THE TWENTIETH COLLOQUIUMFEST

Title: *Pushing back the barrier of imperfection*

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Abstract:
The word "imperfection" in our title not only refers to fields that are not perfect, but also to the defect of valued field extensions; the latter is not necessarily directly connected with imperfect fields but may always appear when at least the residue field of a valued field has positive characteristic. For important open problems in algebraic geometry in positive characteristic, such as resolution of singularities and its local form, local uniformization, both forms of imperfection are a severe hurdle. The same is true for the model theory of valued fields, in particular the open question whether the elementary theory of the imperfect Laurent Series Field $F_p((t))$ is decidable; this problem is also open for the perfect hull of $F_p((t))$, which admits extensions with nontrivial defect. Another only partially answered question is under which conditions the existence of a rational place for a function field $F|K$ implies that K is existentially closed in F; here one can nicely study the interplay of imperfection, the notion of "large field", model theory and resolution of singularities.

Beating or avoiding imperfection, as is done in the theory of "tame valued fields", leads to partial answers to the above oben problems. One possible way of generalizing these answers is to push back the barrier of imperfection, that is, to consider notions of potentially imperfect valued fields with otherwise strong properties, such as the "extremal valued fields", or of perfect valued fields which allow only defects of a sort that we can still handle, such as "deeply ramified fields" (in the sense of Gabber-Ramero) or "separably tame fields".

We will give a survey on what is known and what (hopefully) could be done next. The results I will mention come from various projects in which the following coauthors were involved: Sylvy Anscombe, Salih Azgin, Anna Blaszczok, Hagen Knaf, Koushik Pal, Florian Pop.