BIRS Workshop
Orthogonal Polynomials; Interdisciplinary Aspects
March 27 to April 1, 2004

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

	Sunday	Monday	Tuesday	Wednesday	Thursday	
7:00-9:00	Continental Breakfast, 2nd floor lounge, Corbett Hall					
9:00-9:50	Luc Vinet	Andreas Ruffing	Arno Kuijlaars	Marteen Vanlessen	X	
9:50-10:40	Christian Berg	Walter Van Assche	Peter Miller	Mark Adler		
10:40-11:00	Coffee Break, 2nd floor lounge, Corbett Hall					
11:00-11:50	Sergei K. Suslov	John Harnad	Hans Lundmark	Anatol Odzijewicz	X	
11:50-12:00	X	X	Group Photo[1]	X	X	
11:30-13:30	Buffet Lunch, Donald Cameron Hall					
13:00-14:00	cancelled[2]	X	free afternoon	Guided tour		
14:00-14:50	Richard Askey	Alberto Grunbaum	free afternoon	Jorge Arvesu	X	
14:50-15:40	Francisco Marcellan	Percy Deift	free afternoon	Natig Atakishiyev	X	
15:40-16:10	Coffee Break, 2nd floor lounge, Corbett Hall (except Tues.)				X	
16:10-17:00	Yuan Xu	Mourad Ismail	free afternoon	Alexei Borodin	X	
17:00-17:50	Jeff Geronimo	Dong Won Lee	free afternoon	Lance Littlejohn	X	
17:30-19:30	Buffet Dinner, Donald Cameron Hall					X
19:30-20:20	X	X	Marco Bertola			
20:20-21:10	X	X	Ken McLaughlin			

[1] A group photo will be taken on Tuesday at 11:45 am, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

[2] A free guided tour of The Banff Centre is offered to all participants and their guests on Sunday starting at 1:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Mark Adler** (Brandeis University)
Title: *Title*
Abstract: The talk will discuss pde's for the joint distributions of the Dyson, Airy and Sine processes, the latter two being continuous stationary processes, arising as the limit of the first process, respectively at the edge and in the bulk. We use the pde for the Airy process to settle the asymptotic behavior of the joint distribution at different times, t_1, t_2, as their difference goes to infinity.

Speaker: **Jorge Arvesu** (Universidad Carlos III de Madrid)
Title: *Multiple q-discrete orthogonal polynomials*
Abstract: The multiple orthogonal polynomials are very fascinating mathematical subject at the border of many areas, mainly approximation theory, number theory, differential and difference equations, real and complex analysis, and special functions among others. Consequently, many techniques have been developed and are available.

Nevertheless, some of these techniques fail in the study of multiple q-discrete orthogonal polynomials. The present communication deals with multiple q-discrete orthogonal polynomials of type II. Some interesting results consisting of the Rodrigues-type formula, recurrence relation, difference equation, location of the zeros, asymptotic properties are shown. Also some applications of these multiple orthogonal polynomials are discussed.

Speaker: **Richard Askey** (University of Wisconsin)
Title: *A determinant of Sylvester and Kac and extensions*
Abstract: I will talk about the eigenvalues of a tridiagonal matrix first stated by Sylvester and then by Mark Kac. Above the main diagonal are the integers $1, 2, ..., N$ and below the diagonal are these integers read backwards, $N, N-1, ..., 1$. Extensions will also be given.

Speaker: **N.M. Atakishiyev (speaker) and A.U. Klimyk** (Instituto de Matematicas, UNAM, Cuernavaca)
Title: *On discrete q-ultraspherical polynomials and their duals*
Abstract:

We show that a confluent case of the big q-Jacobi polynomials

$$P_n(x; a, b, c; q) := {}_3\phi_2\,(q^{-n}, abq^{n+1}, x;\; aq, cq;\; q, q),$$

which corresponds to $a = b = -c$, leads to a discrete orthogonality relation for imaginary values of the parameter a (outside of its commonly known domain $0 < a < q^{-1}$). Since $P_n(x; q^\alpha, q^\alpha, -q^\alpha; q)$ tend to Gegenbauer (or ultraspherical) polynomials in the limit as $q \to 1$, this family represents yet another q-extension of these classical polynomials, different from the continuous q-ultraspherical polynomials of Rogers. The dual family with respect to the polynomials $P_n(x; a, a, -a; q)$ (i.e., the dual discrete q-ultraspherical polynomials) corresponds to the indeterminate moment problem, that is, these polynomials have infinitely many orthogonality relations. We find orthogonality relations for these polynomials, which have not been considered before. In particular, some of extremal orthogonality measures for these polynomials are derived.

Speaker: **Christian Berg** (University of Copenhagen)
Title: *Orthogonal polynomials associated to positive definite matrices*
Abstract: To each positive definite infinite matrix $A = (a_{n,m}), n, m = 0, 1, \dots$ we can associate an inner product on the linear space of polynomials $\mathbb{P}$ by the requirement $\langle x^n, x^m \rangle = a_{n,m}$. By the Gram-Schmidt procedure we can replace the monomials by a sequence of orthonormal polynomials $(p_n)_n$, uniquely determined by the orthonormality and the requirement that p_n is of degree n with positive leading coefficient.

We consider the truncated matrices A_n of A of size $n \times n, n \geq 1$. Since A_n is positive definite for each n, we can write its eigenvalues

$$0 < \lambda_{1,n} \leq \lambda_{2,n} \leq \ldots \leq \lambda_{n,n},$$

repeated according to their multiplicity.

We shall relate the beviour of the sequences $\lambda_{1,n}, \lambda_{2,n}, \dots$ to boundedness properties of the operator $T : \mathbb{P} \to \mathbb{P}$ defined by $T(x^n) = p_n, n = 0, 1, \dots$, the norm being $\langle \cdot, \cdot \rangle^{1/2}$.

The asymptotic behaviour of $\lambda_{1,n}$ in the case of A being the Hankel matrix of a positive measure μ with moments of any order has been examined for various classical polynomials by Szegő, Widom and Wilf, Chen and Laurence. In [3] it was proved that $\lambda_{1,n} \to 0$ iff the measure is determinate.

We shall discuss various extensions of this involving the eigenvalues $\lambda_{2,n}, \lambda_{3,n}$ etc. and also extend some of the results to orthogonal matrix polynomials.

Some of the questions involve the index of determinacy $\mathrm{ind}_z(\mu)$ for a determinate measure μ, defined in [1] by

$$\mathrm{ind}_z(\mu) = \sup\{k \in \mathbb{N} \mid |t - z|^{2k}\mu \text{ is determinate}\}.$$

References

[1] C. Berg, A.J. Durán, *The index of determinacy for measures and the ℓ^2-norm of orthonormal polynomials*, Trans. A.M.S. **347** (1995), 2795–2811.

[2] C. Berg, A.J. Durán, *Orthogonal polynomials and analytic functions associated to positive definite matrices*. Manuscript.

[3] C. Berg, Y. Chen, M.E.H. Ismail, *Small eigenvalues of large Hankel matrices: The indeterminate case*. Math. Scand. **91** (2002), 67–81.

Speaker: **Marco Bertola** (Concordia University)
Title: *Random Matrices and Isomonodromic Tau functions*
Abstract: We consider the most general semiclassical orthogonal polynomials and the associated "Hermitean" Random Matrix model (in fact it is a normal matrix model in general, although not in the common sense). Any two consecutive polynomials satisfy a Frobenius compatible overdetermined system of ODE(w.r.t. the spectral parameter) + PDEs (w.r.t. the deformations of the measure) + Difference Equations (w.r.t. the choice of degree of the polynomials). The deformations describe infinitesimal gauge transformations of the ODE, preserving its (generalized) monodromy. The corresponding isomonodromic tau function á la Miwa-Jimbo-Ueno is explicitly computed in terms of the partition function, showing that the two objects are essentially the same (up to an explicit function of the measure which we compute as well).

Speaker: **Alexei Borodin** (California Institute of Technology)
Title: *Applications of discrete orthogonal polynomials in asymptotic representation theory*
Abstract: The goal of the talk is to explain how computing the asymptotics of discrete orthogonal polynomials of hypergeometric type (Charlier, Meixner, Hahn, Racah and their generalizations) leads to solving certain problems of representation theory of the so-called big groups, which include inductive limits of symmetric groups and unitary groups. This is joint work with Grigori Olshanski.

Speaker: **Percy Deift** (Courant Institute)
Title: *Universality for Orthogonal and Symplectic Ensembles*
Abstract: The speaker will show how to prove universality for basic statistical quantities arising in the theory of orthogonal and symplectic ensembles of matrices. We follow the approach of Harold Widom and do not use the theory of skew-orthogonal polynomials. This is joint work with Dimitri Gioev. Speaker:

Jeff Geronimo (Georgia Tech)
Title: *Two variable stable polynomials and applications*
Abstract: A stable polynomial in one variable is one that has all its zeros exterior to the closed unit disk. Such polynomials are closely related to polynomials orthogonal on the unit circle and play an important role in finding causal solutions for autoregressive models. In two variables stable polynomials i.e. those that are nonzero inside and on the bicircle play a similar role. We will discuss these relations as well as applications.

Speaker: **Alberto Grunbaum** (University of California at Berkeley)
Title: *Urn models in statistical physics and classical orthogonal polynomials*
Abstract:I review the classical facts about models like those of Bernoulli-Laplace, as well as the one of Ehrenfest and their relation with Hahn and Krawtchouk polynomials. I take a second look at some of these issues in view of certain matrix valued extensions of these polynomials, along the lines of my joint and ongoing work with Duran, Pacharoni and Tirao.

Speaker: **John Harnad** (Concordia University)
Title: *Biorthogonal Polynomials in 2-Matrix Models*
Abstract: The spectral statistics of 2-matrix models with conjugation invariant measures naturally lead to the study of biorthogonal polynomials. The theory behind this will be reviewed, with special emphasis on the case of bi-measures that are determined by the exponentials of a pair of polynomial "potentials", for which the biorthogonal polynomials satisfy generalized Christoffel-Darboux relations. The emphasis will be on the deformation theory, both with respect to the coefficients defining these measures, and the size N of the matrices. The results to be discussed will include some selection of the following topics: 1) the relation with "dual" isomonodromic deformations 2) spacing distributions and polynomials supported on unions of intervals 3) large x and y asymptotics 4) large N limits and their relation to dispersionlesss hierarchies, and 5) character expansions of matrix integrals.

Speaker: **Mourad Ismail** (University of South Florida)
Title: *The Tau Function and Discriminants*
Abstract: We show how the tau function is related to discriminants of orthogonal polynomials and equilibrium energy in an electrostatic equilibrium problem.

Speaker: **Pavel Bleher and Arno Kuijlaars (speaker)** (Katholiek Universiteit Leuven)
Title: *Gaussian random matrices with external source*
Abstract: Because of their connection to orthogonal polynomials, it has been possible to analyze unitary random matrix ensembles

$$\frac{1}{Z_n}\exp(-ntrV(M))$$

in great detail. Deift, Kriecherbauer, McLaughlin, Venakides and Zhou studied the large n limit using the Riemann-Hilbert problem for orthogonal polynomials, and established the universal behavior of eigenvalue spacings in the bulk for real-analytic V.

Given a fixed Hermitian matrix A (the external source), the random matrix ensemble

$$\frac{1}{Z_n}\exp(-ntr(V(M)-AM))dM$$

is connected with multiple orthogonal polynomials. Following Brezin and Hikami, we consider the case that the external source A has only two eigenvalues $\pm a$ with equal multiplicity. Then the multiple orthogonal polynomials have a characterization in terms of a $3x3$-matrix Riemann-Hilbert problem, first given by Van Assche, Geronimo and Kuijlaars. We apply the Deift/Zhou steepest descent method to the Gaussian case $V(M) = (1/2)M^2$. We recover the usual universality of eigenvalue spacings in the bulk (sine kernel) and at the edge (Airy kernel), except for the value $a = 1$. In that case a phase transition takes place at the origin. In the double scaling limit new kernels appear that are built out of Pearcey integrals.

Speaker: **K. H. Kwon (speaker) and D.W. Lee** (Kyungpook National University)

Title: *On a non-compactly supported Bochner-Krall problem*

Abstract:

Consider a linear differential equation of order $N \geq 1$ of the form

$$L_N[y](x) := \sum_{i=0}^{N} \ell_i(x) y^{(i)}(x) = \lambda_n y(x), \quad \lambda_n \in \mathbb{R}, \tag{1}$$

where $\ell_i(x)$'s are polynomials of degree $\leq i$. The classification problem of orthogonal polynomial systems (OPS's) satisfying a differential equation (1) are now well known by the Bochner-Krall problem whose root goes back to Bochner [1]. Such orthogonal polynomials are called the Bochner-Krall polynomials.

In 1929, all five polynomial sequences (up to a linear change of variables) that satisfy the differential equation (1) with $N = 2$ were found. Four of them are Jacobi, Bessel, Laguerre, and Hermite polynomials, which are now called the classical OPS. In 1938, H. L. Krall [3] found a necessary and sufficient condition for the equation (1) to have an OPS of solution and then classified all fourth order differential equations. For a long time many authors try to solve the Bochner-Krall problem in various method. Based on many results, many people conjectured that the orthogonalizing weight for Bochner-Krall polynomials should be a special form: the classical weight plus pertubations on the boundary of the classical weight. More precisely, if w is a weight for Bochner-Krall polynomials, then it must be one of the two cases: if w has a compact support, then

$$(1-x)^{\alpha}(1+x)^{\beta} H(1-x^2) + A\delta(x-1) + B\delta(x+1) \tag{2}$$

and if w has a non-compact support, then

$$x^{\gamma} e^{-x} H(x) + C\delta(x),$$

where $\alpha, \beta, \gamma > -1$, $\delta(x)$ is the Dirac delta function, $H(x)$ is the Heaviside step function, and A, B, C are constants.

In compactly supported case, authors [4] recently proved that the Bochner-Krall polynomials must be a generalized Jacobi polynomials introduced by Koorwinder [2] and so the conjecture was proved as true. They found six equivalent conditions for Bochner-Krall polynomials including the equation (2) of distributional orthogonalizing weight.

In this talk, we first investigate the ratio asymptotics of orthogonal polynomials on the real line, which will be used in the Bochner-Krall problem for non-compact case. For example, we prove that if

$$\lim_{n\to\infty} \frac{[P_n(\xi_n z)]'}{nP_n(\xi_n z)} = g(z) \tag{3}$$

uniformly for any compact subset of $\mathbb{C} \setminus \mathbb{R}$, then for any integer $m \geq 1$,

$$\lim_{n\to\infty} n\Big[\frac{[P_n(\xi_n z)]^{(m)}}{n^m P_n(\xi_n z)} - \Big(\frac{[P_n(\xi_n z)]'}{nP_n(\xi_n z)}\Big)^m\Big] = \frac{m(m-1)}{2} g^{m-2}(z) g'(z)$$

uniformly for any compact subset of $\mathbb{C} \setminus \mathbb{R}$, where $\{\xi_n\}_{n=0}^{\infty}$ is a sequence going to ∞ as n goes to ∞.

In Bochner-Krall problem, we use the fact that the limit relation (3) was strongly related to the coefficients of the three term recurrence relation.

We also prove that $\deg(\ell_N) = N$ is the equivalence condition for the Bochner-Krall polynomial to be orthogonal relative to a compactly supported weight. Moreover, we proved using the asymptotics that every non-compactly supported Bochner-Krall polynomial must be a generalized Nevai class, that is, if $\{P_n(x)\}_{n=0}^{\infty}$ satisfies

$$xP_n(x) = P_{n+1}(x) + b_nP_n(x) + a_n^2P_{n-1}(x), \quad n \geq 1,$$

where $P_{-1}(x) \equiv 0$, then the coefficients $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ have to converge,

$$\lim_{n\to\infty} \frac{a_n}{n^\alpha} = \frac{a}{2} \quad \text{and} \quad \lim_{n\to\infty} \frac{b_n}{n^\alpha} = b,$$

where $a^2 + b^2 \neq 0$ and $\alpha > 0$. This result is well coincide with the Magnus' conjecture ([5]) which asserts that the coefficients of the three term recurrence relation for Bochner-Krall polynomials should be a rational function on the index n. More precisely, we showed that

(a) if $b \neq 0$, then

$$a^2 = b^2 \quad \text{and} \quad \alpha = \max_{r+1\leq i\leq N} \frac{i-r}{i-\deg(\ell_i)} \in \{1, 2, \cdots, N-1\};$$

(b) if $b = 0$, then

$$\alpha = \max_{r+1\leq i\leq N} \frac{i-r}{i-\deg(\ell_i)} \in \{\frac{1}{2}, 1, \frac{3}{2}, \cdots, \frac{N-1}{2}\}.$$

Here, r is the largest index such that $i = \deg(\ell_i)$. Lastly, the cases $\alpha = 1$ and $\alpha = \frac{1}{2}$ are investigated in details. In particular, if $\alpha = \frac{1}{2}$, then the Bochner-Krall polynomials must be the classical Hermite polynomials on the real line.

References

[1] S. Bochner, *Über Sturm-Liouvillesche Polynomsysteme*, Math. Z., **29** (1929), 730-736.

[2] T. H. Koornwinder, *Orthogonal polynomials with weight function* $(1-x)^\alpha(1+x)^\beta + M\delta(x+1) + N\delta(x-1)$, Canad. Math. Bull., **27(2)** (1984), 205-214.

[3] H. L. Krall, *Certain differential equations for Tchebycheff polynomials*, Duke Math. J., **4** (1938), 705-718.

[4] K. H. Kwon and D. W. Lee, *Characterizations of Bochner-Krall orthogonal polynomials of Jacobi type*, Constr. Approx., **19** (2003), 599-619.

[5] A. P. Magnus, *Open problems*, Orthogonal Polynomials and Their Applications, Eds C. Brezinski et al., J.C. Baltzer AG, Scientific Publishing Co. IMACS (1991), 417-419.

Speaker: **Lance L. Littlejohn** (Utah State University)

Title: *Laguerre Polynomials and Self-Adjoint Operators*

Abstract:

When $\alpha > -1$ it is well known, and classical, on how to construct the self-adjoint operator A_α, generated by the second-order Laguerre differential expression

$$\ell_\alpha[y](x) = \frac{1}{x^\alpha e^{-x}}[(x^{\alpha+1}e^{-x}y'(x))' + kx^\alpha e^{-x}y(x)] \quad (x \in (0,\infty); k \geq 0 \text{ fixed}),$$

in the Hilbert space $L^2((0,\infty);x^\alpha e^{-x})$ having the Laguerre polynomials $\{L_n^\alpha\}_{n=0}^\infty$ as eigenfunctions; indeed, this is an direct consequence of the Glazman-Krein-Naimark theory of self-adjoint extensions of formally Lagrangian symmetric differential expressions. However, when $\alpha \leq -1$, this spectral theory is less understood although it is well known that when $\alpha < -1$ and $-\alpha$ is not a positive integer, the appropriate function space setting is necessarily a Krein space. In this lecture, we consider the case when $-\alpha$ is a positive integer. In this case, by Favard's Theorem, the Laguerre polynomials $\{L_n^\alpha\}_{n=0}^\infty$ cannot be classically orthogonal with respect to a bilinear form of the type

$$(f,g)_\mu := \int_R f(x)\overline{g}(x)d\mu$$

for any (signed) Borel measure μ. However, Kwon and Littlejohn showed that these Laguerre polynomials are orthogonal with respect to a Sobolev inner product. In this lecture, we discuss this Sobolev orthogonality and construct a self-adjoint operator in the associated Sobolev space having these Laguerre polynomials as a complete set of eigenfunctions.

References

[1] K. H. Kwon and L. L. Littlejohn, *"The orthogonality of the Laguerre polynomials* $\{L_n^{-k}(x)\}$ for positive integers k*", Annals of Numerical Mathematics, 2(1995), 289-303.*

[2] W. N. Everitt, L. L. Littlejohn, and R. Wellman, *"The Sobolev orthogonality and spectral analysis of the Laguerre polynomials* $\{L_n^{-k}\}$ for positive integers k*", J. Comput. Appl. Math.,* to appear.

Speaker: **Hans Lundmark** (Linkoeping University)
Title: *Degasperis–Procesi peakons and the discrete "cubic string'*
Abstract: The selfadjoint eigenvalue problem $\phi''(y) = zg(y)\phi(y)$, $\phi(-1) = \phi(1) = 0$ occurs when separating variables in the wave equation for a string with inhomogeneous mass density $g(y)$. The inverse problem of determining $g(y)$ from suitable spectral data was solved by Krein around 1950. In the particular case of a discrete string (consisting only of a finite number of point masses) the solution can be expressed in terms of Stieltjes continued fractions, which connects the problem to the classical theory of orthogonal polynomials. This result was used by Beals, Sattinger and Szmigielski to explicitly compute so-called multipeakon solutions of the Camassa–Holm shallow water wave equation. This is a particular class of weak solutions made up by nonlinear superposition of peaked waves interacting like solitons.

The Degasperis–Procesi equation is a recently discovered integrable PDE similar to the Camassa–Holm equation. It also admits multipeakon solutions, but this time related to the discrete "cubic string" equation $\phi'''(y) = -zg(y)\phi(y)$, $\phi(-1) = \phi'(-1) = \phi(1) = 0$. This problem is not selfadjoint. Nevertheless, the spectrum is real and simple, which can be proved in an indirect way using the isospectral deformation induced by the Degasperis–Procesi equation. Moreover, the inverse spectral problem has an explicit solution which is an interesting and nontrivial generalization of the Stieltjes–Krein method.

This is joint work with Jacek Szmigielski.

Speaker: **Francisco Marcellan** (Universidad Carlos III de Madrid)
Title: *Darboux transform, Hessenberg matrices and orthogonal polynomials. Applications to spectral transformations*
Abstract: Darboux transform has a wide set of applications in the theory of nonlinear integrable systems. In particular, the connection between the discrete Darboux transformation and the perturbation of spectral measures supported on the real line is very well known taking into account the LU factorization of the corresponding Jacobi matrix. In this talk we will analyze the LU factorization of Hessenberg matrices associated with the representation of the multiplication operator in terms of general orthogonal polynomials relative to a bilinear functional. The Darboux transform will be introduced in this framework. Furthermore, for some special cases of inner products the corresponding spectral analysis of the Darboux transform will be presented.

This is a joint work with M. Bueno.

Speaker: **Ken McLaughlin** (University of North Carolina, Chapel Hill)
Title: *On some applications of asymptotic expansions for orthogonal polynomials*
Abstract: I will talk about applications of asymptotic expansions for orthogonal polynomials to (1) an asymptotic expansion for the partition function of random matrices, and (2) singular limits of Toda lattices. This work is joint with Nick Ercolani, and Virgil Pierce (University of Arizona).

Speaker: **Peter Miller** ((University of Michigan, Ann Arbour)
Title: *A dbar steepest descent method for Riemann-Hilbert problems.*
Abstract: Riemann-Hilbert problems with jump matrices that are sufficiently smooth but nonanalytic along the jump contour arise frequently in practice, for example in the inverse-scattering analysis of initial-value problems for integrable PDE with nonanalytic initial data, and also in the theory of orthogonal polynomials with nonanalytic weights. This talk outlines a systematic approach to singular asymptotic calculations for such problems of the sort that would be tractable by the steepest descent method of Deift and Zhou were the jump matrices analytic. The approach is based on simple nonanalytic extensions of smooth functions, at the cost of replacing the Riemann-Hilbert problem by a mixed Riemann-Hilbert-dbar problem. Great advantage is taken of the fact that the Cauchy kernel is less singular in two-dimensional integration than in contour integration. The method can be used to extend universality in certain random matrix models beyond analytic classes of weights. This is joint work with K. McLaughlin.

Speaker: **Anatol Odzijewicz** (University of Bialystok, Bialystok)
Title: *Orthogonal Polynomials and integrable multibosons systems*
Abstract: The method based on the quantum reduction leads from N-degree of freedom Heisenberg algebra to one–degree of freedom operator algebras, which are not in general Lie algebras. These algebras could be considered as "symmetry algebras" of some multiboson systems. The Schrödinger operators of these systems after proper choice of basis are represented by infinite Jacobi matrix. In such a way one reduces their eigenproblem to the solution of the moment problem of the corresponding class of the orthogonal polynomials. We shall present boson quantum physical systems (including systems appearing in the non–linear quantum optics) integrable by applaying:

1. classical orthogonal polynomials (Jacobi, Laguerre, Hermite)
2. q-Hahn orthogonal polynomials (Big q–Jacobi, Big q–Laguerre, Little q–Jacobi, Little q–Laguerre/Wall, Alterbative q–Charkier, Discrete q–Hermite I, Al-Salaman-Carlitz I)
3. finite orthogonal polynomials (Hahn, Dual Hahn, Krawtchouk)

References

[O-H-T] A. Odzijewicz, M. Horowski, A.Tereszkiewicz "Integrable multi-boson systems and orthogonal polynomials" J. Phys. A: Math. Gen. 34, (2001) 4353-4376

[H-O-T] M. Horowski, A.Odzijewicz and A.Tereszkiewicz, "Some integrable system in nonlinear quantum optics", J.Math.Phys. 44, (2003) 2, (480-506), arXiv:math-ph/0207031 v1 23 Jul 2002

[CH-H-O-T] G. Chadzitaskos, M. Horowski, A. Odzijewicz, A. Tereszkiewicz, "The exact solution of the eigenproblem for the parametric down conversion process in the Kerr medium", Cz.J.Phys.,53, (2003) 11

[H-CH-O-T] M. Horowski, G. Chadzitaskos, A. Odzijewicz, A. Tereszkiewicz, "Systems with intensity dependent conversion integrable by finite orthogonal polynomials"-to appear, arXiv:math-ph/0310058 v2 18 dec 2003

Speaker: **Andreas Ruffing** (Arizona State University Affiliation)
Title: *On the Meaning of Moment Problems to Mathematical Modelling in Quantum Optics*
Abstract: Ladder operators play a fundamental role in the understanding of analytic models within quantum optics. In this context, the concept of coherent states is reviewed. Different moment problems which arise in the investigation of coherent states are presented. They are closely related to the spectral properties of the underlying difference operators. Some new aspects are contributed to the spectral theory of ladder operators and discrete Schroedinger operators on q-quadratic grids.

Speaker: **Sergei K. Suslov** (Arizona State University)
Title: *Convergence of Basic Fourier Series*
Abstract: We discuss convergence of basic Fourier series and its application in q-Fourier analysis.

Speaker: **Walter Van Assche** (Katholieke Universiteit Leuven)
Title: *Some applications of multiple orthogonal polynomialsTitle*
Abstract: Multiple orthogonal polynomials are polynomials of one variable that satisfy orthogonality relations with respect to r measures (where $r \geq 1$). The case $r = 1$ gives the usual orthogonal polynomials. I will briefly give some properties of these multiple orthogonal polynomials, in particular a Riemann-Hilbert problem for matrices of order $r + 1$. The main part of the talk deals with applications of these multiple orthogonal polynomials: 1) Simultaneous rational approximation 2) Irrationality and transcendence of certain numbers, in particular $\zeta(2n + 1)$ and its q-analogs 3) Random matrices with external source and coupled random matrices 4) Simultaneous Gauss quadrature

Speaker: **Maarten Vanlessen** (Katholieke Universiteit Leuven)
Title: *Universality for eigenvalue correlations at the origin of the spectrum.*
Abstract: In this talk I will consider the local eigenvalue correlations in unitary random matrix ensembles $\frac{1}{Z_n}|\det M|^{2\alpha}e^{-nV(M)}dM$ near the origin of the spectrum. If V is even, and if the recurrence coefficients of the orthogonal polynomials associated with $|x|^{2\alpha}e^{-nV(x)}$ have a regular limiting behavior, then it is known, using Shohat's method, that the local eigenvalue correlations have universal behavior in terms of Bessel functions. I will explain how to extend this result to a much wider class of confinement potentials V using the Riemann-Hilbert method of Deift and Zhou. This is joint work with A.B.J. Kuijlaars

Speaker: **Luc Vinet** (Mcgill University)
Title: *Linear operator pencils on Lie algebras and Laurent biorthogonal polynomials*
Abstract: We study operator pencils on generators of the Lie algebras sl2 and the oscillator algebra. These pencils are linear in a spectral parameter z. The corresponding generalized eigenvalue problem gives rise to some sets of orthogonal polynomials and Laurent biorthogonal polynomials (LBP) expressed in terms of the Gauss 2F1 and degenerate 1F1 hypergeometric functions. For special choices of parameters of the pencils, we identify the resulting polynomials with the Hendriksen-van Rossum LBP which are widely believed to be the biorthogonal analogues of the classical orthogonal polynomials. This places these examples under the umbrella of the generalized bispectral problem which is considered here. This is joint work with F.A. Grunbaum and Alexei Zhedanov.

Speaker: **Yuan Xu** (University of Oregon)
Title: *On discrete orthogonal polynomials of several variables*
Abstract: For a given set V of isolated points in $\mathbb{R}^d$ and a positive function W on V, we consider discrete orthogonal polynomials with respect to the inner product $\langle f, g\rangle = \sum_{x\in V} f(x)g(x)W(x)$. Unlike the case of one variable, the polynomial subspace generated by the discrete orthogonal polynomials depends on the structure of V. We identify this polynomial subspace and show that the three-term relation and Favard's theorem hold for discrete orthogonal polynomials in this general setting.