8. Using (1),

$$m = \lim_{x\to 4} \frac{\sqrt{2x+1}-\sqrt{2(4)+1}}{x-4} = \lim_{x\to 4} \frac{\sqrt{2x+1}-3}{x-4} \cdot \frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3}$$

$$= \lim_{x\to 4} \frac{(2x+1)-3^2}{(x-4)(\sqrt{2x+1}+3)} = \lim_{x\to 4} \frac{2(x-4)}{(x-4)(\sqrt{2x+1}+3)}$$

$$= \lim_{x\to 4} \frac{2}{(\sqrt{2x+1}+3)} = \frac{2}{3+3} = \frac{1}{3}.$$

Tangent line: $y-3=\frac{1}{3}(x-4) \quad \Leftrightarrow \quad y-3=\frac{1}{3}x-\frac{4}{3} \quad \Leftrightarrow \quad y=\frac{1}{3}x+\frac{5}{3}$

9. Using (1) with $f(x)=\dfrac{x-1}{x-2}$ and $P(3,2)$,

$$m = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to 3} \frac{\frac{x-1}{x-2}-2}{x-3} = \lim_{x\to 3} \frac{x-1-2(x-2)}{x-2} = \lim_{x\to 3} \frac{3-x}{(x-2)(x-3)}$$

$$= \lim_{x\to 3} \frac{-1}{x-2} = \frac{-1}{1} = -1.$$

Tangent line: $y-2=-1(x-3) \quad \Leftrightarrow \quad y-2=-x+3 \quad \Leftrightarrow \quad y=-x+5$

10. Using (1), $m = \lim_{x\to 0} \dfrac{\frac{2x}{(x+1)^2}-0}{x-0} = \lim_{x\to 0} \dfrac{2x}{x(x+1)^2} = \lim_{x\to 0} \dfrac{2}{(x+1)^2} = \dfrac{2}{1^2} = 2.$

Tangent line: $y-0=2(x-0) \quad \Leftrightarrow \quad y=2x$

11. (a) $m = \lim_{x\to a} \dfrac{f(x)-f(a)}{x-a} = \lim_{x\to a} \dfrac{2/(x+3)-2/(a+3)}{x-a} = \lim_{x\to a} \dfrac{2(a+3)-2(x+3)}{(x-a)(x+3)(a+3)}$

$$= \lim_{x\to a} \frac{2(a-x)}{(x-a)(x+3)(a+3)} = \lim_{x\to a} \frac{-2}{(x+3)(a+3)} = \frac{-2}{(a+3)^2}$$

(b) (i) $a=-1 \quad \Rightarrow \quad m=\dfrac{-2}{(-1+3)^2}=-\dfrac{1}{2}$ (ii) $a=0 \quad \Rightarrow \quad m=\dfrac{-2}{(0+3)^2}=-\dfrac{2}{9}$

(iii) $a=1 \quad \Rightarrow \quad m=\dfrac{-2}{(1+3)^2}=-\dfrac{1}{8}$

12. (a) Using (1),

$$m = \lim_{x\to a} \frac{(1+x+x^2)-(1+a+a^2)}{x-a} = \lim_{x\to a} \frac{x+x^2-a-a^2}{x-a} = \lim_{x\to a} \frac{x-a+(x-a)(x+a)}{x-a}$$

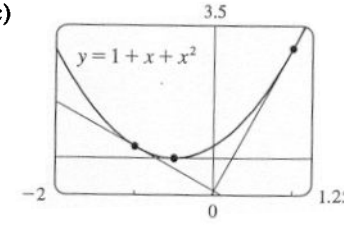
$$= \lim_{x\to a} \frac{(x-a)(1+x+a)}{x-a} = \lim_{x\to a} (1+x+a) = 1+2a$$

(b) (i) $x=-1 \quad \Rightarrow \quad m=1+2(-1)=-1$

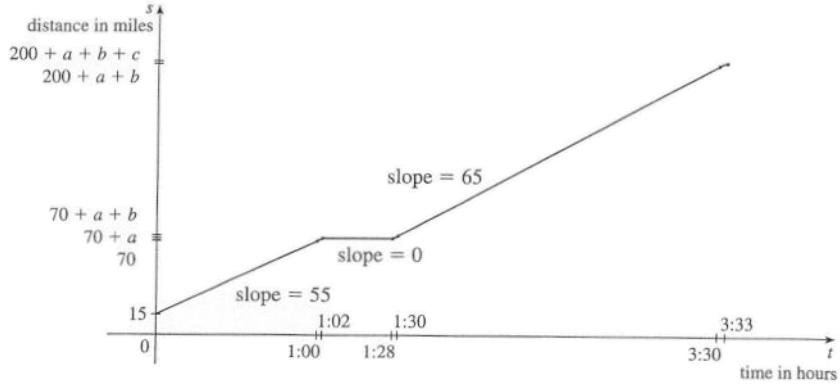
(ii) $x=-\frac{1}{2} \quad \Rightarrow \quad m=1+2\left(-\frac{1}{2}\right)=0$

(iii) $x=1 \quad \Rightarrow \quad m=1+2(1)=3$

(c)



16. Let a denote the distance traveled from 1:00 to 1:02, b from 1:28 to 1:30, and c from 3:30 to 3:33, where all the times are relative to $t = 0$ at the beginning of the trip.



17. Let $s(t) = 40t - 16t^2$.

$$v(2) = \lim_{t\to 2} \frac{s(t) - s(2)}{t - 2} = \lim_{t\to 2} \frac{(40t - 16t^2) - 16}{t - 2} = \lim_{t\to 2} \frac{-16t^2 + 40t - 16}{t - 2} = \lim_{t\to 2} \frac{-8(2t^2 - 5t + 2)}{t - 2}$$

$$= \lim_{t\to 2} \frac{-8(t - 2)(2t - 1)}{t - 2} = -8 \lim_{t\to 2} (2t - 1) = -8(3) = -24$$

Thus, the instantaneous velocity when $t = 2$ is -24 ft/s.

18. (a) $v(1) = \lim_{h\to 0} \dfrac{H(1 + h) - H(1)}{h}$

$$= \lim_{h\to 0} \frac{(58 + 58h - 0.83 - 1.66h - 0.83h^2) - 57.17}{h} = \lim_{h\to 0} (56.34 - 0.83h) = 56.34 \text{ m/s}$$

(b) $v(a) = \lim_{h\to 0} \dfrac{H(a + h) - H(a)}{h}$

$$= \lim_{h\to 0} \frac{(58a + 58h - 0.83a^2 - 1.66ah - 0.83h^2) - (58a - 0.83a^2)}{h}$$

$$= \lim_{h\to 0} (58 - 1.66a - 0.83h) = 58 - 1.66a \text{ m/s}$$

(c) The arrow strikes the moon when the height is 0, that is, $58t - 0.83t^2 = 0 \iff t(58 - 0.83t) = 0 \iff t = \dfrac{58}{0.83} \approx 69.9$ s (since t can't be 0).

(d) Using the time from part (c), $v\left(\dfrac{58}{0.83}\right) = 58 - 1.66\left(\dfrac{58}{0.83}\right) = -58$ m/s. Thus, the arrow will have a velocity of -58 m/s.

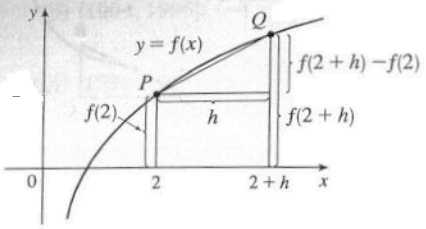
19. $v(a) = \lim_{h\to 0} \dfrac{s(a + h) - s(a)}{h} = \lim_{h\to 0} \dfrac{4(a + h)^3 + 6(a + h) + 2 - (4a^3 + 6a + 2)}{h}$

$$= \lim_{h\to 0} \frac{4a^3 + 12a^2h + 12ah^2 + 4h^3 + 6a + 6h + 2 - 4a^3 - 6a - 2}{h}$$

$$= \lim_{h\to 0} \frac{12a^2h + 12ah^2 + 4h^3 + 6h}{h} = \lim_{h\to 0} (12a^2 + 12ah + 4h^2 + 6) = (12a^2 + 6) \text{ m/s}$$

So $v(1) = 12(1)^2 + 6 = 18$ m/s, $v(2) = 12(2)^2 + 6 = 54$ m/s, and $v(3) = 12(3)^2 + 6 = 114$ m/s.

2.8 Derivatives

1. 

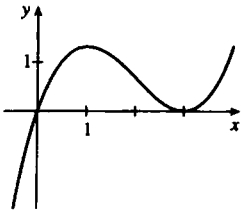
The line from $P(2, f(2))$ to $Q(2+h, f(2+h))$ is the line that has slope $\dfrac{f(2+h)-f(2)}{h}$.

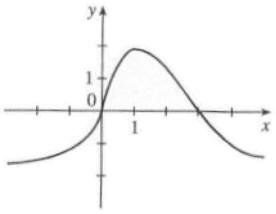
2. As h decreases, the line PQ becomes steeper, so its slope increases. So $0 < \dfrac{f(4)-f(2)}{4-2} < \dfrac{f(3)-f(2)}{3-2} < \lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}$. Thus, $0 < \frac{1}{2}[f(4)-f(2)] < f(3)-f(2) < f'(2)$.

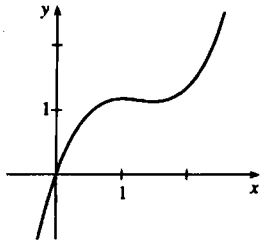
3. $g'(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$. Thus, $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$.

4. Since $(4,3)$ is on $y=f(x)$, $f(4)=3$. The slope of the tangent line between $(0,2)$ and $(4,3)$ is $\frac{1}{4}$, so $f'(4)=\frac{1}{4}$.

5. We begin by drawing a curve through the origin at a slope of 3 to satisfy $f(0)=0$ and $f'(0)=3$. Since $f'(1)=0$, we will round off our figure so that there is a horizontal tangent directly over $x=1$. Lastly, we make sure that the curve has a slope of -1 as we pass over $x=2$. Two of the many possibilities are shown.





6. 

7. Using Definition 2 with $f(x)=3x^2-5x$ and the point $(2,2)$, we have

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{[3(2+h)^2-5(2+h)]-2}{h}$$

$$=\lim_{h\to 0}\frac{(12+12h+3h^2-10-5h)-2}{h}=\lim_{h\to 0}\frac{3h^2+7h}{h}=\lim_{h\to 0}(3h+7)=7.$$

So an equation of the tangent line at $(2,2)$ is $y-2=7(x-2)$ or $y=7x-12$.

8. Using Definition 2 with $g(x)=1-x^3$ and the point $(0,1)$, we have

$$g'(0)=\lim_{h\to 0}\frac{g(0+h)-g(0)}{h}=\lim_{h\to 0}\frac{[1-(0+h)^3]-1}{h}=\lim_{h\to 0}\frac{(1-h^3)-1}{h}=\lim_{h\to 0}(-h^2)=0.$$

So an equation of the tangent line is $y-1=0(x-0)$ or $y=1$.

16. $f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \dfrac{\dfrac{(a+h)^2+1}{(a+h)-2} - \dfrac{a^2+1}{a-2}}{h}$

$$= \lim_{h\to 0} \frac{(a^2+2ah+h^2+1)(a-2)-(a^2+1)(a+h-2)}{h(a+h-2)(a-2)}$$

$$= \lim_{h\to 0} \frac{(a^3-2a^2+2a^2h-4ah+ah^2-2h^2+a-2)-(a^3+a^2h-2a^2+a+h-2)}{h(a+h-2)(a-2)}$$

$$= \lim_{h\to 0} \frac{a^2h-4ah+ah^2-2h^2-h}{h(a+h-2)(a-2)} = \lim_{h\to 0} \frac{h\,(a^2-4a+ah-2h-1)}{h(a+h-2)(a-2)}$$

$$= \lim_{h\to 0} \frac{a^2-4a+ah-2h-1}{(a+h-2)(a-2)} = \frac{a^2-4a-1}{(a-2)^2}$$

17. Use Definition 2 with $f(x) = 1/\sqrt{x+2}$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{(a+h)+2}} - \dfrac{1}{\sqrt{a+2}}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{\sqrt{a+2}-\sqrt{a+h+2}}{\sqrt{a+h+2}\sqrt{a+2}}}{h} = \lim_{h\to 0} \left[\frac{\sqrt{a+2}-\sqrt{a+h+2}}{h\sqrt{a+h+2}\sqrt{a+2}} \cdot \frac{\sqrt{a+2}+\sqrt{a+h+2}}{\sqrt{a+2}+\sqrt{a+h+2}}\right]$$

$$= \lim_{h\to 0} \frac{(a+2)-(a+h+2)}{h\sqrt{a+h+2}\sqrt{a+2}\,(\sqrt{a+2}+\sqrt{a+h+2})}$$

$$= \lim_{h\to 0} \frac{-h}{h\sqrt{a+h+2}\sqrt{a+2}\,(\sqrt{a+2}+\sqrt{a+h+2})}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{a+h+2}\sqrt{a+2}\,(\sqrt{a+2}+\sqrt{a+h+2})}$$

$$= \frac{-1}{(\sqrt{a+2})^2\,(2\sqrt{a+2})} = -\frac{1}{2\,(a+2)^{3/2}}$$

18. $f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \dfrac{\sqrt{3\,(a+h)+1}-\sqrt{3a+1}}{h}$

$$= \lim_{h\to 0} \frac{(\sqrt{3a+3h+1}-\sqrt{3a+1})(\sqrt{3a+3h+1}+\sqrt{3a+1})}{h\,(\sqrt{3a+3h+1}+\sqrt{3a+1})}$$

$$= \lim_{h\to 0} \frac{(3a+3h+1)-(3a+1)}{h\,(\sqrt{3a+3h+1}+\sqrt{3a+1})} = \lim_{h\to 0} \frac{3h}{h\,(\sqrt{3a+3h+1}+\sqrt{3a+1})}$$

$$= \lim_{h\to 0} \frac{3}{\sqrt{3a+3h+1}+\sqrt{3a+1}} = \frac{3}{2\sqrt{3a+1}}$$

Note that the answers to Exercises 19–24 are not unique.

19. By Definition 2, $\lim_{h\to 0} \dfrac{(1+h)^{10}-1}{h} = f'(1)$, where $f(x) = x^{10}$ and $a = 1$.

Or: By Definition 2, $\lim_{h\to 0} \dfrac{(1+h)^{10}-1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

20. By Definition 2, $\lim_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h} = f'(16)$, where $f(x) = \sqrt[4]{x}$ and $a = 16$.

Or: By Definition 2, $\lim_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h} = f'(0)$, where $f(x) = \sqrt[4]{16+x}$ and $a = 0$.

21. By Equation 3, $\lim_{x\to 5} \dfrac{2^x - 32}{x-5} = f'(5)$, where $f(x) = 2^x$ and $a = 5$.

22. By Equation 3, $\lim_{x\to \pi/4} \dfrac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.

23. By Definition 2, $\lim_{h\to 0} \dfrac{\cos(\pi+h)+1}{h} = f'(\pi)$, where $f(x) = \cos x$ and $a = \pi$.

Or: By Definition 2, $\lim_{h\to 0} \dfrac{\cos(\pi+h)+1}{h} = f'(0)$, where $f(x) = \cos(\pi + x)$ and $a = 0$.

24. By Equation 3, $\lim_{t\to 1} \dfrac{t^4+t-2}{t-1} = f'(1)$, where $f(t) = t^4 + t$ and $a = 1$.

25.
$$v(2) = f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\left[(2+h)^2 - 6(2+h) - 5\right] - \left[2^2 - 6(2) - 5\right]}{h}$$
$$= \lim_{h\to 0} \frac{\left(4+4h+h^2-12-6h-5\right)-(-13)}{h} = \lim_{h\to 0} \frac{h^2-2h}{h} = \lim_{h\to 0}(h-2) = -2 \text{ m/s}$$

26.
$$v(2) = f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$
$$= \lim_{h\to 0} \frac{\left[2(2+h)^3 - (2+h) + 1\right] - \left[2(2)^3 - 2 + 1\right]}{h}$$
$$= \lim_{h\to 0} \frac{\left(2h^3+12h^2+24h+16-2-h+1\right)-15}{h}$$
$$= \lim_{h\to 0} \frac{2h^3+12h^2+23h}{h} = \lim_{h\to 0}\left(2h^2+12h+23\right) = 23 \text{ m/s}$$

27. (a) $f'(x)$ is the rate of change of the production cost with respect to the number of ounces of gold produced. units are dollars per ounce.

(b) After 800 ounces of gold have been produced, the rate at which the production cost is increasing is \$17/ou So the cost of producing the 800th (or 801st) ounce is about \$17.

(c) In the short term, the values of $f'(x)$ will decrease because more efficient use is made of start-up costs as increases. But eventually $f'(x)$ might increase due to large-scale operations.

28. (a) $f'(5)$ is the rate of growth of the bacteria population when $t = 5$ hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

29. (a) $f'(v)$ is the rate at which the fuel consumption is changing with respect to the speed. Its units are (gal/h)/(mi/h).

(b) The fuel consumption is decreasing by 0.05 (gal/h)/(mi/h) as the car's speed reaches 20 mi/h. So if you increase your speed to 21 mi/h, you could expect to decrease your fuel consumption by about 0.05 (gal/h)/(mi/h).

36. Since $f(x) = x^2 \sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$f'(0) = \lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \dfrac{h^2 \sin(1/h) - 0}{h} = \lim_{h \to 0} h \sin(1/h)$. Since $-1 \leq \sin \dfrac{1}{h} \leq 1$, we hav

$-|h| \leq |h| \sin \dfrac{1}{h} \leq |h| \quad \Rightarrow \quad -|h| \leq h \sin \dfrac{1}{h} \leq |h|$. Because $\lim_{h \to 0} (-|h|) = 0$ and $\lim_{h \to 0} |h| = 0$, we know

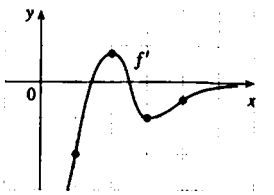
$\lim_{h \to 0} \left(h \sin \dfrac{1}{h} \right) = 0$ by the Squeeze Theorem. Thus, $f'(0) = 0$.

2.9 The Derivative as a Function

1. *Note:* Your answers may vary depending on your estimates. By estimating the slopes of tangent lines on the graph of f, it appears that

(a) $f'(1) \approx -2$ (b) $f'(2) \approx 0.8$

(c) $f'(3) \approx -1$ (d) $f'(4) \approx -0.5$

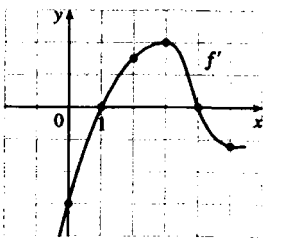


2. *Note:* Your answers may vary depending on your estimates. By estimating the slopes of tangent lines on the graph of f, it appears that

(a) $f'(0) \approx -3$ (b) $f'(1) \approx 0$

(c) $f'(2) \approx 1.5$ (d) $f'(3) \approx 2$

(e) $f'(4) \approx 0$ (f) $f'(5) \approx -1.2$



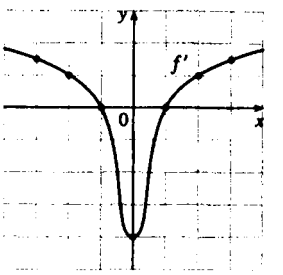
3. It appears that f is an odd function, so f' will be an even function—that is, $f'(-a) = f'(a)$.

(a) $f'(-3) \approx 1.5$ (b) $f'(-2) \approx 1$

(c) $f'(-1) \approx 0$ (d) $f'(0) \approx -4$

(e) $f'(1) \approx 0$ (f) $f'(2) \approx 1$

(g) $f'(3) \approx 1.5$



4. (a)′ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then posit then 0, then negative again. The actual function values in graph II follow the same pattern.

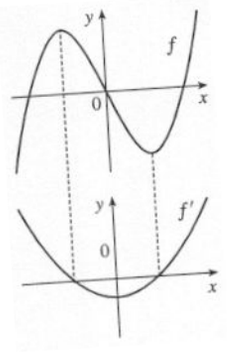
(b)′ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, th suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in slopes of the tangents.

(c)′ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)′ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, the then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

Hints for Exercises 5–13: First plot x-intercepts on the graph of f' for any horizontal tangents on the graph of f. Look for any negative) slope, the graph of f — there will be a discon negative). he graph of f'. On any interval where f has a tangent with positive (or nega the graph

5.



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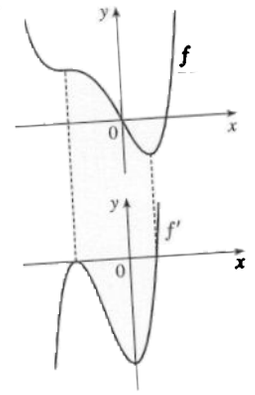

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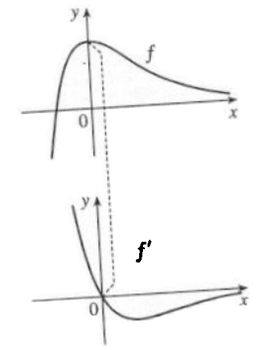

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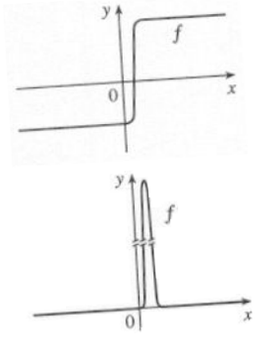

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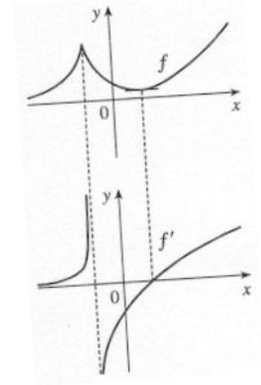

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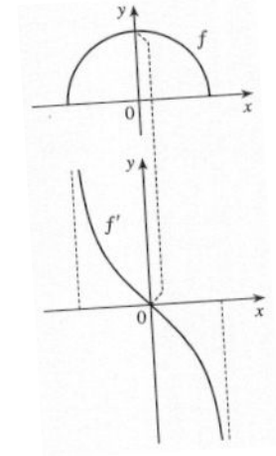

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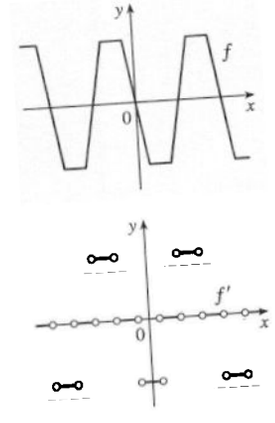

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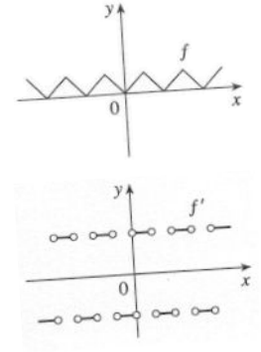
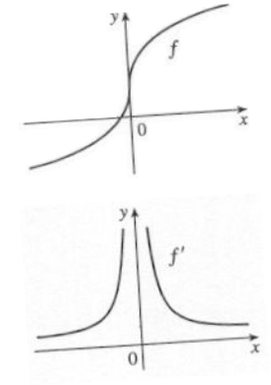


13.



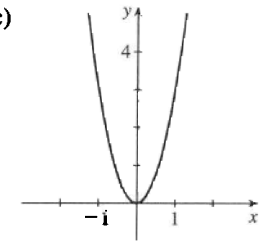
(d) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$

$= \lim_{h\to 0} \dfrac{(x^2+2hx+h^2)-x^2}{h} = \lim_{h\to 0} \dfrac{2hx+h^2}{h} = \lim_{h\to 0} \dfrac{h(2x+h)}{h} = \lim_{h\to 0}(2x+h) = 2x$

20. (a) By zooming in, we estimate that $f'(0) = 0$, $f'\left(\frac{1}{2}\right) \approx 0.75$, $f'(1) \approx 3$, $f'(2) \approx 12$, and $f'(3) \approx 27$.

(b) By symmetry, $f'(-x) = f'(x)$. So $f'\left(-\frac{1}{2}\right) \approx 0.75$, $f'(-1) \approx 3$, $f'(-2) \approx 12$, and $f'(-3) \approx 27$.

(c)



(d) Since $f'(0) = 0$, it appears that f' may have the form $f'(x) = ax^2$. Using $f'(1) = 3$, we have $a = 3$, so $f'(x) = 3x^2$.

(e) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \dfrac{(x^3+3x^2h+3xh^2+h^3)-x^3}{h}$

$= \lim_{h\to 0} \dfrac{3x^2h+3xh^2+h^3}{h} = \lim_{h\to 0} \dfrac{h\left(3x^2+3xh+h^2\right)}{h} = \lim_{h\to 0}\left(3x^2+3xh+h^2\right) = 3x^2$

21. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{37-37}{h} = \lim_{h\to 0} \dfrac{0}{h} = \lim_{h\to 0} 0 = 0$

Domain of f = domain of f' = $\mathbb{R}$.

22. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{[12+7(x+h)]-(12+7x)}{h}$

$= \lim_{h\to 0} \dfrac{12+7x+7h-12-7x}{h} = \lim_{h\to 0} \dfrac{7h}{h} = \lim_{h\to 0} 7 = 7$

Domain of f = domain of f' = $\mathbb{R}$.

23. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{[1-3(x+h)^2]-(1-3x^2)}{h}$

$= \lim_{h\to 0} \dfrac{[1-3(x^2+2xh+h^2)]-(1-3x^2)}{h} = \lim_{h\to 0} \dfrac{1-3x^2-6xh-3h^2-1+3x^2}{h}$

$= \lim_{h\to 0} \dfrac{-6xh-3h^2}{h} = \lim_{h\to 0} \dfrac{h(-6x-3h)}{h} = \lim_{h\to 0}(-6x-3h) = -6x$

Domain of f = domain of f' = $\mathbb{R}$.

24. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{[5(x+h)^2+3(x+h)-2]-(5x^2+3x-2)}{h}$

$= \lim_{h\to 0} \dfrac{5x^2+10xh+5h^2+3x+3h-2-5x^2-3x+2}{h} = \lim_{h\to 0} \dfrac{10xh+5h^2+3h}{h}$

$= \lim_{h\to 0} \dfrac{h(10x+5h+3)}{h} = \lim_{h\to 0}(10x+5h+3) = 10x+3$

Domain of f = domain of f' = $\mathbb{R}$.

3 ❑ DIFFERENTIATION RULES

3.1 Derivatives of Polynomials and Exponential Functions

1. (a) e is the number such that $\lim_{h \to 0} \dfrac{e^h - 1}{h} = 1$.

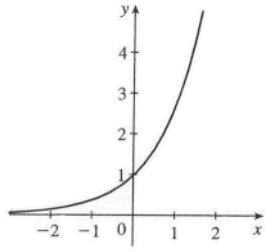
(b)

x	$(2.7^x - 1)/x$
−0.001	0.9928
−0.0001	0.9932
0.001	0.9937
0.0001	0.9933

x	$(2.8^x - 1)/x$
−0.001	1.0291
−0.0001	1.0296
0.001	1.0301
0.0001	1.0297

From the tables (to two decimal places), $\lim_{h \to 0} \dfrac{2.7^h - 1}{h} = 0.99$ and $\lim_{h \to 0} \dfrac{2.8^h - 1}{h} = 1.03$. Since $0.99 < 1 < 1.03$, $2.7 < e < 2.8$.

2. (a)



The function value at $x = 0$ is 1 and the slope at $x = 0$ is 1.

(b) $f(x) = e^x$ is an exponential function and $g(x) = x^e$ is a power function. $\dfrac{d}{dx}(e^x) = e^x$ and $\dfrac{d}{dx}(x^e) = ex^{e-1}$

(c) $f(x) = e^x$ grows more rapidly than $g(x) = x^e$ when x is large.

3. $f(x) = 186.5$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

4. $f(x) = \sqrt{30}$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

5. $f(x) = 5x - 1 \;\Rightarrow\; f'(x) = 5 - 0 = 5$

6. $F(x) = -4x^{10} \;\Rightarrow\; F'(x) = -4\left(10x^{10-1}\right) = -40x^9$

7. $f(x) = x^2 + 3x - 4 \;\Rightarrow\; f'(x) = 2x^{2-1} + 3 - 0 = 2x + 3$

8. $g(x) = 5x^8 - 2x^5 + 6 \;\Rightarrow\; g'(x) = 5\left(8x^{8-1}\right) - 2\left(5x^{5-1}\right) + 0 = 40x^7 - 10x^4$

9. $f(t) = \frac{1}{4}(t^4 + 8) \;\Rightarrow\; f'(t) = \frac{1}{4}(t^4 + 8)' = \frac{1}{4}(4t^{4-1} + 0) = t^3$

10. $f(t) = \frac{1}{2}t^6 - 3t^4 + t \;\Rightarrow\; f'(t) = \frac{1}{2}(6t^5) - 3(4t^3) + 1 = 3t^5 - 12t^3 + 1$

11. $y = x^{-2/5} \;\Rightarrow\; y' = -\frac{2}{5}x^{(-2/5)-1} = -\frac{2}{5}x^{-7/5} = -\dfrac{2}{5x^{7/5}}$

12. $y = 5e^x + 3 \;\Rightarrow\; y' = 5(e^x) + 0 = 5e^x$

13. $V(r) = \frac{4}{3}\pi r^3 \;\Rightarrow\; V'(r) = \frac{4}{3}\pi(3r^2) = 4\pi r^2$

14. $R(t) = 5t^{-3/5} \;\Rightarrow\; R'(t) = 5\left[-\frac{3}{5}t^{(-3/5)-1}\right] = -3t^{-8/5}$

15. $Y(t) = 6t^{-9} \Rightarrow Y'(t) = 6(-9)t^{-10} = -54t^{-10}$

16. $R(x) = \dfrac{\sqrt{10}}{x^7} = \sqrt{10}\,x^{-7} \Rightarrow R'(x) = -7\sqrt{10}\,x^{-8} = -\dfrac{7\sqrt{10}}{x^8}$

17. $G(x) = \sqrt{x} - 2e^x = x^{1/2} - 2e^x \Rightarrow G'(x) = \frac{1}{2}x^{-1/2} - 2e^x = \dfrac{1}{2\sqrt{x}} - 2e^x$

18. $y = \sqrt[3]{x} = x^{1/3} \Rightarrow y' = \frac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

19. $F(x) = (\frac{1}{2}x)^5 = (\frac{1}{2})^5 x^5 = \frac{1}{32}x^5 \Rightarrow F'(x) = \frac{1}{32}(5x^4) = \frac{5}{32}x^4$

20. $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2} \Rightarrow f'(t) = \frac{1}{2}t^{-1/2} - \left(-\frac{1}{2}t^{-3/2}\right) = \dfrac{1}{2\sqrt{t}} + \dfrac{1}{2t\sqrt{t}}$

21. $g(x) = x^2 + \dfrac{1}{x^2} = x^2 + x^{-2} \Rightarrow g'(x) = 2x + (-2)x^{-3} = 2x - \dfrac{2}{x^3}$

22. $y = \sqrt{x}\,(x-1) = x^{3/2} - x^{1/2} \Rightarrow y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}(3x-1)$ [factor out $\frac{1}{2}x^{-1/2}$] or $y' = \dfrac{3x-1}{2\sqrt{x}}$.

23. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2} \Rightarrow$

$y' = \frac{3}{2}x^{1/2} + 4(\frac{1}{2})x^{-1/2} + 3(-\frac{1}{2})x^{-3/2} = \frac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x\sqrt{x}}$

[note that $x^{3/2} = x^{2/2} \cdot x^{1/2} = x\sqrt{x}$]

24. $y = \dfrac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2} \Rightarrow y' = 1 - 2(-\frac{1}{2})x^{-3/2} = 1 + 1/(x\sqrt{x})$

25. $y = 4\pi^2 \Rightarrow y' = 0$ since $4\pi^2$ is a constant.

26. $g(u) = \sqrt{2}\,u + \sqrt{3u} = \sqrt{2}\,u + \sqrt{3}\,\sqrt{u} \Rightarrow g'(u) = \sqrt{2}\,(1) + \sqrt{3}\left(\frac{1}{2}u^{-1/2}\right) = \sqrt{2} + \sqrt{3}/(2\sqrt{u})$

27. $y = ax^2 + bx + c \Rightarrow y' = 2ax + b$

28. $y = ae^v + \dfrac{b}{v} + \dfrac{c}{v^2} = ae^v + bv^{-1} + cv^{-2} \Rightarrow y' = ae^v - bv^{-2} - 2cv^{-3} = ae^v - \dfrac{b}{v^2} - \dfrac{2c}{v^3}$

29. $v = t^2 - \dfrac{1}{\sqrt[4]{t^3}} = t^2 - t^{-3/4} \Rightarrow v' = 2t - \left(-\frac{3}{4}\right)t^{-7/4} = 2t + \dfrac{3}{4t^{7/4}} = 2t + \dfrac{3}{4t\sqrt[4]{t^3}}$

30. $u = \sqrt[3]{t^2} + 2\sqrt{t^3} = t^{2/3} + 2t^{3/2} \Rightarrow u' = \frac{2}{3}t^{-1/3} + 2\left(\frac{3}{2}\right)t^{1/2} = \dfrac{2}{3\sqrt[3]{t}} + 3\sqrt{t}$

31. $z = \dfrac{A}{y^{10}} + Be^y = Ay^{-10} + Be^y \Rightarrow z' = -10Ay^{-11} + Be^y = -\dfrac{10A}{y^{11}} + Be^y$

32. $y = e^{x+1} + 1 = e^x e^1 + 1 = e \cdot e^x + 1 \Rightarrow y' = e \cdot e^x = e^{x+1}$