WORKSHOP
on
"VALUATIONS ON RATIONAL FUNCTION FIELDS"
Abstracts

Speaker: **Eberhard Becker**

Title: $C(M, S^n)$ *for real function fields and applications*

Abstract:
Let K denote a formally real field and M its space of real places $\lambda : K \to \mathbb{R} \cup \infty =: \mathbb{P}^1$. For each $a \in K$ the evaluation map $\hat{a} : M \to \mathbb{P}^1, \lambda \mapsto \lambda(a)$ is continuous and we get a map

$$\Phi : K \to C(M, \mathbb{P}^1), a \mapsto \hat{a}$$

where all function spaces in this talk are endowed with the compact-open topology. The closure of the image admits a nice description but it remains an open question whether the image is always dense.

The subring

$$H := \{a \in K \mid \hat{a}(M) \subseteq \mathbb{R}\}$$

is called the **real holomorphy ring of** K. Using this ring we study an infinite series of representations

$$\Phi_n : S^n(H) \to C(M, S^n), (a_0, \ldots, a_n) \mapsto (\widehat{a_0}, \ldots, \widehat{a_n}),$$

$$S^n \text{ the standard n-sphere and } S^n(H) = \{(a_0, \ldots, a_n) \in H^{n+1} \mid \sum_0^n a_i^2 = 1\}.$$

The map $\Phi : K \to C(M, \mathbb{P}^1)$ can be identified with Φ_1.

The talk will focus on real function fields F over $\mathbb{R}$ where if can be proven that the image of Φ_n is dense if $n \geq tr(F|\mathbb{R})$. The proof uses resolution of singularities and recent results of Kucharz et al. on regular maps on real varieties.

Applications to the study of sums of squares and sums of higher powers in real function fields will be presented.

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Speaker: **Anna Blaszczok**

Title: *Distances of elements in valued function fields*

Abstract:
The defect of valued field extensions is the main obstacle to the solution of deep open problems like local uniformization or questions in the model theory of valued fields. These problems are linked through the structure theory of valued function fields. Therefore a better understanding of the structure of defect extensions, in particular defect extensions of valued function fields, is crucial.

A useful tool for the study of defect extensions is the notion of distance, which measures how well an element in an immediate extension can be approximated by elements from the base field. In connection with the study of the structure of defect extensions of function fields arose the question how many essentially distinct distances of generators of Artin-Schreier defect extensions exist over a fixed valued field.

With the use of the classification of Artin-Schreier defect extensions, we show that in several situations the number of essentially distinct distances in fixed extensions, or even just over a fixed base field, is finite, and we compute upper bounds. We also apply this result to the special case of valued functions fields over perfect base fields.

This is joint work with Franz-Viktor Kuhlmann.

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Speaker: **Olga Kashcheyeva**

Title: *Key polynomials and truncated valuations*

Abstract:
Given a rational rank 1 zero dimensional valuation ν centered in a polynomial ring R we construct a sequence of polynomials $\{Q_i\}$ in R such that ν is approximated by valuations 'monomial' in Q_i's.

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Speaker: **Wael Mahboub**

Title: *Valuations over regular local rings of dimension two*

Abstract:
Let R be a regular local ring of dimension two and let V denote the space of all normalized valuations centered at R. When $R = \mathbb{C}[[x, y]]$, C. Favre and M. Jonsson

proved that V has the structure of a parametrized, rooted, non-metric tree. Their proof is based on associating a sequence of key polynomials to each element of V.

A. Granja generalized this result to the case when R is any two-dimensional regular local ring. A. Granja gives a proof based on associating to each valuation in V a sequence of point blowing ups.

In this talk, we will describe the use of key polynomials for the construction of the valuative tree for two-dimensional regular local rings.

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Speaker: **Edward Mosteig**

Title: *Reversely Well-Ordered Valuations on Polynomial Rings in Two Variables*

Abstract:
Gröbner bases are the primary computational tool for solving the ideal membership problem in polynomial rings. We give a brief overview of Gröbner bases, which heavily rely on monomial orders, and discuss how valuations can be used in place of monomial orders. We consider classes of valuations on a rational function field $K(x, y)$ that are suitable for this use and analyze their behavior when restricting to an underlying polynomial ring $K[x, y]$. Interestingly, such images of polynomial rings are not always well-behaved. For example, we will see examples of when the images are non-positive, and yet are not reversely well-ordered.

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Speaker: **Josnei Novacoski**

Title: *(Abstract) Key polynomials*

Abstract:
(Joint work with Mark Spivakovsky) In this talk we introduce a new concept of key polynomials for a given valuation ν on $K[x]$ (such key polynomials are also called abstract key polynomials). We present some basic properties of these polynomials, for instance, that they are irreducible and that the truncation of ν associated to each key polynomial is a valuation. An important result that we discuss is that every valuation ν on $K[x]$ admits a sequence of key polynomials that completely determines ν. Finally, we establish the relation between these key polynomials and pseudo-convergent sequences defined by Kaplansky.

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Speaker: **Josnei Novacoski**

Title: *Key polynomials and minimal pairs*

Abstract:
In this talk we establish the relation between key polynomials and minimal pairs of definition of a valuation. We also discuss truncations of valuations on a polynomial ring $K[x]$. We show that a valuation ν is equal to its truncation on some polynomial if and only if ν is valuation-transcendental.

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Speaker: **Miguel Ángel Olalla Acosta**

Title: *Valuations in algebraic field extensions*

Abstract:
This talk is based on the article with the same title in Journal of Algebra 312 (2007) 10331074, with F.J. Herrera Govantes and M. Spivakovsky.

Let $K \to L$ be an algebraic field extension and ν a valuation of K. Let ν' a valuation of L that extends ν.

First we'll use a refined version of MacLanes key polynomials to describe ν'. We'll say that a set of key polynomials Q_i is *complete* if the valuation ν' is completely determined by the data $\{Q_i, \nu'(Q_i)\}$.

Then we'll give an algorithm to describe the totality of extensions ν' of ν to L.

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Speaker: **Giulio Peruginelli**

Title: *Transcendental extensions of a valuation domain of rank one*

Abstract:
Recently, Loper and Werner exhibit a Prüfer domain in between $\mathbb{Z}[X]$ and $\text{Int}(\mathbb{Z})$, the ring of integer-valued polynomials, by intersecting $\mathbb{Q}[X]$ with a suitable class of valuation domains of $\mathbb{Q}(X)$ extending the DVRs of $\mathbb{Q}$, i.e., localizations of $\mathbb{Z}$ at prime integers. In this talk we give an explicit description of a class of valuation domains W of the field of rational functions $K(X)$ which extend a given rank one valuation domain V of a field K which generalizes the class of valuation domains considered by Loper and Werner. These valuation domains are indexed by elements α of the algebraic closure of $\widehat{K}$, the V-adic completion of K (plus the point at infinity) in the following sense: each such a valuation domain W is equal to W_α, the ring of rational functions whose value at such an α is integral over $\widehat{V}$, the completion of V. In case V is discrete, we show that these valuation domains are uniquely characterized

by these two properties: i) the degree of the extension of the residue fields of W and V is finite; ii) $\pi W = M^e$ for some $e \geq 1$, where M is the maximal ideal of W and π a uniformizer of V. We also show that, for α_1, α_2 elements as above, $W_{\alpha_1} = W_{\alpha_2}$ if and only if α_1 and α_2 are conjugated over $\widehat{K}$. Furthermore, this class of valuation domains endowed with the Zariski topology is homeomorphic to the set of irreducible polynomials over $\widehat{K}$ endowed with an ultrametric distance à la Krasner (i.e., the distance between two such polynomials p and q is the minimum of the distances between the roots of p and the roots of q).

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Speaker: **Bernd Schober**

Title: *Constructing overweight deformation using polyhedra*

Abstract:
The topic of my talk is in the context of Teissier's approach of resolving singularities with a single toric morphism. More precisely, I will discuss the following question: Given a variety embedded into a regular ambient space, is it possible to find a re-embedding into a possibly larger regular ambient space such that, in there, our original variety can be considered as an overweight deformation of a binomial variety? I will explain the ideas how this can be achieved using polyhedra in particular cases. This is joint work with Hussein Mourtada.

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Speaker: **Bernard Teissier**

Title: *Geometry and dynamics of intersections* (Institute Colloquium)

Abstract: to be announced.
In intersection theory, typically when proving Bzouts theorem, one defines the local intersection multiplicity or cycle of two varieties which do not intersect transversally at non singular points (a situation called general position) by making a one parameter deformation of one of them to obtain general position and counting the points or looking at the intersection cycle. I will examine the information obtained when studying the multiscale dynamics of the intersection points or cycles when the deformation parameter tends to zero.

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Speaker: **Bernard Teissier**

Title: *On key polynomials and the valuative Cohen theorem*

Abstract: to be announced.
The evaluative Cohen theorem describes the relation between a complete equicharacteristic noetherian local domain endowed with a rational valuation and the associated graded ring with respect to the valuation. In certain cases it is possible to establish a relationship with the key polynomials which describe extensions of a valuation of a field K to a monogeneous extension.

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Speaker: **Michel Vaquie**

Title: *Augmented valuation, key polynomial and admissible families* (3 lectures)

Abstract:
In these lectures we show how we can associate to a valuation of $K[X]$ a family of valuations, called an admissible family, such that any valuation of this family is obtained either as an augmented valuation or a limit augmented valuation of the polynomial ring $K[X]$.
This family is essentially unique and contains some information on the extension of valued fields $(K[X]/K)$.
Moreover this family depends only of the field $K(X)$, and not on the choice of the generating element X.

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