Wild Ramification Kinks

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Setup

- Suppose we are given a finite branched cover $f : Y \to X$ of smooth projective curves over a non-archimedean field.
- Can “analytify” to get a finite map of (formal, rigid, Berkovich) spaces $f^{an} : Y^{an} \to X^{an}$.
- Let $D \subseteq X^{an}$ be a rigid open disk.
- We can ask: When is $f^{-1}(D)$ also (geometrically) an open disk?

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Examples

- $f : \mathbb{P}^1 \to \mathbb{P}^1$ over $\mathbb{Q}_p$ given by $z \mapsto z^m$, $p \nmid m$.
  - $f^{-1}(D(0, r))$ is an open disk for all r.
  - $f^{-1}(D(1, r))$ is not when $r < 1$.
  - Indeed, $f^{-1}(D)$ is an open disk iff D contains exactly one of 0 or ∞.
- $f : \mathbb{P}^1 \to \mathbb{P}^1$ over $K/\mathbb{Q}_p$ finite extension given by $z \mapsto z^p$
  - $f^{-1}(D(0, r))$ is an open disk for all r.
  - $f^{-1}(D(1, r))$ is not when $r \leq p^{-p/(p-1)}$, but is when $r > p^{-p/(p-1)}$, even when $r < 1$.
  - In Berkovich terms, the "topological ramification locus" contains more than just the path from 0 to ∞.

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(A simplified version of) the problem

- Let $\mathcal{F} : Y \to X \times \mathcal{A}$ be a flat family of G-Galois branched covers of X over a mixed-characteristic non-archimedean field K, parameterized by an affinoid $\mathcal{A}$.
- Let $D \subseteq X^{an}$ be an open disk — fix an origin and a metric.
- For $r > 0$, let $D(r) \subseteq D$ be the open disk of radius p^{-r}.

Theorem

Suppose there exists a sequence $r_1, r_2, \ldots$ decreasing to 0 and $a_1, a_2, \ldots \in \mathcal{A}(\overline{K})$ such that $(\mathcal{F}|_{a_i})^{-1}(D(r_i))$ is an open disk for all i. Then there exists $a \in \mathcal{A}(\overline{K})$ such that $(\mathcal{F}|_a)^{-1}(D)$ is an open disk.

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Motivation: the local lifting problem (LLP)

- The LLP asks: given a G-extension $k[[z]]/k[[t]]$, when k is algebraically closed of characteristic p, does there exist a characteristic zero DVR R with residue field k, and a G-Galois extension $R[[Z]]/R[[T]]$ lifting $k[[z]]/k[[t]]$?
- Can think of $R[[T]]$ as the ring of functions on an open unit disk.
- Idea is to find a G-extension of $\mathbb{K} := \mathrm{Frac}(R[[T]])$ in which normalization of $R[[T]]$ is $R[[Z]]$ (another disk).
- New method of me and Wewers: Come up with an affinoid family of possible extensions. Show that, for a sequence $r_1, r_2, \ldots$ decreasing to 0, one can find an extension in the family such that normalization of $\mathrm{Frac}(R[[p^{-r}T]])$ in the extension is a power series ring over R.
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Why valuations?

- Simplify: Let $X = \mathbb{P}^1$, $G = \mathbb{Z}/p$. Base field K contains μ_p. Write $K(X) = K(T)$. $f : Y \to X$ given by extracting a pth root of F.
- Our "families of covers" are more or less families of rational functions F as above (Kummer theory).
- Take $r \in (0, \infty)$. Let v_r be the "Gauss valuation" on $K(T)$ with respect to $D[r]$ (i.e., $v_r(T) = r$).
- Completion of $K(T)$ at v_r is DVR with *imperfect* residue field $k(t)$, with t the reduction of $p^{-r}T$.
- Want to measure "geometric" wild ramification of v_r in $K(Y) = K(T, \sqrt[p]{F})$ (comes from inseparable residue field extension).

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Valuations, continued

- For each $F \in K(X)^\times/(K(X)^\times)^p$, we can define a piecewise-linear, continuous function $\delta_F : [0, \infty) \to [0, \infty)$ using Kato's *depth Swan conductor* (this essentially measures the different of $K(Y)/K(X)$ relative to v_r).
- KEY POINT: The inverse image of $D(r)$ is a disk iff the right-slope of δ_F at r is one less than the number of branch points of f inside $D(r)$.
- So we can detect disks using the kinks in the function δ_F (hence the title).

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Sketch of the proof of the theorem

- The function $\delta_F(r)$ can be read off directly from F so long as the normalized residue of $F - 1 \pmod{v_r}$ is not a pth power. Call such an F in "standard form."
- Let $\mathcal{A}$ be the affinoid parameterizing our covers. For $a \in \mathcal{A}$, $f_a : Y_a \to X$ is the corresponding cover.
- First step: Find a family of rational functions F_a for $a \in \mathcal{A}$ such that F_a is a Kummer representative for f_a in standard form (might require finite cover of $\mathcal{A}$).
- Thus δ_{F_a} can be read off easily.

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Sketch of the proof of the theorem (continued)

- Fix $R > 0$ such that no branch point of any f_a lies in $D \backslash D(r)$. Let m be the total number of branch points in $D(r)$ of one (equiv. all) f_a.
- Let r_a be the supremum over all $r \in [0, R]$ such that the right-slope of δ_{F_a} at r is not equal to $m - 1$. Then r_a (as a function of a) can be expressed as the valuation of an analytic function on $\mathcal{A}$ (really need a finite cover here again — but r_a descends to $\mathcal{A}$ as a function).
- By maximum principle, r_a achieves its minimum on $\mathcal{A}$.
- By input to the theorem, this minimum must be 0! So $r_b = 0$ for some $b \in \mathcal{A}$.
- Then $f_b^{-1}(D)$ is an open disk.

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- Fix $R > 0$ such that no branch point of any f_a lies in $D \backslash D(r)$. Let m be the total number of branch points in $D(r)$ of one (equiv. all) f_a.
- Let r_a be the supremum over all $r \in [0, R]$ such that the right-slope of δ_{F_a} at r is not equal to $m - 1$. Then r_a (as a function of a) can be expressed as the valuation of an analytic function on $\mathcal{A}$ (really need a finite cover here again — but r_a descends to $\mathcal{A}$ as a function).
- By maximum principle, r_a achieves its minimum on $\mathcal{A}$.
- By input to the theorem, this minimum must be 0! So $r_b = 0$ for some $b \in \mathcal{A}$.
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- Thank you!