On Kelvin's 1880 problem and exact solutions to the Navier-Stokes equations

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Abstract Exact solutions to the steady Euler hydrodynamics equations are derived for which a classification of knots in $\mathbb{R}^3$ formed by the closed vorticity lines is found (Kelvin’s problem of 1880).

Using the Alexander polynomial (that is the main topological invariant of any knot in $\mathbb{R}^3$), it is shown which torus knots $K_{p,q}$ are not realized for the constructed exact solutions and which are realized by the closed vorticity lines.

Exact solutions to the non-stationary Navier-Stokes equations are presented describing dynamics of a viscous incompressible fluid in $\mathbb{R}^3$. The solutions depend on an arbitrary vector field tangent to the two dimensional unit sphere $S^2 \subset \mathbb{R}^3$ and on an arbitrary measure on $S^2$. It is shown that dynamics of the fluid for these solutions is not turbulent in both Eulerian and Lagrangian senses, in spite of the fact that the corresponding Reynolds numbers can be arbitrarily large.