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Von Neumann inequality and sums of squares

Von Neumann inequality states that if r is a rational function in one variable over \mathbb{C} which is regular on the closed unit disc in the complex plane, and T is a contractive (norm less than or equal to one) linear operator on a Hilbert space, then $r(T)$ is again contractive. This fact — which goes back a little over half a century — turned out to play a central role in many problems of operator theory, operator algebras, and system theory; a search for possible generalizations to higher dimensions (functions of several variables) is an active research area.

For a variety of reasons, the case when $r(z) = z^n \overline{p(1/\bar{z})}/p(z)$, where p is a polynomial of degree n with no zeroes on the closed unit disc, is of special importance. In this case, von Neumann inequality turns out to be closely related to decompositions of a certain positive polynomial as a sum of squares. I will discuss this connection, with an emphasis on higher dimensional generalizations.