Spaces of $\mathbb{R}$-places

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An $\mathbb{R}$-place is a place of some field with residue field inside the field $\mathbb{R}$ of real numbers. In our talk, we will give a survey on some recent developments in the theory of $\mathbb{R}$-places.

T. Craven showed in 1975 that every boolean space can be realized as the space of orderings of some field. In contrast to this, it is an open problem whether every compact Hausdorff space can be realized as the space of $\mathbb{R}$-places of some field. Here, the topology on the space of $\mathbb{R}$-places is the one induced by the Harrison topology of the space of orderings of that field, via the map that associates to every ordering its canonical $\mathbb{R}$-place.

The talk will concentrate on two problems: metrizability and realizibility of spaces of $\mathbb{R}$-places. We will show that the class of compact Hausdorff spaces which can be realized as spaces of $\mathbb{R}$-places is closed under finite disjoint unions, closed subsets, and direct products with Boolean spaces (joint work with Ido Efrat, [1]). It is obvious that the space of $\mathbb{R}$-places of any countable field if metrizable. The same is true for any function field of transcendence degree 1 over a totally Archimedean field. We will show that for a function field F over a real closed field R of higher transcendence degree than 1 the space $M(F)$ is metrizable iff R is countable (joint work with Murray Marshall and Michal Machura, [2]). For transcendence degree 1 the situation is more complicated. The necessary condition for metrizability of the space $M(F)$ is that R contains a countable dense subfield. It is also a sufficient condition in the case of F being a rational function field over R (joint work with F.-V. Kuhlmann and M. Machura, [3]). We will end our talk by a description of the structure of the space $M(R(X))$ over any non-Archimedean real closed field R. Finally, we will mention some open problems.

References

[1] I. Efrat, and K. Osiak. Topological spaces as spaces of $\mathbb{R}$-places. *J. Pure Appl. Alg.* **215** (5), 2011

[2] M. Machura, M. Marshall and K. Osiak. Metrizability of the space of R-places of a real function field. *Math. Zeitschrift* **266** (1), 2010

[3] F.-V. Kuhlmann, M. Machura, K. Osiak. Metrizability of spaces of $\mathbb{R}$-places of function fields of transcendence degree 1 over real closed fields. *Comm. in Alg.* **39** (9), 2011