Defect extensions of prime degree

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joint work with Franz-Viktor Kuhlmann

The Twentieth Colloquiumfest

May 20, 2017

(K, v) a valued field
vK the value group, Kv the residue field.

If $(L|K, v)$ is a finite extension of valued fields and the extension of v from K to L is unique, then

$$[L : K] = p^n (vL : vK)[Lv : Kv],$$

where $p = \text{char} Kv$ if it is positive and $p = 1$ otherwise.

$d(L|K, v) := p^n$ - the **defect** of $(L|K, v)$.

If $p^n > 1$, then $(L|K, v)$ is called a **defect extension**. Otherwise it is called a **defectless extension**.

A henselian field (K, v) is called **defectless** if every finite extension $(L|K, v)$ is defectless, i.e.,

$$[L : K] = (vL : vK)[Lv : Kv].$$

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A valued field extension $(L|K,v)$ is called immediate if

$$(vL : vK) = [Lv : Kv] = 1.$$

A valued field which admits no nontrivial immediate algebraic (separable-algebraic) extension is called **algebraically maximal** (**separable-algebraically maximal**).

- algebraically maximal $\not\Rightarrow$ defectless

Theorem 1 (F.-V. Kuhlmann)

A valued field of positive characteristic is henselian and defectless if and only if it is separable-algebraically maximal and admits no purely inseparable defect extension.

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a characterization of defectless fields

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$(L|K, v)$ - a finite extension of henselian fields,
char$Kv = p > 0$.

$$d(L|K, v) = d(L.N|N, v)$$

- If char$K = p$, then Galois extensions of degree p are Artin-Schreier extensions, i.e., extensions generated by roots of polynomials $X^p - X - a$, for $a \in K$.

defect extensions

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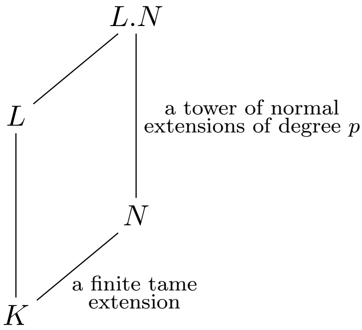

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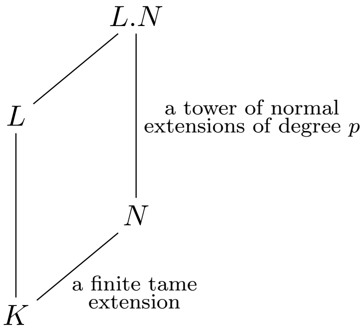

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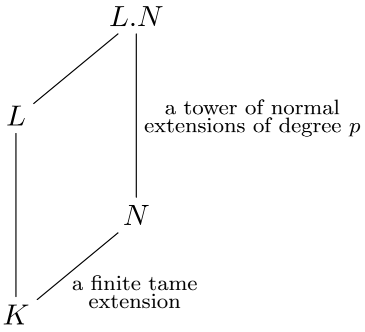

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(K, v) a valued field of characteristic $p > 0$,

$(K(\vartheta)|K, v)$ an Artin-Schreier defect extension, where ϑ is an **Artin-Schreier generator**, that is, a root of a polynomial

$$X^p - X - a$$

for some $a \in K$.

- $(K(\vartheta)|K, v)$ is an immediate extension,
- $v(\vartheta - K) := \{v(\vartheta - c) \mid c \in K\}$ is an initial segment of vK,
- $v(\vartheta - K) \subseteq vK^{<0}$ and does not depend on the choice of ϑ.

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$(K(\vartheta)|K,v)$ an Artin-Schreier defect extension

If there is a purely inseparable defect extension $(K(\eta)|K,v)$ of degree p such that

$$v(\eta-\vartheta) > v(\vartheta - K),$$

then $(K(\vartheta)|K,v)$ is called a **dependent Artin-Schreier defect extension**. Otherwise $(K(\vartheta)|K,v)$ is called an **independent Artin-Schreier defect extension**.

Proposition 2

$(K(\vartheta)|K,v)$ is an independent Artin-Schreier defect extension if and only if the smallest initial segment of the divisible hull $\widetilde{vK}$ of vK containing $v(\vartheta - K)$ is equal to

$$\{\alpha \in \widetilde{vK} \mid \alpha < H\}$$

for some proper convex subgroup H of $\widetilde{vK}$.

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Theorem

A valued field of positive characteristic is henselian and defectless if and only if it is separable-algebraically maximal and admits no purely inseparable defect extension.

- (K, v) admits no purely inseparable defect extensions

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  every finite extension of K admits no purely inseparable defect extensions
  (hence also no dependent Artin-Schreier defect extensions);

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Kummer defect extensions

(K, v) a henselian field; char$K = 0$, char $Kv = p > 0$;
$\varepsilon_p \in K$, where ε_p is a primitive p-th root of unity.

$(L|K, v)$ a Galois defect extension of degree p. Then:

- $L = K(a)$, where $\boldsymbol{a^p \in K}$;
- $(K(a)|K, v)$ is an immediate extension;
- we can choose $\boldsymbol{a \in 1 + \mathcal{M}_L}$.

Lemma 3

The set $v(a - K)$ is an initial segment of vK and does not depend on the choice of the generator a which satisfies the above assumptions.

- $v(a - K) \nsubseteq vK^{<0}$

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$h_a(X) := X^p - a^p$ - the minimal polynomial of a over K,

Take $C \in \tilde{\mathbb{Q}}$ such that $C^{p-1} = -p$.

- Since K is henselian and $\varepsilon_p \in K$, we obtain that $C \in K$.

Consider the transformation $X = CY + 1$ for h_a and divide the polynomial by C^p.
We then obtain the polynomial

$$f_a(Y) = Y^p + g(Y) - Y - \frac{a^p - 1}{C^p},$$

where $g(Y)$ has all coefficients in $\mathcal{M}_K$.

Set $\vartheta_a := \frac{a-1}{C}$. Then

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$$f_a(Y) = Y^p + g(Y) - Y - \frac{a^p - 1}{C^p},$$

where $g(Y)$ has all coefficients in $\mathcal{M}_K$.

Set $\vartheta_a := \frac{a-1}{C}$. Then

$$K(a) = K(\vartheta_a) \text{ and } f_a(\vartheta_a) = 0.$$

$h_a(X) := X^p - a^p$ - the minimal polynomial of a over K,

Take $C \in \tilde{\mathbb{Q}}$ such that $C^{p-1} = -p$.

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The initial segment $v(\vartheta_a - K)$ of vK does not depend on the choice of the generator ϑ_a and

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$(K(\vartheta_a)|K, v)$ is called an **independent Kummer defect extension** if the smallest initial segment of $\widetilde{vK}$ containing $v(\vartheta_a - K)$ is equal to

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Otherwise $(K(\vartheta_a)|K, v)$ is called a **dependent Kummer defect extension**.

- Both types of Kummer defect extensions exist.

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$$Tr_{L|K}(\mathcal{M}_L) = \{d \in K \mid vd \in \Lambda\}$$

Theorem 7

Assume that $(L|K, v)$ is an independent Artin-Schreier/Kummer defect extension. Then

$$\mathrm{Tr}_{L|K}(\mathcal{M}_L) = \{d \in K \mid vd > H\}$$

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Proposition 8

Assume that (K, v) is a valued field of characteristic 0 and positive characteristic p and $\varepsilon_p \in K$. If (K, v) is algebraically maximal, then every finite defectless extension of K admits no independent Kummer defect extensions.

(K, v) of residue char. $p > 0$ is called **deeply ramified** if

- vK is p-divisible,
- the Frobenius homomorphism $\mathcal{O}/p\mathcal{O} \to \mathcal{O}/p\mathcal{O}$ is surjective.

If char$K = p$, then:

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Theorem 10

Assume that (K, v) is a valued field of characteristic 0 and positive characteristic p. If (K, v) is deeply ramified and algebraically maximal, then it is defectless.

THANK YOU
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