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Valued modules over Ore polynomial rings

Let K be a field, φ an endomorphism of $(K, +, \times, 0, 1)$ and R the ring generated by an indeterminate t and by constants from the field K subject to the relation $at = t\varphi(a)$, for $a \in K$. We note $R = K[t; \varphi]$. This ring is right euclidien, and in particular is right Ore. If K is of characteristic $p > 0$, then any extension of K can be viewed as a (right) module over the ring $K[t; x \mapsto x^p]$, that is the ring of additive polynomials with coefficients from the field K.

In this talk we will axiomatize valued R-modules as two-sorted structures. We will first define an action of K on a chain Δ which defines a valuation v_K on K. Then we will extend this action to R and call Δ an R-chain. We will equip an R-module with an R-chain to define valued R-modules. If K is a valued field of characteristic $p > 0$, then any extension of $(K \subset L, v_L)$ will be a valued $K[t; x \mapsto x^p]$ valued module. We will introduce notions of "henselianness", "maximality", "affine maximality" for valued R-modules like their analogous in the theory of valued fields. We will also replace "Kaplasky's hypothesis A" by the notion of "residual divisibilty" and prove uniqueness of the immediate maximal extension in this case. An Ax-Kochen and Ershov type result for the class of residually divisible and affinely maximal valued R-modules is established. We will see that whenever the above valuation v_K is not trivial on K, the hypothesis of being residually divisible is necessary to prove our Ax-Kochen and Ershov type result.