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Algebraization of formal ideals

Let X be a possibly singular complete algebraic variety, defined over a field k of characteristic zero. X is nonsingular at $p \in X$ if $O_{X,p}$ is a regular local ring. The problem of resolution of singularities is to show that there exists a nonsingular complete variety X, which birationally dominates X. Resolution of singularities (in characteristic zero) was proved by Hironaka in 1964. Let v be a valuation of the function field of X, v dominates a unique point p, on any complete variety Y, which birationally dominates X. The problem of local uniformization is to show that, given a valuation v of the function field of X, there exists a complete variety Y, which birationally dominates X, such that the center of v on Y is a regular local ring. Zariski proved local uniformization (in characteristic zero) in 1944. His proof gives a very detailed analysis of rank 1 valuations, and produces a resolution which reflects invariants of the valuation.

We extend these methods to higher rank to give a proof of local uniformization which reflects important properties of the valuation. We simultaneously resolve the centers of all the composite valuations, and resolve certain formal ideals associated to the valuation.