

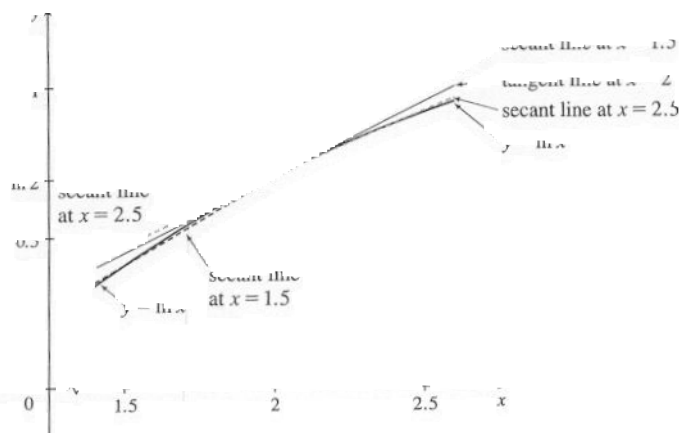
4. For the curve  $y = \ln x$  and the point  $P(2, \ln 2)$ :

(a)

	$x$	$Q$	$m_{PQ}$
(i)	1.5	(1.5, 0.405465)	0.575364
(ii)	1.9	(1.9, 0.641854)	0.512933
(iii)	1.99	(1.99, 0.688135)	0.501254
(iv)	1.999	(1.999, 0.692647)	0.500125
(v)	2.5	(2.5, 0.916291)	0.446287
(vi)	2.1	(2.1, 0.741937)	0.487902
(vii)	2.01	(2.01, 0.698135)	0.498754
(viii)	2.001	(2.001, 0.693647)	0.499875

(b) The slope appears to be  $\frac{1}{2}$ .(c)  $y - \ln 2 = \frac{1}{2}(x - 2)$  or  
 $y = \frac{1}{2}x - 1 + \ln 2$ 

(d)



5. (a)  $y = y(t) = 40t - 16t^2$ . At  $t = 2$ ,  $y = 40(2) - 16(2)^2 = 16$ . The average velocity between times 2 and  $2 + h$  is  $v_{\text{ave}} = \frac{y(2+h) - y(2)}{(2+h) - 2} = \frac{[40(2+h) - 16(2+h)^2] - 16}{h} = \frac{-24h - 16h^2}{h} = -24 - 16h$ , if  $h \neq 0$ .

(i)  $[2, 2.5]: h = 0.5, v_{\text{ave}} = -32$  ft/s      (ii)  $[2, 2.1]: h = 0.1, v_{\text{ave}} = -25.6$  ft/s(iii)  $[2, 2.05]: h = 0.05, v_{\text{ave}} = -24.8$  ft/s      (iv)  $[2, 2.01]: h = 0.01, v_{\text{ave}} = -24.16$  ft/s(b) The instantaneous velocity when  $t = 2$  ( $h$  approaches 0) is  $-24$  ft/s.6. The average velocity between  $t$  and  $t + h$  seconds is

$$\frac{58(t+h) - 0.83(t+h)^2 - (58t - 0.83t^2)}{h} = \frac{58h - 1.66th - 0.83h^2}{h} = 58 - 1.66t - 0.83h \text{ if } h \neq 0.$$

(a) Here  $t = 1$ , so the average velocity is  $58 - 1.66 - 0.83h = 56.34 - 0.83h$ .(i)  $[1, 2]: h = 1, 55.51$  m/s(ii)  $[1, 1.5]: h = 0.5, 55.925$  m/s(iii)  $[1, 1.1]: h = 0.1, 56.257$  m/s(iv)  $[1, 1.01]: h = 0.01, 56.3317$  m/s(v)  $[1, 1.001]: h = 0.001, 56.33917$  m/s(b) The instantaneous velocity after 1 second is  $56.34$  m/s.



(d)  $\lim_{t \rightarrow 2^-} y(t) = 2$

(e)  $\lim_{t \rightarrow 2^+} g(t) = 0$

(f)  $\lim_{t \rightarrow 2} g(t)$  does not exist because the limits in part (d) and part (e) are not equal

(g)  $g(2) =$

(h)  $\lim_{t \rightarrow 4} g(t) = 3$

8. (a)  $\lim_{x \rightarrow 2} h(x) = \infty$

(b)  $\lim_{x \rightarrow 5} R(x) = \infty$

(c)  $\lim_{x \rightarrow -3^-} R(x) = -\infty$

(d)  $\lim_{x \rightarrow -3^+} h(x) = \infty$

(e) The equations of the vertical asymptotes are  $x = -3$ ,  $x = 2$ , and  $x = 5$ .

9. (a)  $\lim_{x \rightarrow -1} f(x) = \infty$

(b)  $\lim_{x \rightarrow -3} f(x) = \infty$

(c)  $\lim_{x \rightarrow 0} f(x) = \infty$

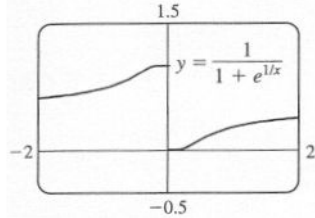
(d)  $\lim_{x \rightarrow 6^-} f(x) = \infty$

(e)  $\lim_{x \rightarrow 6^+} f(x) = \infty$

(f) The equations of the vertical asymptotes are  $x = -7$ ,  $x = -3$ ,  $x = 0$ , and  $x = 6$ .

10.  $\lim_{t \rightarrow 12^-} f(t) = 150$  mg and  $\lim_{t \rightarrow 12^+} f(t) = 300$  mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at  $t = 12$  h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

11.

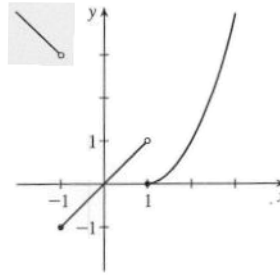


(a)  $\lim_{x \rightarrow 0^-} f(x) = 1$

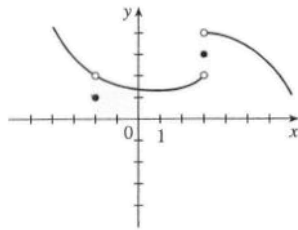
(b)  $\lim_{x \rightarrow 0^+} f(x) = 0$

(c)  $\lim_{x \rightarrow 0} f(x)$  does not exist because the limits in part (a) and part (b) are not equal.

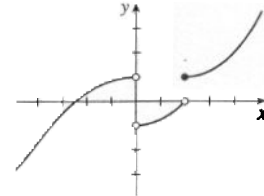
12.  $\lim_{x \rightarrow a} f(x)$  exists for all  $a$  except  $a = \pm 1$ .



13.  $\lim_{x \rightarrow 3^+} f(x) = 4$ ,  $\lim_{x \rightarrow 3^-} f(x) = 2$ ,  
 $\lim_{x \rightarrow -2} f(x) = 2$ ,  $f(3) = 3$ ,  $f(-2) = 1$



14.  $\lim_{x \rightarrow 0^-} f(x) = 1$ ,  $\lim_{x \rightarrow 0^+} f(x) = -1$ ,  
 $\lim_{x \rightarrow 2^-} f(x) = 0$ ,  $\lim_{x \rightarrow 2^+} f(x) = 1$ ,  $f(2) = 1$ ,  
 $f(0)$  is undefined



15. For  $f(x) = \frac{x^2 - 2x}{x^2 - x - 2}$ :

$x$	$f(x)$	$x$	$f(x)$
2.5	0.714286	1.9	0.655172
2.1	0.677419	1.95	0.661017
2.05	0.672131	1.99	0.665552
2.01	0.667774	1.995	0.666110
2.005	0.667221	1.999	0.666556
2.001	0.666778		

It appears that  $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - x - 2} = 0.\bar{6} = \frac{2}{3}$ .

17. For  $f(x) = \frac{e^x - 1 - x}{x^2}$ :

$x$	$f(x)$	$x$	$f(x)$
1	0.718282	-1	0.367879
0.5	0.594885	-0.5	0.426123
0.1	0.517092	-0.1	0.483742
0.05	0.508439	-0.05	0.491770
0.01	0.501671	-0.01	0.498337

It appears that  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = 0.5 = \frac{1}{2}$ .

19. For  $f(x) = \frac{\sqrt{x+4} - 2}{x}$ :

$x$	$f(x)$	$x$	$f(x)$
1	0.236068	-1	0.267949
0.5	0.242641	-0.5	0.258343
0.1	0.248457	-0.1	0.251582
0.05	0.249224	-0.05	0.250786
0.01	0.249844	-0.01	0.250156

It appears that  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} = 0.25 = \frac{1}{4}$ .

16. For  $f(x) = \frac{x^2 - 2x}{x^2 - x - 2}$ :

$x$	$f(x)$	$x$	$f(x)$
0	0	-2	2
-0.5	-1	-1.5	3
-0.9	-9	-1.1	11
-0.95	-19	-1.01	101
-0.99	-99	-1.001	1001
-0.999	-999		

It appears that  $\lim_{x \rightarrow -1} \frac{x^2 - 2x}{x^2 - x - 2}$  does not exist

since  $f(x) \rightarrow -\infty$  as  $x \rightarrow -1^-$  and  $f(x) \rightarrow \infty$  as  $x \rightarrow -1^+$ .

18. For  $f(x) = x \ln(x + x^2)$ :

$x$	$f(x)$
1	0.693147
0.5	-0.143841
0.1	-0.220727
0.05	-0.147347
0.01	-0.045952
0.005	-0.026467
0.001	-0.006907

It appears that  $\lim_{x \rightarrow 0^+} x \ln(x + x^2) = 0$ .

20. For  $f(x) = \frac{\tan 3x}{\tan 5x}$ :

$x$	$f(x)$
$\pm 0.2$	0.439279
$\pm 0.1$	0.566236
$\pm 0.05$	0.591893
$\pm 0.01$	0.599680
$\pm 0.001$	0.599997

It appears that  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x} = 0.6 = \frac{3}{5}$ .

1. For  $f(x) = \frac{x^6 - 1}{x^{10} - 1}$ :

$x$	$f(x)$	$x$	$f(x)$
0.5	0.985337	1.5	0.183369
0.9	0.719397	1.1	0.484119
0.95	0.660186	1.05	0.540783
0.99	0.612018	1.01	0.588022
0.999	0.601200	1.001	0.598800

It appears that  $\lim_{x \rightarrow 1} \frac{x^6 - 1}{x^{10} - 1} = 0.6 = \frac{3}{5}$ .

22. For  $f(x) = \frac{9^x - 5^x}{x}$ :

$x$	$f(x)$	$x$	$f(x)$
0.5	1.527864	-0.5	0.227761
0.1	0.711120	-0.1	0.485984
0.05	0.646496	-0.05	0.534447
0.01	0.599082	-0.01	0.576706
0.001	0.588906	-0.001	0.586669

It appears that  $\lim_{x \rightarrow 0} \frac{9^x - 5^x}{x} = 0.59$ . Later we will be able to show that the exact value is  $\ln(9/5)$ .

23.  $\lim_{x \rightarrow 5^+} \frac{6}{x-5} = \infty$  since  $(x-5) \rightarrow 0$  as  $x \rightarrow 5^+$  and  $\frac{6}{x-5} > 0$  for  $x > 5$ .

24.  $\lim_{x \rightarrow 5^-} \frac{6}{x-5} = -\infty$  since  $(x-5) \rightarrow 0$  as  $x \rightarrow 5^-$  and  $\frac{6}{x-5} < 0$  for  $x < 5$ .

25.  $\lim_{x \rightarrow 1} \frac{2-x}{(x-1)^2} = \infty$  since the numerator is positive and the denominator approaches 0 through positive values as  $x \rightarrow 1$ .

26.  $\lim_{x \rightarrow 0} \frac{x-1}{x^2(x+2)} = -\infty$  since  $x^2 \rightarrow 0$  as  $x \rightarrow 0$  and  $\frac{x-1}{x^2(x+2)} < 0$  for  $0 < x < 1$  and for  $-2 < x < 0$ .

27.  $\lim_{x \rightarrow -2^+} \frac{x-1}{x^2(x+2)} = -\infty$  since  $(x+2) \rightarrow 0$  as  $x \rightarrow -2^+$  and  $\frac{x-1}{x^2(x+2)} < 0$  for  $-2 < x < 0$ .

28.  $\lim_{x \rightarrow \pi^-} \csc x = \lim_{x \rightarrow \pi^-} (1/\sin x) = \infty$  since  $\sin x \rightarrow 0$  as  $x \rightarrow \pi^-$  and  $\sin x > 0$  for  $0 < x < \pi$ .

29.  $\lim_{x \rightarrow (-\pi/2)^-} \sec x = \lim_{x \rightarrow (-\pi/2)^-} (1/\cos x) = -\infty$  since  $\cos x \rightarrow 0$  as  $x \rightarrow (-\pi/2)^-$  and  $\cos x < 0$  for  $-\pi < x < -\pi/2$ .

30.  $\lim_{x \rightarrow 5^+} \ln(x-5) = -\infty$  since  $x-5 \rightarrow 0^+$  as  $x \rightarrow 5^+$ .

31. (a)  $f(x) = 1/(x^3 - 1)$

$x$	$f(x)$
0.5	-1.14
0.9	-3.69
0.99	-33.7
0.999	-333.7
0.9999	-3333.7
0.99999	-33,333.7

$x$	$f(x)$
1.5	0.42
1.1	3.02
1.01	33.0
1.001	333.0
1.0001	3333.0
1.00001	33,333.3

From these calculations, it seems that  $\lim_{x \rightarrow 1^-} f(x) = -\infty$  and  $\lim_{x \rightarrow 1^+} f(x) = \infty$ .

## 2.3 Calculating Limits: Using the Limit Laws

$$1. (a) \lim_{x \rightarrow a} [f(x) + h(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} h(x) \\ = -3 + 8 = 5$$

$$(c) \lim_{x \rightarrow a} \sqrt[3]{h(x)} = \sqrt[3]{\lim_{x \rightarrow a} h(x)} = \sqrt[3]{8} = 2$$

$$(e) \lim_{x \rightarrow a} \frac{f(x)}{h(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} h(x)} = \frac{-3}{8} = -\frac{3}{8}$$

(g) The limit does not exist, since  $\lim_{x \rightarrow a} g(x) = 0$  but  $\lim_{x \rightarrow a} f(x) \neq 0$ .

$$(h) \lim_{x \rightarrow a} \frac{2f(x)}{h(x) - f(x)} = \frac{2 \lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} h(x) - \lim_{x \rightarrow a} f(x)} = \frac{2(-3)}{8 - (-3)} = -\frac{1}{1}$$

$$(b) \lim_{x \rightarrow a} [f(x)]^2 = [\lim_{x \rightarrow a} f(x)]^2 = (-3)^2 = 9$$

$$(d) \lim_{x \rightarrow a} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow a} f(x)} = \frac{1}{-3} = -\frac{1}{3}$$

$$(f) \lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{\lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} f(x)} = \frac{1}{-3} = -\frac{1}{3}$$

$$2. (a) \lim_{x \rightarrow 2} [f(x) + g(x)] = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 2 + 0 = 2$$

(b)  $\lim_{x \rightarrow 1} g(x)$  does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

$$(c) \lim_{x \rightarrow 0} [f(x)g(x)] = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) = 0 \cdot 1.3 = 0$$

(d) Since  $\lim_{x \rightarrow -1} g(x) = 0$  and  $g$  is in the denominator, but  $\lim_{x \rightarrow -1} f(x) = -1 \neq 0$ , the given limit does not exist.

$$(e) \lim_{x \rightarrow 0} x^3 f(x) = [\lim_{x \rightarrow 0} x^3] [\lim_{x \rightarrow 0} f(x)] = 2^3 \cdot 2 = 16$$

$$(f) \lim_{x \rightarrow 1} \sqrt{3 + f(x)} = \sqrt{3 + \lim_{x \rightarrow 1} f(x)} = \sqrt{3 + 1} = 2$$

$$3. \lim_{x \rightarrow -2} (3x^4 + 2x^2 - x + 1) = \lim_{x \rightarrow -2} 3x^4 + \lim_{x \rightarrow -2} 2x^2 - \lim_{x \rightarrow -2} x + \lim_{x \rightarrow -2} 1 \\ = 3 \lim_{x \rightarrow -2} x^4 + 2 \lim_{x \rightarrow -2} x^2 - \lim_{x \rightarrow -2} x + \lim_{x \rightarrow -2} 1 \\ = 3(-2)^4 + 2(-2)^2 - (-2) + 1 \\ = 48 + 8 + 2 + 1 = 59$$

[Limit Laws 1 and 2]

[3]

[9, 8, and 7]

$$4. \lim_{x \rightarrow 2} \frac{2x^2 + 1}{x^2 + 6x - 4} = \frac{\lim_{x \rightarrow 2} (2x^2 + 1)}{\lim_{x \rightarrow 2} (x^2 + 6x - 4)}$$

[Limit Law 5]

$$= \frac{2 \lim_{x \rightarrow 2} x^2 + \lim_{x \rightarrow 2} 1}{\lim_{x \rightarrow 2} x^2 + 6 \lim_{x \rightarrow 2} x - \lim_{x \rightarrow 2} 4}$$

[2, 1, and 3]

$$= \frac{2(2)^2 + 1}{(2)^2 + 6(2) - 4} = \frac{9}{12} = \frac{3}{4}$$

[9, 7, and 8]

$$5. \lim_{x \rightarrow 3} (x^2 - 4)(x^3 + 5x - 1) = \lim_{x \rightarrow 3} (x^2 - 4) \cdot \lim_{x \rightarrow 3} (x^3 + 5x - 1)$$

[Limit Law 4]

$$= \left( \lim_{x \rightarrow 3} x^2 - \lim_{x \rightarrow 3} 4 \right) \cdot \left( \lim_{x \rightarrow 3} x^3 + 5 \lim_{x \rightarrow 3} x - \lim_{x \rightarrow 3} 1 \right)$$

[2, 1, and 3]

$$= (3^2 - 4) \cdot (3^3 + 5 \cdot 3 - 1)$$

[7, 8, and 9]

$$= 5 \cdot 41 = 205$$

$$\begin{aligned}
 6. \lim_{t \rightarrow -1} (t^2 + 1)^3 (t + 3)^5 &= \lim_{t \rightarrow -1} (t^2 + 1)^3 \cdot \lim_{t \rightarrow -1} (t + 3)^5 && \text{[Limit Law 4]} \\
 &= \left[ \lim_{t \rightarrow -1} (t^2 + 1) \right]^3 \cdot \left[ \lim_{t \rightarrow -1} (t + 3) \right]^5 && [6] \\
 &= \left[ \lim_{t \rightarrow -1} t^2 + \lim_{t \rightarrow -1} 1 \right]^3 \cdot \left[ \lim_{t \rightarrow -1} t + \lim_{t \rightarrow -1} 3 \right]^5 && [1] \\
 &= [(-1)^2 + 1]^3 \cdot [-1 + 3]^5 = 8 \cdot 32 = 256 && [9, 7, \text{ and } 8]
 \end{aligned}$$

$$\begin{aligned}
 7. \lim_{x \rightarrow 1} \left( \frac{1 + 3x}{1 + 4x^2 + 3x^4} \right)^3 &= \left( \lim_{x \rightarrow 1} \frac{1 + 3x}{1 + 4x^2 + 3x^4} \right)^3 && [6] \\
 &= \left[ \frac{\lim_{x \rightarrow 1} (1 + 3x)}{\lim_{x \rightarrow 1} (1 + 4x^2 + 3x^4)} \right]^3 && [5] \\
 &= \left[ \frac{\lim_{x \rightarrow 1} 1 + 3 \lim_{x \rightarrow 1} x}{\lim_{x \rightarrow 1} 1 + 4 \lim_{x \rightarrow 1} x^2 + 3 \lim_{x \rightarrow 1} x^4} \right]^3 && [2, 1, \text{ and } 3] \\
 &= \left[ \frac{1 + 3(1)}{1 + 4(1)^2 + 3(1)^4} \right]^3 = \left[ \frac{4}{8} \right]^3 = \left( \frac{1}{2} \right)^3 = \frac{1}{8} && [7, 8, \text{ and } 9]
 \end{aligned}$$

$$\begin{aligned}
 8. \lim_{u \rightarrow -2} \sqrt{u^4 + 3u + 6} &= \sqrt{\lim_{u \rightarrow -2} (u^4 + 3u + 6)} && [11] \\
 &= \sqrt{\lim_{u \rightarrow -2} u^4 + 3 \lim_{u \rightarrow -2} u + \lim_{u \rightarrow -2} 6} && [1, 2, \text{ and } 3] \\
 &= \sqrt{(-2)^4 + 3(-2) + 6} && [9, 8, \text{ and } 7] \\
 &= \sqrt{16 - 6 + 6} = \sqrt{16} = 4
 \end{aligned}$$

$$9. \lim_{x \rightarrow 4^-} \sqrt{16 - x^2} = \sqrt{\lim_{x \rightarrow 4^-} (16 - x^2)} \quad [11]$$

$$= \sqrt{\lim_{x \rightarrow 4^-} 16 - \lim_{x \rightarrow 4^-} x^2} \quad [2]$$

$$= \sqrt{16 - (4)^2} = 0 \quad [7 \text{ and } 9]$$

10. (a) The left-hand side of the equation is not defined for  $x = 2$ , but the right-hand side is.

(b) Since the equation holds for all  $x \neq 2$ , it follows that both sides of the equation approach the same limit as  $x \rightarrow 2$ , just as in Example 3. Remember that in finding  $\lim_{x \rightarrow a} f(x)$ , we never consider  $x = a$ .

$$11. \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x + 3)(x - 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 3) = 2 + 3 = 5$$

$$12. \lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4} = \lim_{x \rightarrow -4} \frac{(x + 4)(x + 1)}{(x + 4)(x - 1)} = \lim_{x \rightarrow -4} \frac{x + 1}{x - 1} = \frac{-4 + 1}{-4 - 1} = \frac{-3}{-5} = \frac{3}{5}$$

$$13. \lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x - 2} \text{ does not exist since } x - 2 \rightarrow 0 \text{ but } x^2 - x + 6 \rightarrow 8 \text{ as } x \rightarrow 2.$$

$$14. \lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \rightarrow 4} \frac{x(x - 4)}{(x - 4)(x + 1)} = \lim_{x \rightarrow 4} \frac{x}{x + 1} = \frac{4}{4 + 1} = \frac{4}{5}$$

$$15. \lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3} = \lim_{t \rightarrow -3} \frac{(t + 3)(t - 3)}{(2t + 1)(t + 3)} = \lim_{t \rightarrow -3} \frac{t - 3}{2t + 1} = \frac{-3 - 3}{2(-3) + 1} = \frac{-6}{-5} = \frac{6}{5}$$

$$16. \lim_{x \rightarrow -1} \frac{x^2 - 4x}{x^2 - 3x - 4} \text{ does not exist since } x^2 - 3x - 4 \rightarrow 0 \text{ but } x^2 - 4x \rightarrow 5 \text{ as } x \rightarrow -1.$$

$$17. \lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h} = \lim_{h \rightarrow 0} \frac{(16 + 8h + h^2) - 16}{h} = \lim_{h \rightarrow 0} \frac{8h + h^2}{h} = \lim_{h \rightarrow 0} \frac{h(8+h)}{h} = \lim_{h \rightarrow 0} (8+h) = 8+0 = 8$$

$$18. \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x+1} = \frac{1^2 + 1 + 1}{1+1} = \frac{3}{2}$$

$$19. \lim_{h \rightarrow 0} \frac{(1+h)^4 - 1}{h} = \lim_{h \rightarrow 0} \frac{(1 + 4h + 6h^2 + 4h^3 + h^4) - 1}{h} = \lim_{h \rightarrow 0} \frac{4h + 6h^2 + 4h^3 + h^4}{h} \\ = \lim_{h \rightarrow 0} \frac{h(4 + 6h + 4h^2 + h^3)}{h} = \lim_{h \rightarrow 0} (4 + 6h + 4h^2 + h^3) = 4 + 0 + 0 + 0 = 4$$

$$20. \lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h} = \lim_{h \rightarrow 0} \frac{(8 + 12h + 6h^2 + h^3) - 8}{h} = \lim_{h \rightarrow 0} \frac{12h + 6h^2 + h^3}{h} \\ = \lim_{h \rightarrow 0} (12 + 6h + h^2) = 12 + 0 + 0 = 12$$

$$21. \lim_{t \rightarrow 9} \frac{9-t}{3-\sqrt{t}} = \lim_{t \rightarrow 9} \frac{(3+\sqrt{t})(3-\sqrt{t})}{3-\sqrt{t}} = \lim_{t \rightarrow 9} (3+\sqrt{t}) = 3 + \sqrt{9} = 6$$

$$22. \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1} = \lim_{h \rightarrow 0} \frac{(1+h) - 1}{h(\sqrt{1+h} + 1)} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{1+h} + 1)} \\ = \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{\sqrt{1+1} + 1} = \frac{1}{2}$$

$$23. \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7} = \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7} \cdot \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} = \lim_{x \rightarrow 7} \frac{(x+2) - 9}{(x-7)(\sqrt{x+2} + 3)} \\ = \lim_{x \rightarrow 7} \frac{x-7}{(x-7)(\sqrt{x+2} + 3)} = \lim_{x \rightarrow 7} \frac{1}{\sqrt{x+2} + 3} = \frac{1}{\sqrt{9+3} + 3} = \frac{1}{6}$$

$$24. \lim_{x \rightarrow 2} \frac{x^4 - 16}{x-2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)(x^2+4)}{x-2} = \lim_{x \rightarrow 2} (x+2)(x^2+4) = \lim_{x \rightarrow 2} (x+2) \lim_{x \rightarrow 2} (x^2+4) \\ = (2+2)(2^2+4) = 32$$

$$25. \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x} = \lim_{x \rightarrow -4} \frac{\frac{x+4}{4x}}{4+x} = \lim_{x \rightarrow -4} \frac{x+4}{4x(4+x)} = \lim_{x \rightarrow -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$$

$$26. \lim_{t \rightarrow 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right) = \lim_{t \rightarrow 0} \frac{(t^2 + t) - t}{t(t^2 + t)} = \lim_{t \rightarrow 0} \frac{t^2}{t \cdot t(t+1)} = \lim_{t \rightarrow 0} \frac{1}{t+1} = \frac{1}{0+1} = 1$$

$$27. \lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3} = \lim_{x \rightarrow 9} \frac{(x-9)(x+9)}{\sqrt{x} - 3} = \lim_{x \rightarrow 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)(x+9)}{\sqrt{x}-3} \quad \left[ \text{factor } x-9 \text{ as a} \right. \\ \left. \text{difference of squares} \right] \\ = \lim_{x \rightarrow 9} [(\sqrt{x}+3)(x+9)] = (\sqrt{9}+3)(9+9) = 6 \cdot 18 = 108$$

$$28. \lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h \rightarrow 0} \frac{3 - (3+h)}{h(3+h)3} = \lim_{h \rightarrow 0} \frac{-h}{h(3+h)3} \\ = \lim_{h \rightarrow 0} \left[ -\frac{1}{3(3+h)} \right] = -\frac{1}{\lim_{h \rightarrow 0} [3(3+h)]} = -\frac{1}{3(3+0)} = -\frac{1}{9}$$

36.  $3x \leq f(x) \leq x^3 + 2$  for  $0 \leq x \leq 2$ . Now  $\lim_{x \rightarrow 1} 3x = 3$  and  $\lim_{x \rightarrow 1} (x^3 + 2) = \lim_{x \rightarrow 1} x^3 + \lim_{x \rightarrow 1} 2 = 1^3 + 2 = 3$ .

Therefore, by the Squeeze Theorem,  $\lim_{x \rightarrow 1} f(x) = 3$ .

37.  $-1 \leq \cos(2/x) \leq 1 \Rightarrow -x^4 \leq x^4 \cos(2/x) \leq x^4$ . Since  $\lim_{x \rightarrow 0} (-x^4) = 0$  and  $\lim_{x \rightarrow 0} x^4 = 0$ , we have

$\lim_{x \rightarrow 0} [x^4 \cos(2/x)] = 0$  by the Squeeze Theorem.

38.  $-1 \leq \sin(\pi/x) \leq 1 \Rightarrow e^{-1} \leq e^{\sin(\pi/x)} \leq e^1 \Rightarrow \sqrt{x}/e \leq \sqrt{x} e^{\sin(\pi/x)} \leq \sqrt{x} e$ . Since

$\lim_{x \rightarrow 0^+} (\sqrt{x}/e) = 0$  and  $\lim_{x \rightarrow 0^+} (\sqrt{x} e) = 0$ , we have  $\lim_{x \rightarrow 0^+} [\sqrt{x} e^{\sin(\pi/x)}] = 0$  by the Squeeze Theorem.

39. If  $x > -4$ , then  $|x + 4| = x + 4$ , so  $\lim_{x \rightarrow -4^+} |x + 4| = \lim_{x \rightarrow -4^+} (x + 4) = -4 + 4 = 0$ .

If  $x < -4$ , then  $|x + 4| = -(x + 4)$ , so  $\lim_{x \rightarrow -4^-} |x + 4| = \lim_{x \rightarrow -4^-} -(x + 4) = -(-4 + 4) = 0$ .

Since the right and left limits are equal,  $\lim_{x \rightarrow -4} |x + 4| = 0$ .

40. If  $x < -4$ , then  $|x + 4| = -(x + 4)$ , so  $\lim_{x \rightarrow -4^-} \frac{|x + 4|}{x + 4} = \lim_{x \rightarrow -4^-} \frac{-(x + 4)}{x + 4} = \lim_{x \rightarrow -4^-} (-1) = -1$ .

41. If  $x > 2$ , then  $|x - 2| = x - 2$ , so  $\lim_{x \rightarrow 2^+} \frac{|x - 2|}{x - 2} = \lim_{x \rightarrow 2^+} \frac{x - 2}{x - 2} = \lim_{x \rightarrow 2^+} 1 = 1$ . If  $x < 2$ , then

$|x - 2| = -(x - 2)$ , so  $\lim_{x \rightarrow 2^-} \frac{|x - 2|}{x - 2} = \lim_{x \rightarrow 2^-} \frac{-(x - 2)}{x - 2} = \lim_{x \rightarrow 2^-} -1 = -1$ . The right and left limits are

different, so  $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$  does not exist.

42. If  $x > \frac{3}{2}$ , then  $|2x - 3| = 2x - 3$ , so

$\lim_{x \rightarrow 1.5^+} \frac{2x^2 - 3x}{|2x - 3|} = \lim_{x \rightarrow 1.5^+} \frac{2x^2 - 3x}{2x - 3} = \lim_{x \rightarrow 1.5^+} \frac{x(2x - 3)}{2x - 3} = \lim_{x \rightarrow 1.5^+} x = 1.5$ . If  $x < \frac{3}{2}$ , then

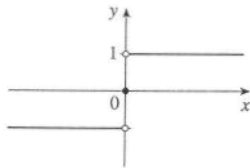
$|2x - 3| = 3 - 2x$ , so  $\lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|} = \lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{-(2x - 3)} = \lim_{x \rightarrow 1.5^-} \frac{x(2x - 3)}{-(2x - 3)} = \lim_{x \rightarrow 1.5^-} -x = -1.5$ .

The right and left limits are different, so  $\lim_{x \rightarrow 1.5} \frac{2x^2 - 3x}{|2x - 3|}$  does not exist.

43. Since  $|x| = -x$  for  $x < 0$ , we have  $\lim_{x \rightarrow 0^-} \left( \frac{1}{x} - \frac{1}{|x|} \right) = \lim_{x \rightarrow 0^-} \left( \frac{1}{x} - \frac{1}{-x} \right) = \lim_{x \rightarrow 0^-} \frac{2}{x}$ , which does not exist since the denominator approaches 0 and the numerator does not.

44. Since  $|x| = x$  for  $x > 0$ , we have  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{|x|} \right) = \lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{x} \right) = \lim_{x \rightarrow 0^+} 0 = 0$ .

45. (a)



(b) (i) Since  $\operatorname{sgn} x = 1$  for  $x > 0$ ,  $\lim_{x \rightarrow 0^+} \operatorname{sgn} x = \lim_{x \rightarrow 0^+} 1 = 1$ .

(ii) Since  $\operatorname{sgn} x = -1$  for  $x < 0$ ,  $\lim_{x \rightarrow 0^-} \operatorname{sgn} x = \lim_{x \rightarrow 0^-} -1 = -1$ .

(iii) Since  $\lim_{x \rightarrow 0^-} \operatorname{sgn} x \neq \lim_{x \rightarrow 0^+} \operatorname{sgn} x$ ,  $\lim_{x \rightarrow 0} \operatorname{sgn} x$  does not exist.

(iv) Since  $|\operatorname{sgn} x| = 1$  for  $x \neq 0$ ,  $\lim_{x \rightarrow 0} |\operatorname{sgn} x| = \lim_{x \rightarrow 0} 1 = 1$ .

3. On the left side of  $x = 2$ , we need  $|x - 2| < \left| \frac{10}{7} - 2 \right| = \frac{4}{7}$ . On the right side, we need  $|x - 2| < \left| \frac{10}{3} - 2 \right| = \frac{4}{3}$ .

For both of these conditions to be satisfied at once, we need the more restrictive of the two to hold, that is,

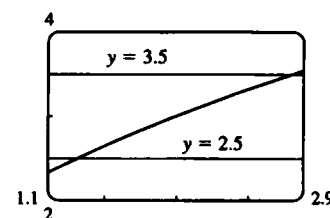
$|x - 2| < \frac{4}{7}$ . So we can choose  $\delta = \frac{4}{7}$ , or any smaller positive number.

4. On the left side, we need  $|x - 5| < |4 - 5| = 1$ . On the right side, we need  $|x - 5| < |5.7 - 5| = 0.7$ . For both conditions to be satisfied at once, we need the more restrictive condition to hold; that is,  $|x - 5| < 0.7$ . So we can choose  $\delta = 0.7$ , or any smaller positive number.

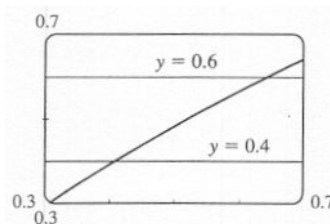
5. The leftmost question mark is the solution of  $\sqrt{x} = 1.6$  and the rightmost,  $\sqrt{x} = 2.4$ . So the values are  $1.6^2 = 2.56$  and  $2.4^2 = 5.76$ . On the left side, we need  $|x - 4| < |2.56 - 4| = 1.44$ . On the right side, we need  $|x - 4| < |5.76 - 4| = 1.76$ . To satisfy both conditions, we need the more restrictive condition to hold—namely,  $|x - 4| < 1.44$ . Thus, we can choose  $\delta = 1.44$ , or any smaller positive number.

6. The left-hand question mark is the positive solution of  $x^2 = \frac{1}{2}$ , that is,  $x = \frac{1}{\sqrt{2}}$ , and the right-hand question mark is the positive solution of  $x^2 = \frac{3}{2}$ , that is,  $x = \sqrt{\frac{3}{2}}$ . On the left side, we need  $|x - 1| < \left| \frac{1}{\sqrt{2}} - 1 \right| \approx 0.292$  (rounded down to be safe). On the right side, we need  $|x - 1| < \left| \sqrt{\frac{3}{2}} - 1 \right| \approx 0.224$ . The more restrictive of these two conditions must apply, so we choose  $\delta = 0.224$  (or any smaller positive number).

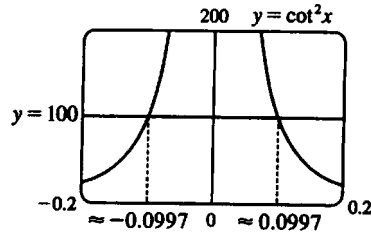
7.  $|\sqrt{4x+1} - 3| < 0.5 \Leftrightarrow 2.5 < \sqrt{4x+1} < 3.5$ . We plot the three parts of this inequality on the same screen and identify the  $x$ -coordinates of the points of intersection using the cursor. It appears that the inequality holds for  $1.3125 \leq x \leq 2.8125$ . Since  $|2 - 1.3125| = 0.6875$  and  $|2 - 2.8125| = 0.8125$ , we choose  $0 < \delta < \min\{0.6875, 0.8125\} = 0.6875$ .



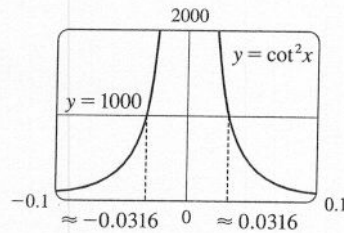
8.  $|\sin x - \frac{1}{2}| < 0.1 \Leftrightarrow 0.4 < \sin x < 0.6$ . From the graph, we see that for this inequality to hold, we need  $0.42 \leq x \leq 0.64$ . So since  $|0.5 - 0.42| = 0.08$  and  $|0.5 - 0.64| = 0.14$ , we choose  $0 < \delta \leq \min\{0.08, 0.14\} = 0.08$ .



12. For  $M = 100$ , we need  $-0.0997 < x < 0$  or  $0 < x < 0.0997$ . Thus, we choose  $\delta = 0.0997$  (or any smaller positive number) so that if  $0 < |x| < \delta$ , then  $\cot^2 x > 100$ .



For  $M = 1000$ , we need  $-0.0316 < x < 0$  or  $0 < x < 0.0316$ . Thus, we choose  $\delta = 0.0316$  (or any smaller positive number) so that if  $0 < |x| < \delta$ , then  $\cot^2 x > 1000$ .



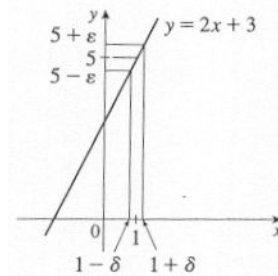
13. (a)  $A = \pi r^2$  and  $A = 1000 \text{ cm}^2 \Rightarrow \pi r^2 = 1000 \Rightarrow r^2 = \frac{1000}{\pi} \Rightarrow$   
 $r = \sqrt{\frac{1000}{\pi}} \quad [r > 0] \approx 17.8412 \text{ cm}.$
- (b)  $|A - 1000| \leq 5 \Rightarrow -5 \leq \pi r^2 - 1000 \leq 5 \Rightarrow 1000 - 5 \leq \pi r^2 \leq 1000 + 5 \Rightarrow$   
 $\sqrt{\frac{995}{\pi}} \leq r \leq \sqrt{\frac{1005}{\pi}} \Rightarrow 17.7966 \leq r \leq 17.8858. \quad \sqrt{\frac{1000}{\pi}} - \sqrt{\frac{995}{\pi}} \approx 0.04466$  and  
 $\sqrt{\frac{1005}{\pi}} - \sqrt{\frac{1000}{\pi}} \approx 0.04455.$  So if the machinist gets the radius within 0.0445 cm of 17.8412, the area will be within 5  $\text{cm}^2$  of 1000.
- (c)  $x$  is the radius,  $f(x)$  is the area,  $a$  is the target radius given in part (a),  $L$  is the target area (1000),  $\varepsilon$  is the tolerance in the area (5), and  $\delta$  is the tolerance in the radius given in part (b).
14. (a)  $T = 0.1w^2 + 2.155w + 20$  and  $T = 200 \Rightarrow$   
 $0.1w^2 + 2.155w + 20 = 200 \Rightarrow$  [by the quadratic formula or from the graph]  $w \approx 33.0$  watts ( $w > 0$ )
- 202 ( $^{\circ}\text{C}$ )

$T = 201$

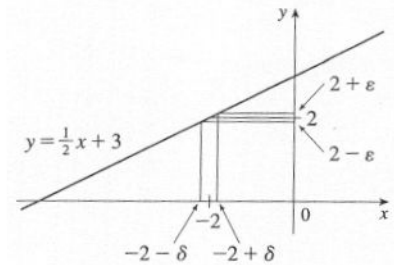
$T = 200$

$T = 199$
- (b) From the graph,  $199 \leq T \leq 201 \Rightarrow 32.89 < w < 33.11.$
- (c)  $x$  is the input power,  $f(x)$  is the temperature,  $a$  is the target input power given in part (a),  $L$  is the target temperature (200),  $\varepsilon$  is the tolerance in the temperature (1), and  $\delta$  is the tolerance in the power input in watts indicated in part (b) (0.11 watts).

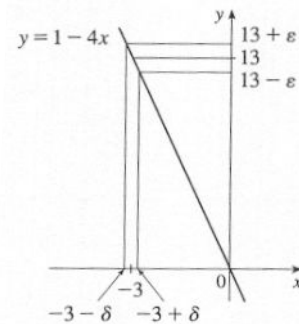
15. Given  $\varepsilon > 0$ , we need  $\delta > 0$  such that if  $0 < |x - 1| < \delta$ , then  $|(2x + 3) - 5| < \varepsilon$ . But  $|(2x + 3) - 5| < \varepsilon \Leftrightarrow |2x - 2| < \varepsilon \Leftrightarrow 2|x - 1| < \varepsilon \Leftrightarrow |x - 1| < \varepsilon/2$ . So if we choose  $\delta = \varepsilon/2$ , then  $0 < |x - 1| < \delta \Rightarrow |(2x + 3) - 5| < \varepsilon$ . Thus,  $\lim_{x \rightarrow 1} (2x + 3) = 5$  by the definition of a limit.



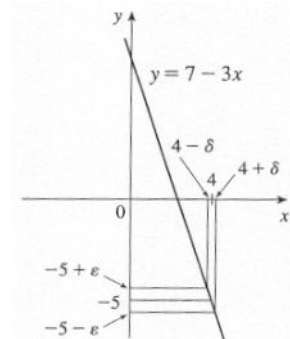
16. Given  $\varepsilon > 0$ , we need  $\delta > 0$  such that if  $0 < |x - (-2)| < \delta$ , then  $|(\frac{1}{2}x + 3) - 2| < \varepsilon$ . But  $|(\frac{1}{2}x + 3) - 2| < \varepsilon \Leftrightarrow |\frac{1}{2}x + 1| < \varepsilon \Leftrightarrow \frac{1}{2}|x + 2| < \varepsilon \Leftrightarrow |x - (-2)| < 2\varepsilon$ . So if we choose  $\delta = 2\varepsilon$ , then  $0 < |x - (-2)| < \delta \Rightarrow |(\frac{1}{2}x + 3) - 2| < \varepsilon$ . Thus,  $\lim_{x \rightarrow -2} (\frac{1}{2}x + 3) = 2$  by the definition of a limit.



17. Given  $\varepsilon > 0$ , we need  $\delta > 0$  such that if  $0 < |x - (-3)| < \delta$ , then  $|(1 - 4x) - 13| < \varepsilon$ . But  $|(1 - 4x) - 13| < \varepsilon \Leftrightarrow |-4x - 12| < \varepsilon \Leftrightarrow |-4||x + 3| < \varepsilon \Leftrightarrow |x - (-3)| < \varepsilon/4$ . So if we choose  $\delta = \varepsilon/4$ , then  $0 < |x - (-3)| < \delta \Rightarrow |(1 - 4x) - 13| < \varepsilon$ . Thus,  $\lim_{x \rightarrow -3} (1 - 4x) = 13$  by the definition of a limit.



18. Given  $\varepsilon > 0$ , we need  $\delta > 0$  such that if  $0 < |x - 4| < \delta$ , then  $|(7 - 3x) - (-5)| < \varepsilon$ . But  $|(7 - 3x) - (-5)| < \varepsilon \Leftrightarrow |-3x - 12| < \varepsilon \Leftrightarrow |-3||x + 4| < \varepsilon \Leftrightarrow |x - 4| < \varepsilon/3$ . So if we choose  $\delta = \varepsilon/3$ , then  $0 < |x - 4| < \delta \Rightarrow |(7 - 3x) - (-5)| < \varepsilon$ . Thus,  $\lim_{x \rightarrow 4} (7 - 3x) = -5$  by the definition of a limit.



19. Given  $\varepsilon > 0$ , we need  $\delta > 0$  such that if  $0 < |x - 3| < \delta$ , then  $|\frac{x}{5} - \frac{3}{5}| < \varepsilon \Leftrightarrow \frac{1}{5}|x - 3| < \varepsilon \Leftrightarrow |x - 3| < 5\varepsilon$ . So choose  $\delta = 5\varepsilon$ . Then  $0 < |x - 3| < \delta \Rightarrow |x - 3| < 5\varepsilon \Rightarrow \frac{|x - 3|}{5} < \varepsilon \Rightarrow |\frac{x}{5} - \frac{3}{5}| < \varepsilon$ . By the definition of a limit,  $\lim_{x \rightarrow 3} \frac{x}{5} = \frac{3}{5}$ .

(b)

$x$	$f(x)$
10,000	0.135308
100,000	0.135333
1,000,000	0.135335

From the table, we estimate that  $\lim_{x \rightarrow \infty} f(x) = 0.1353$   
(to four decimal places.)

$$11. \lim_{x \rightarrow \infty} \frac{3x^2 - x + 4}{2x^2 + 5x - 8} = \lim_{x \rightarrow \infty} \frac{(3x^2 - x + 4)/x^2}{(2x^2 + 5x - 8)/x^2}$$

[divide both the numerator and denominator by  $x^2$   
(the highest power of  $x$  that appears in the denominator)]

$$= \frac{\lim_{x \rightarrow \infty} (3 - 1/x + 4/x^2)}{\lim_{x \rightarrow \infty} (2 + 5/x - 8/x^2)}$$

[Limit Law 5]

$$= \frac{\lim_{x \rightarrow \infty} 3 - \lim_{x \rightarrow \infty} (1/x) + \lim_{x \rightarrow \infty} (4/x^2)}{\lim_{x \rightarrow \infty} 2 + \lim_{x \rightarrow \infty} (5/x) - \lim_{x \rightarrow \infty} (8/x^2)}$$

[Limit Laws 1 and 2]

$$= \frac{3 - \lim_{x \rightarrow \infty} (1/x) + 4 \lim_{x \rightarrow \infty} (1/x^2)}{2 + 5 \lim_{x \rightarrow \infty} (1/x) - 8 \lim_{x \rightarrow \infty} (1/x^2)}$$

[Limit Laws 7 and 3]

$$= \frac{3 - 0 + 4(0)}{2 + 5(0) - 8(0)}$$

[Theorem 5 of Section 2.5]

$$= \frac{3}{2}$$

$$12. \lim_{x \rightarrow \infty} \sqrt{\frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}} = \sqrt{\lim_{x \rightarrow \infty} \frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}}$$

[Limit Law 11]

$$= \sqrt{\lim_{x \rightarrow \infty} \frac{12 - 5/x^2 + 2/x^3}{1/x^3 + 4/x + 3}}$$

[divide by  $x^3$ ]

$$= \sqrt{\frac{\lim_{x \rightarrow \infty} (12 - 5/x^2 + 2/x^3)}{\lim_{x \rightarrow \infty} (1/x^3 + 4/x + 3)}}$$

[Limit Law 5]

$$= \sqrt{\frac{\lim_{x \rightarrow \infty} 12 - \lim_{x \rightarrow \infty} (5/x^2) + \lim_{x \rightarrow \infty} (2/x^3)}{\lim_{x \rightarrow \infty} (1/x^3) + \lim_{x \rightarrow \infty} (4/x) + \lim_{x \rightarrow \infty} 3}}$$

[Limit Laws 1 and 2]

$$= \sqrt{\frac{12 - 5 \lim_{x \rightarrow \infty} (1/x^2) + 2 \lim_{x \rightarrow \infty} (1/x^3)}{\lim_{x \rightarrow \infty} (1/x^3) + 4 \lim_{x \rightarrow \infty} (1/x) + 3}}$$

[Limit Laws 7 and 3]

$$= \sqrt{\frac{12 - 5(0) + 2(0)}{0 + 4(0) + 3}}$$

[Theorem 5 of Section 2.5]

$$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

$$13. \lim_{x \rightarrow \infty} \frac{1}{2x + 3} = \lim_{x \rightarrow \infty} \frac{1/x}{(2x + 3)/x} = \frac{\lim_{x \rightarrow \infty} (1/x)}{\lim_{x \rightarrow \infty} (2 + 3/x)} = \frac{\lim_{x \rightarrow \infty} (1/x)}{\lim_{x \rightarrow \infty} 2 + 3 \lim_{x \rightarrow \infty} (1/x)} = \frac{0}{2 + 3(0)} = \frac{0}{2} = 0$$

$$14. \lim_{x \rightarrow \infty} \frac{3x+5}{x-4} = \lim_{x \rightarrow \infty} \frac{(3x+5)/x}{(x-4)/x} = \lim_{x \rightarrow \infty} \frac{3+5/x}{1-4/x} = \frac{\lim_{x \rightarrow \infty} 3+5 \lim_{x \rightarrow \infty} \frac{1}{x}}{\lim_{x \rightarrow \infty} 1-4 \lim_{x \rightarrow \infty} \frac{1}{x}} = \frac{3+5(0)}{1-4(0)} = 3$$

$$15. \lim_{x \rightarrow -\infty} \frac{1-x-x^2}{2x^2-7} = \lim_{x \rightarrow -\infty} \frac{(1-x-x^2)/x^2}{(2x^2-7)/x^2} = \frac{\lim_{x \rightarrow -\infty} (1/x^2 - 1/x - 1)}{\lim_{x \rightarrow -\infty} (2 - 7/x^2)}$$

$$= \frac{\lim_{x \rightarrow -\infty} (1/x^2) - \lim_{x \rightarrow -\infty} (1/x) - \lim_{x \rightarrow -\infty} 1}{\lim_{x \rightarrow -\infty} 2 - 7 \lim_{x \rightarrow -\infty} (1/x^2)} = \frac{0 - 0 - 1}{2 - 7(0)} = -\frac{1}{2}$$

$$16. \lim_{y \rightarrow \infty} \frac{2-3y^3}{5y^2+4y} = \lim_{y \rightarrow \infty} \frac{(2-3y^2)/y^2}{(5y^2+4y)/y^2} = \frac{\lim_{y \rightarrow \infty} (2/y^2 - 3)}{\lim_{y \rightarrow \infty} (5+4/y)} = \frac{2 \lim_{y \rightarrow \infty} (1/y^2) - \lim_{y \rightarrow \infty} 3}{\lim_{y \rightarrow \infty} 5 + 4 \lim_{y \rightarrow \infty} (1/y)} = \frac{2(0) - 3}{5 + 4(0)} = -\frac{3}{5}$$

17. Divide both the numerator and denominator by  $x^3$  (the highest power of  $x$  that occurs in the denominator).

$$\lim_{x \rightarrow \infty} \frac{x^3+5x}{2x^3-x^2+4} = \lim_{x \rightarrow \infty} \frac{\frac{x^3+5x}{x^3}}{\frac{2x^3-x^2+4}{x^3}} = \lim_{x \rightarrow \infty} \frac{1+\frac{5}{x^2}}{2-\frac{1}{x}+\frac{4}{x^3}} = \frac{\lim_{x \rightarrow \infty} \left(1+\frac{5}{x^2}\right)}{\lim_{x \rightarrow \infty} \left(2-\frac{1}{x}+\frac{4}{x^3}\right)}$$

$$= \frac{\lim_{x \rightarrow \infty} 1 + 5 \lim_{x \rightarrow \infty} \frac{1}{x^2}}{\lim_{x \rightarrow \infty} 2 - \lim_{x \rightarrow \infty} \frac{1}{x} + 4 \lim_{x \rightarrow \infty} \frac{1}{x^3}} = \frac{1+5(0)}{2-0+4(0)} = \frac{1}{2}$$

$$18. \lim_{t \rightarrow -\infty} \frac{t^2+2}{t^3+t^2-1} = \lim_{t \rightarrow -\infty} \frac{(t^2+2)/t^3}{(t^3+t^2-1)/t^3} = \lim_{t \rightarrow -\infty} \frac{1/t+2/t^3}{1+1/t-1/t^3} = \frac{0+0}{1+0-0} = 0$$

19. First, multiply the factors in the denominator. Then divide both the numerator and denominator by  $u^4$ .

$$\lim_{u \rightarrow \infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)} = \lim_{u \rightarrow \infty} \frac{4u^4+5}{2u^4-5u^2+2} = \lim_{u \rightarrow \infty} \frac{\frac{4u^4+5}{u^4}}{\frac{2u^4-5u^2+2}{u^4}} = \lim_{u \rightarrow \infty} \frac{4+\frac{5}{u^4}}{2-\frac{5}{u^2}+\frac{2}{u^4}}$$

$$= \frac{\lim_{u \rightarrow \infty} \left(4+\frac{5}{u^4}\right)}{\lim_{u \rightarrow \infty} \left(2-\frac{5}{u^2}+\frac{2}{u^4}\right)} = \frac{\lim_{u \rightarrow \infty} 4+5 \lim_{u \rightarrow \infty} \frac{1}{u^4}}{\lim_{u \rightarrow \infty} 2-5 \lim_{u \rightarrow \infty} \frac{1}{u^2}+2 \lim_{u \rightarrow \infty} \frac{1}{u^4}} = \frac{4+5(0)}{2-5(0)+2(0)}$$

$$= \frac{4}{2} = 2$$

$$20. \lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{9x^2+1}} = \lim_{x \rightarrow \infty} \frac{(x+2)/x}{\sqrt{9x^2+1}/x} = \lim_{x \rightarrow \infty} \frac{1+2/x}{\sqrt{9+1/x^2}} = \frac{1+0}{\sqrt{9+0}} = \frac{1}{3}$$

$$21. \lim_{x \rightarrow \infty} \frac{\sqrt{9x^6-x}}{x^3+1} = \lim_{x \rightarrow \infty} \frac{\sqrt{9x^6-x}/x^3}{(x^3+1)/x^3} = \frac{\lim_{x \rightarrow \infty} \sqrt{(9x^6-x)/x^6}}{\lim_{x \rightarrow \infty} (1+1/x^3)} \quad [\text{since } x^3 = \sqrt{x^6} \text{ for } x > 0]$$

$$= \frac{\lim_{x \rightarrow \infty} \sqrt{9-1/x^5}}{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} (1/x^3)} = \frac{\sqrt{\lim_{x \rightarrow \infty} 9 - \lim_{x \rightarrow \infty} (1/x^5)}}{1+0}$$

$$= \sqrt{9-0} = 3$$

$$\begin{aligned}
 22. \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}/x^3}{(x^3 + 1)/x^3} = \frac{\lim_{x \rightarrow -\infty} -\sqrt{(9x^6 - x)/x^6}}{\lim_{x \rightarrow -\infty} (1 + 1/x^3)} \quad [\text{since } x^3 = -\sqrt{x^6} \text{ for } x \\
 &= \frac{\lim_{x \rightarrow -\infty} -\sqrt{9 - 1/x^5}}{\lim_{x \rightarrow -\infty} 1 + \lim_{x \rightarrow -\infty} (1/x^3)} = \frac{-\sqrt{\lim_{x \rightarrow -\infty} 9 - \lim_{x \rightarrow -\infty} (1/x^5)}}{1 + 0} \\
 &= -\sqrt{9 - 0} = -3
 \end{aligned}$$

$$\begin{aligned}
 23. \lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{9x^2 + x} - 3x)(\sqrt{9x^2 + x} + 3x)}{\sqrt{9x^2 + x} + 3x} = \lim_{x \rightarrow \infty} \frac{(\sqrt{9x^2 + x})^2 - (3x)^2}{\sqrt{9x^2 + x} + 3x} \\
 &= \lim_{x \rightarrow \infty} \frac{(9x^2 + x) - 9x^2}{\sqrt{9x^2 + x} + 3x} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{9x^2 + x} + 3x} \cdot \frac{1/x}{1/x} \\
 &= \lim_{x \rightarrow \infty} \frac{x/x}{\sqrt{9x^2/x^2 + x/x^2} + 3x/x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{9 + 1/x} + 3} = \frac{1}{\sqrt{9 + 3}} = \frac{1}{3 + 3} =
 \end{aligned}$$

$$\begin{aligned}
 24. \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x}) &= \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x}) \left[ \frac{x - \sqrt{x^2 + 2x}}{x - \sqrt{x^2 + 2x}} \right] = \lim_{x \rightarrow -\infty} \frac{x^2 - (x^2 + 2x)}{x - \sqrt{x^2 + 2x}} \\
 &= \lim_{x \rightarrow -\infty} \frac{-2x}{x - \sqrt{x^2 + 2x}} \cdot \frac{-2}{-2} = \frac{-2}{1 + \sqrt{1 + 2/x}} = \frac{-2}{1 + \sqrt{1 + 2(0)}} = -1
 \end{aligned}$$

Note: In dividing numerator and denominator by  $x$ , we used the fact that for  $x < 0$ ,  $x = -\sqrt{x^2}$ .

$$\begin{aligned}
 25. \lim_{x \rightarrow \infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx}) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + ax} - \sqrt{x^2 + bx})(\sqrt{x^2 + ax} + \sqrt{x^2 + bx})}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} \\
 &= \lim_{x \rightarrow \infty} \frac{(x^2 + ax) - (x^2 + bx)}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} = \lim_{x \rightarrow \infty} \frac{[(a - b)x]/x}{(\sqrt{x^2 + ax} + \sqrt{x^2 + bx})/\sqrt{x^2}} \\
 &= \lim_{x \rightarrow \infty} \frac{a - b}{\sqrt{1 + a/x} + \sqrt{1 + b/x}} = \frac{a - b}{\sqrt{1 + 0} + \sqrt{1 + 0}} = \frac{a - b}{2}
 \end{aligned}$$

26.  $\lim_{x \rightarrow \infty} \cos x$  does not exist because as  $x$  increases  $\cos x$  does not approach any one value, but oscillates between 1 and  $-1$ .

27.  $\sqrt{x}$  is large when  $x$  is large, so  $\lim_{x \rightarrow \infty} \sqrt{x} = \infty$ .

28.  $\sqrt[3]{x}$  is large negative when  $x$  is large negative, so  $\lim_{x \rightarrow -\infty} \sqrt[3]{x} = -\infty$ .

29.  $\lim_{x \rightarrow \infty} (x - \sqrt{x}) = \lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x} - 1) = \infty$  since  $\sqrt{x} \rightarrow \infty$  and  $\sqrt{x} - 1 \rightarrow \infty$  as  $x \rightarrow \infty$ .

$$\begin{aligned}
 30. \lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{5 - 2x^2} &= \lim_{x \rightarrow \infty} \frac{(x^3 - 2x + 3)/x^2}{(5 - 2x^2)/x^2} \quad [\text{divide by the highest power of } x \text{ in the denominator}] \\
 &= \lim_{x \rightarrow \infty} \frac{x - 2/x + 3/x^2}{5/x^2 - 2} = -\infty \text{ because } x - 2/x + 3/x^2 \rightarrow \infty \text{ and } 5/x^2 - 2 \rightarrow -2 \text{ as } x \rightarrow \infty.
 \end{aligned}$$

31.  $\lim_{x \rightarrow -\infty} (x^4 + x^5) = \lim_{x \rightarrow -\infty} x^5(\frac{1}{x} + 1)$  [factor out the largest power of  $x$ ] =  $-\infty$  because  $x^5 \rightarrow -\infty$  and  $1/x + 1 \rightarrow 1$  as  $x \rightarrow -\infty$ .