^i \partial/\partial B^i - 2p\partial/\partial p$$

Action of symmetries, submodel 2 ($a^2 + b^2 \neq 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

	ang. mom. α_j	gen. mom. β_j	incomp. cond.	solen. cond.
rotations a_i	ang. mom. $\tilde{\alpha}_j = \epsilon_j^{im} \alpha_i a_m$	gen. mom. $\tilde{\beta}_j = \epsilon_j^{im} \beta_i a_m$	0	0
space shifts b_i	gen. mom. $\tilde{\beta}_j = \epsilon_j^{im} \alpha_i b_m$	0	0	0
pressure shift	0	0	0	0
scaling	ang. mom. $\tilde{\alpha}_j = \alpha_j$	0	incomp. cond.	solen. cond.

	gen. mag. fluxes $f(\mathbf{x})$
rotations a_i	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = \epsilon_k^{im} x^k \nabla_i f(\mathbf{x}) a_m$
space shifts b_i	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = b^i \nabla_i f(\mathbf{x})$
pressure shift	0
scaling	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = x^k \nabla_k f(\mathbf{x}) - f(\mathbf{x})$

Minimal generating set of conservation laws, submodel 2 ($a^2 + b^2 \neq 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

- projection of angular momentum on arbitrary direction
- infinite family of generalized magnetic fluxes
- incompressibility condition
- solenoidal condition

Conservation laws of the submodel 3 $(a^2 + b^2 = 0)$

angular momenta

$$T[\alpha_i] = \alpha_i \epsilon^i_{\ km} x^k v^m$$
$$\Phi^j[\alpha_i] = \alpha_i \epsilon^i_{\ km} \left(x^k v^m v^j - x^k B^m B^j + \delta^{mj} x^k \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) \right)$$

generalized momenta

$$T[\beta_i(t)] = \beta_i v^i$$
$$\Phi^j[\beta_i(t)] = \beta_i \left(v^i v^j - B^i B^j + \delta^{ij} \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) \right) - v^j x^i \partial_t \beta_i$$

generalized magnetic fluxes

$$T[f(t, \mathbf{x})] = B^k \nabla_k f$$
$$\Phi^j[f(t, \mathbf{x})] = \left(v^j B^m - B^j v^m \right) \nabla_m f - B^j \partial_t f$$

incompressibility condition

$$T[h(t)] = 0$$
$$\Phi^j[h(t)] = h v^j$$

energy

$$T = \frac{1}{2} \delta_{ik} \left(v^i v^k + B^i B^k \right)$$
$$\Phi^j = v^j \left(p + \frac{1}{2} \delta_{ik} v^i v^k \right) + \delta_{ik} B^k (v^j B^i - B^j v^i)$$

mixed helicity

$$T = \delta_{ik} v^i B^k$$
$$\Phi^j = B^j \left(p + \frac{1}{2} \delta_{ik} v^i v^k \right) + \delta_{ik} v^k (v^j B^i - B^j v^i)$$

Point symmetries, submodel 3 ($a^2 + b^2 = 0$)

time translation

$$\mathrm{X} = \partial/\partial t$$

rotations

$$\mathrm{X}[a_i] = a_i \epsilon_k{}^{im}(x^k \partial/\partial x^m + v^k \partial/\partial v^m + B^k \partial/\partial B^m)$$

generalized Galilean boosts

$$\mathrm{X}[g_i(t)] = g^i \partial/\partial x^i + (\partial_t g^i)\partial/\partial v^i - (\partial_t^2 g_i) x^i \partial/\partial p$$

pressure shift

$$\mathrm{X}[w(t)] = w\ \partial/\partial p$$

scaling

$$\mathrm{X} = t\partial/\partial t - v^i \partial/\partial v^i - B^i \partial/\partial B^i - 2p\partial/\partial p$$

dilation

$$\mathrm{X} = t\partial/\partial t + x^i \partial/\partial x^i$$

Action of symmetries I, submodel 3 ($a^2 + b^2 = 0$)

	ang. mom. α_j	gen. mom. $\beta_j(t)$	gen. mag. fluxes $f(t,\mathbf{x})$
time translation	0	gen. mom. $\tilde\beta_j(t) = \partial_t\beta_j(t)$	gen. mag. fluxes $\tilde f(t,\mathbf{x}) = \partial_t f(t,\mathbf{x})$
rotations a_i	ang. mom. $\tilde\alpha_j = \epsilon_j^{\ im}\alpha_i a_m$	gen. mom. $\tilde\beta_j(t) = \epsilon_j^{\ im}\beta_i(t)a_m$	gen. mag. fluxes $\tilde f(t,\mathbf{x}) = \epsilon_k^{\ im}x^k\nabla_i f(t,\mathbf{x})a_m$
generalized Galilean boosts $g_i(t)$	gen. mom. $\tilde\beta_j(t) = \epsilon_j^{\ im}\alpha_i g_m(t)$	incomp. cond. $\tilde h(t) = (g_k(t)\partial_t\beta_j(t) - \beta_j(t)\partial_t g_k(t))\delta^{jk}$	gen. mag. fluxes $\tilde f(t,\mathbf{x}) = g^i(t)\nabla_i f(t,\mathbf{x})$
pressure shift $w(t)$	0	0	0
scaling	ang. mom. $\tilde\alpha_j = -\alpha_j$	gen. mom. $\tilde\beta_j(t) = -\beta_j(t) + t\partial_t\beta_j(t)$	gen. mag. fluxes $\tilde f(t,\mathbf{x}) = -f(t,\mathbf{x}) + t\partial_t f(t,\mathbf{x})$
dilation	ang. mom. $\tilde\alpha_j = 4\alpha_j$	gen. mom. $\tilde\beta_j(t) = 3\beta_j(t) + t\partial_t(\beta_j(t))$	gen. mag. fluxes $\tilde f(t,\mathbf{x}) = 2f(t,\mathbf{x}) + t\partial_t f(t,\mathbf{x}) + x^k\nabla_k f(t,\mathbf{x})$

Action of symmetries II, submodel 3 ($a^2 + b^2 = 0$)

	incomp. cond. $h(t)$	energy	mixed hel.
time translation	incomp. cond. $\tilde{h}(t) = \partial_t h(t)$	0	0
rotations a_i	0	0	0
generalized Galilean boosts $g_i(t)$	0	gen. mom. $\tilde{\beta}_j(t) = \partial_t g_j(t)$	gen. mag. fluxes $\tilde{f}(t, \mathbf{x}) = x^i \partial_t g_i(t)$
pressure shift $w(t)$	0	incomp. cond. $\tilde{h}(t) = w(t)$	gen. mag. fluxes $\tilde{f}(t, \mathbf{x}) = -\int w(t)dt$
scaling	incomp. cond. $\tilde{h}(t) = t\partial_t h(t)$	energy $*(-2)$	mixed hel. $*(-2)$
dilation	incomp. cond. $\tilde{h}(t) = 3h(t) + t\partial_t h(t)$	energy $*3$	mixed hel. $*3$

Minimal generating set of conservation laws, submodel 3 ($a^2 + b^2 = 0$)

- projection of angular momentum on arbitrary direction
- infinite family of generalized magnetic fluxes
- energy
- mixed helicity

Conservation laws of the submodel 4 ($a^2 + b^2 = 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

angular momenta

$$\Phi^j[\alpha_i] = \alpha_i \epsilon^i_{\ km} \left(x^k v^m v^j - x^k B^m B^j + \delta^{mj} x^k \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) \right)$$

generalized momenta

$$\Phi^j[\beta_i] = \beta_i \left(v^i v^j - B^i B^j + \delta^{ij} \left(p + \frac{1}{2} \delta_{ns} B^n B^s \right) \right)$$

generalized magnetic fluxes

$$\Phi^j[f(\mathbf{x})] = \left(v^j B^m - B^j v^m \right) \nabla_m f$$

solenoidal condition

$$\Phi^j = B^j$$

incompressibility condition

$$\Phi^j = v^j$$

energy

$$\Phi^j = v^j \left(p + \frac{1}{2} \delta_{ik} v^i v^k \right) + \delta_{ik} B^k (v^j B^i - B^j v^i)$$

mixed helicity

$$\Phi^j = B^j \left(p + \frac{1}{2} \delta_{ik} v^i v^k \right) + \delta_{ik} v^k (v^j B^i - B^j v^i)$$

Point symmetries, submodel 4 ($a^2 + b^2 = 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

rotations

$$\mathrm{X}[a_i] = a_i \epsilon_k^{\ im} (x^k \partial/\partial x^m + v^k \partial/\partial v^m + B^k \partial/\partial B^m)$$

space shifts

$$\mathrm{X}[c_i] = c^i \partial/\partial x^i$$

pressure shift

$$\mathrm{X} = \partial/\partial p$$

scaling

$$\mathrm{X} = v^i \partial/\partial v^i + B^i \partial/\partial B^i + 2p\partial/\partial p$$

dilation

$$\mathrm{X} = x^i \partial/\partial x^i$$

duality rotation

$$\mathrm{X} = B^i \partial/\partial v^i + v^i \partial/\partial B^i - \delta_{ij} v^i B^j \partial/\partial p$$

Potential symmetries and conservation laws

Magnetic part:

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \implies \mathbf{v} \times \mathbf{B} = \nabla \Psi$$

Two extra symmetries:

$$\mathrm{X}_\Psi = \partial/\partial\Psi$$
$$\mathrm{X}_\infty = M(\Psi)(B^i \partial/\partial v^i + v^i \partial/\partial B^i - \delta_{ij} v^i B^j \partial/\partial p)$$

Extra conservation laws:

$$\Phi^j_\infty = N(\Psi)\Phi^j,$$

where Φ^j are fluxes of generalized magnetic fluxes, the incompressibility condition, the solenoidal condition, energy, mixed helicity, components of generalized momenta normal to the gradient of Ψ and component of angular momenta along it.

Action of symmetries I, submodel 4 ($a^2 + b^2 = 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

	ang. mom. α_j	gen. mom. β_j	gen. mag. fluxes $f(\mathbf{x})$
rotations a_i	ang. mom. $\tilde{\alpha}_j = \epsilon_j^{\ im} \alpha_i a_m$	gen. mom. $\tilde{\beta}_j = \epsilon_j^{\ im} \beta_i a_m$	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = \epsilon_k^{\ im} x^k \nabla_i f(\mathbf{x}) a_m$
space shifts b_i	gen. mom. $\tilde{\beta}_j = \epsilon_j^{\ im} \alpha_i b_m$	0	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = b^i \nabla_i f(\mathbf{x})$
pressure shift	0	0	0
scaling	ang. mom. $\tilde{\alpha}_j = 2\alpha_j$	gen. mom. $\tilde{\beta}_j = 2\beta_j$	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = 2f(\mathbf{x})$
dilation	ang. mom. $\tilde{\alpha}_j = 3\alpha_j$	gen. mom. $\tilde{\beta}_j = 2\beta_j$	gen. mag. fluxes $\tilde{f}(\mathbf{x}) = f(\mathbf{x}) + x^k \nabla_k f(\mathbf{x})$
duality rotation	0	0	0

Action of symmetries II, submodel 4 ($a^2 + b^2 = 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

	incomp. cond.	solen.cond.	energy	mixed hel.
rotations a_i	0	0	0	0
space shifts b_i	0	0	0	0
pressure shift	0	0	incomp. cond.	solen. cond.
scaling	incomp. cond.	mixed hel.	energy $*3$	mixed hel. $*3$
dilation	incomp. cond. $*2$	solen. cond. $*2$	energy $*2$	mixed hel. $*2$
duality rotation	solen. cond.	incomp. cond.	mixed hel.	energy

Minimal generating set of conservation laws, submodel 4 ($a^2 + b^2 = 0$, $\partial_t \mathbf{v} = 0, \partial_t \mathbf{B} = 0, \partial_t p = 0$)

- projection of angular momentum on arbitrary direction
- infinite family of generalized magnetic fluxes
- energy

Conclusions

- symmetry and conservation law classification with respect to parameters (a, b)
- action of symmetry on set of conservation laws
- orbits of action of symmetry on set of conservation laws
- existence of conservation laws who's multipliers depend on first order derivatives of dynamical variables
- factorization of set of conservation laws by subset of trivial conservation laws
- potential conservation laws of extended submodels