APPENDIXES

A Numbers, Inequalities, and Absolute Values

1. $|5-23| = |-18| = 18$

2. $|5| - |-23| = 5 - 23 = -18$

3. $|-\pi| = \pi$ because $\pi > 0$.

4. $|\pi - 2| = \pi - 2$ because $\pi - 2 > 0$.

5. $\left|\sqrt{5} - 5\right| = -\left(\sqrt{5} - 5\right) = 5 - \sqrt{5}$ because $\sqrt{5} - 5 < 0$.

6. $\left||-2| - |-3|\right| = |2 - 3| = |-1| = 1$

7. For $x < 2$, $x - 2 < 0$, so $|x - 2| = -(x-2) = 2 - x$.

8. For $x > 2$, $x - 2 > 0$, so $|x - 2| = x - 2$.

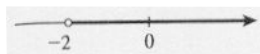
9. $|x+1| = \begin{cases} x+1 & \text{for } x+1 \geq 0 \Leftrightarrow x \geq -1 \\ -(x+1) & \text{for } x+1 < 0 \Leftrightarrow x < -1 \end{cases}$

10. $|2x-1| = \begin{cases} 2x-1 & \text{for } 2x-1 \geq 0 \Leftrightarrow x \geq \frac{1}{2} \\ 1-2x & \text{for } 2x-1 < 0 \Leftrightarrow x < \frac{1}{2} \end{cases}$

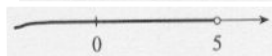
11. $\left|x^2+1\right| = x^2 + 1$ (since $x^2 + 1 \geq 0$ for all x).

12. Determine when $1 - 2x^2 < 0 \Leftrightarrow 1 < 2x^2 \Leftrightarrow x^2 > \frac{1}{2} \Leftrightarrow \sqrt{x^2} > \sqrt{\frac{1}{2}} \Leftrightarrow |x| > \sqrt{\frac{1}{2}} \Leftrightarrow$ $x < -\frac{1}{\sqrt{2}}$ or $x > \frac{1}{\sqrt{2}}$. Thus, $\left|1 - 2x^2\right| = \begin{cases} 1 - 2x^2 & \text{if } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \\ 2x^2 - 1 & \text{if } x < -\frac{1}{\sqrt{2}} \text{ or } x > \frac{1}{\sqrt{2}} \end{cases}$

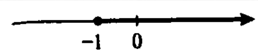
$2x > -4 \Leftrightarrow x > -2$, so $x \in (-2, \infty)$.



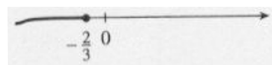
$3x < 15 \Leftrightarrow x < 5$, so $x \in (-\infty, 5)$.



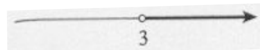
$-x \leq 1 \Leftrightarrow$



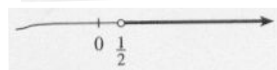
$-3x \geq 2 \Leftrightarrow$



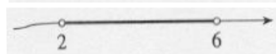
$\Leftrightarrow 9 < 3x$



$\Leftrightarrow 8x > 4$

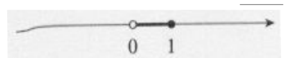


$\Leftrightarrow 4 < 2x < 12$

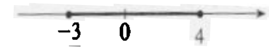


$\Leftrightarrow -3 < 3x \leq 12$

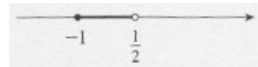
$\Leftrightarrow -1 \leq -x < 0$



22. $-5 \le 3 - 2x \le 9 \quad \Leftrightarrow \quad -8 \le -2x \le 6 \quad \Leftrightarrow \quad 4 \ge x \ge -3$, so $x \in [-3, 4]$.



23. $4x < 2x + 1 \le 3x + 2$. So $4x < 2x + 1 \quad \Leftrightarrow \quad 2x < 1 \quad \Leftrightarrow \quad x < \frac{1}{2}$, and $2x + 1 \le 3x + 2 \quad \Leftrightarrow \quad -1 \le x$. Thus, $x \in \left[-1, \frac{1}{2}\right)$.



24. $2x - 3 < x + 4 < 3x - 2$. So $2x - 3 < x + 4 \quad \Leftrightarrow \quad x < 7$, and $x + 4 < 3x - 2 \quad \Leftrightarrow \quad 6 < 2x \quad \Leftrightarrow \quad 3 < x$, so $x \in (3, 7)$.

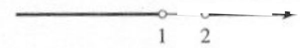
25. $(x - 1)(x - 2) > 0$. *Case 1:* (both factors are positive, so their product is positive)

$x - 1 > 0 \quad \Leftrightarrow \quad x > 1$, and $x - 2 > 0 \quad \Leftrightarrow \quad x > 2$, so $x \in (2, \infty)$.

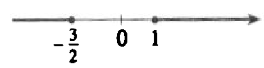
Case 2: (both factors are negative, so their product is positive)

$x - 1 < 0 \quad \Leftrightarrow \quad x < 1$, and $x - 2 < 0 \quad \Leftrightarrow \quad x < 2$, so $x \in (-\infty, 1)$.

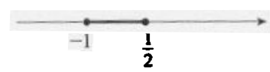
Thus, the solution set is $(-\infty, 1) \cup (2, \infty)$.



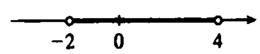
26. $(2x + 3)(x - 1) \ge 0$. *Case 1:* $2x + 3 \ge 0 \quad \Leftrightarrow \quad x \ge -\frac{3}{2}$, and $x - 1 \ge 0 \quad \Leftrightarrow \quad x \ge 1$, so $x \in [1, \infty)$. *Case 2:* $2x + 3 \le 0 \quad \Leftrightarrow \quad x \le -\frac{3}{2}$, and $x - 1 \le 0 \quad \Leftrightarrow \quad x \le 1$, so $x \in \left(-\infty, -\frac{3}{2}\right]$. Thus, the solution set is $\left(-\infty, -\frac{3}{2}\right] \cup [1, \infty)$.



27. $2x^2 + x \le 1 \quad \Leftrightarrow \quad 2x^2 + x - 1 \le 0 \quad \Leftrightarrow \quad (2x - 1)(x + 1) \le 0$. *Case 1:* $2x - 1 \ge 0 \quad \Leftrightarrow \quad x \ge \frac{1}{2}$, and $x + 1 \le 0 \quad \Leftrightarrow \quad x \le -1$, which is an impossible combination. *Case 2:* $2x - 1 \le 0 \quad \Leftrightarrow \quad x \le \frac{1}{2}$, and $x + 1 \ge 0 \quad \Leftrightarrow \quad x \ge -1$, so $x \in \left[-1, \frac{1}{2}\right]$. Thus, the solution set is $\left[-1, \frac{1}{2}\right]$.

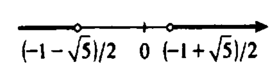


28. $x^2 < 2x + 8 \quad \Leftrightarrow \quad x^2 - 2x - 8 < 0 \quad \Leftrightarrow \quad (x - 4)(x + 2) < 0$. *Case 1:* $x > 4$ and $x < -2$, which is impossible. *Case 2:* $x < 4$ and $x > -2$. Thus, the solution set is $(-2, 4)$.



29. $x^2 + x + 1 > 0 \quad \Leftrightarrow \quad x^2 + x + \frac{1}{4} + \frac{3}{4} > 0 \quad \Leftrightarrow \quad \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0$. But since $\left(x + \frac{1}{2}\right)^2 \ge 0$ for every real x, the original inequality will be true for all real x as well. Thus, the solution set is $(-\infty, \infty)$.

30. $x^2 + x > 1 \quad \Leftrightarrow \quad x^2 + x - 1 > 0$. Using the quadratic formula, we obtain $x^2 + x - 1 = \left(x - \frac{-1 - \sqrt{5}}{2}\right)\left(x - \frac{-1 + \sqrt{5}}{2}\right) > 0$. *Case 1:* $x - \frac{-1 - \sqrt{5}}{2} > 0$ and $x - \frac{-1 + \sqrt{5}}{2} > 0$, so that $x > \frac{-1 + \sqrt{5}}{2}$. *Case 2:* $x - \frac{-1 - \sqrt{5}}{2} < 0$ and $x - \frac{-1 + \sqrt{5}}{2} < 0$, so that $x < \frac{-1 - \sqrt{5}}{2}$. Thus, the solution set is $\left(-\infty, \frac{-1 - \sqrt{5}}{2}\right) \cup \left(\frac{-1 + \sqrt{5}}{2}, \infty\right)$.



69. Observe that the sum, difference and product of two integers is always an integer. Let the rational numbers be represented by $r = m/n$ and $s = p/q$ (where m, n, p and q are integers with $n \neq 0$, $q \neq 0$). Now $r + s = \frac{m}{n} + \frac{p}{q} = \frac{mq + pn}{nq}$, but $mq + pn$ and nq are both integers, so $\frac{mq + pn}{nq} = r + s$ is a rational number by definit. [illegible] nq are both integers, so $\frac{mp}{nq} = r \cdot s$ is a rational number by definition.

70. (a) No. Consider the case of $\sqrt{2}$ and $-\sqrt{2}$. Both are irrational numbers, yet $\sqrt{2} + (-\sqrt{2}) = 0$ and 0, being an integer, is not irrational.

(b) No. Consider the case of $\sqrt{2}$ and $\sqrt{2}$. Both are irrational numbers, yet $\sqrt{2} \cdot \sqrt{2} = 2$ is not irrational.

B Coordinate Geometry and Lines

1. From the Distance Formula with $x_1 = 1$, $x_2 = 4$, $y_1 = 1$, $y_2 = 5$, we find the distance from $(1, 1)$ to $(4, 5)$ to be $\sqrt{(4-1)^2 + (5-1)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

2. The distance from $(1, -3)$ to $(5, 7)$ is $\sqrt{(5-1)^2 + [7-(-3)]^2} = \sqrt{4^2 + 10^2} = \sqrt{116} = 2\sqrt{29}$.

3. $\sqrt{(-1-6)^2 + [3-(-2)]^2} = \sqrt{(-7)^2 + 5^2} = \sqrt{74}$

4. $\sqrt{(-1-1)^2 + [-3-(-6)]^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

5. $\sqrt{(4-2)^2 + (-7-5)^2} = \sqrt{2^2 + (-12)^2} = \sqrt{148} = 2\sqrt{37}$

6. $\sqrt{(b-a)^2 + (a-b)^2} = \sqrt{(a-b)^2 + (a-b)^2} = \sqrt{2(a-b)^2} = \sqrt{2}\,|a-b|$

7. From (2), the slope is $\frac{11-5}{4-1} = \frac{6}{3} = 2$.

8. $m = \frac{-3-6}{4-(-1)} = -\frac{9}{5}$

9. With $P(-3, 3)$ and $Q(-1, -6)$, the slope m of the line through P and Q is $m = \frac{-6-3}{-1-(-3)} = -\frac{9}{2}$.

10. $m = \frac{0-(-4)}{6-(-1)} = \frac{4}{7}$

11. Since $|AC| = \sqrt{(-4-0)^2 + (3-2)^2} = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$ and $|BC| = \sqrt{[-4-(-3)]^2 + [3-(-1)]^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$, the triangle has two sides of equal length, and so is isosceles.

12. (a) $|AB| = \sqrt{(11-6)^2 + [-3-(-7)]^2} = \sqrt{5^2 + 4^2} = \sqrt{41}$,

$|AC| = \sqrt{(2-6)^2 + [-2-(-7)]^2} = \sqrt{(-4)^2 + 5^2} = \sqrt{41}$, and

$|BC| = \sqrt{(2-11)^2 + [-2-(-3)]^2} = \sqrt{(-9)^2 + 1^2} = \sqrt{82}$, so

$|AB|^2 + |AC|^2 = 41 + 41 = 82 = |BC|^2$, and so $\triangle ABC$ is a right triangle.

(b) $m_{AB} = \dfrac{-3-(-7)}{11-6} = \dfrac{4}{5}$ and $m_{AC} = \dfrac{-2-(-7)}{2-6} = -\dfrac{5}{4}$. Thus $m_{AB} \cdot m_{AC} = -1$ and so AB is perpendicular to AC and $\triangle ABC$ must be a right triangle.

(c) Taking lengths from part (a), the base is $\sqrt{41}$ and the height is $\sqrt{41}$. Thus the area is $\frac{1}{2}bh = \frac{1}{2}\sqrt{41}\sqrt{41} = \frac{41}{2}$.

13. Using $A(-2,9)$, $B(4,6)$, $C(1,0)$, and $D(-5,3)$, we have

$|AB| = \sqrt{[4-(-2)]^2+(6-9)^2} = \sqrt{6^2+(-3)^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$,

$|BC| = \sqrt{(1-4)^2+(0-6)^2} = \sqrt{(-3)^2+(-6)^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$,

$|CD| = \sqrt{(-5-1)^2+(3-0)^2} = \sqrt{(-6)^2+3^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$, and

$|DA| = \sqrt{[-2-(-5)]^2+(9-3)^2} = \sqrt{3^2+6^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$. So all sides are of equal length and we have a rhombus. Moreover, $m_{AB} = \dfrac{6-9}{4-(-2)} = -\dfrac{1}{2}$, $m_{BC} = \dfrac{0-6}{1-4} = 2$, $m_{CD} = \dfrac{3-0}{-5-1} = -\dfrac{1}{2}$, and

$m_{DA} = \dfrac{9-3}{-2-(-5)} = 2$, so the sides are perpendicular. Thus, A, B, C, and D are vertices of a square.

14. (a) Using $A(-1,3)$, $B(3,11)$, and $C(5,15)$, we have

$|AB| = \sqrt{[3-(-1)]^2+(11-3)^2} = \sqrt{4^2+8^2} = \sqrt{80} = 4\sqrt{5}$,

$|BC| = \sqrt{(5-3)^2+(15-11)^2} = \sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$, and

$|AC| = \sqrt{[5-(-1)]^2+(15-3)^2} = \sqrt{6^2+12^2} = \sqrt{180} = 6\sqrt{5}$. Thus, $|AC| = |AB| + |BC|$.

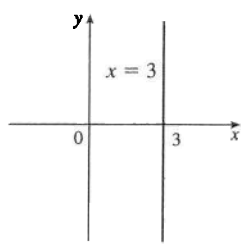
(b) $m_{AB} = \dfrac{11-3}{3-(-1)} = \dfrac{8}{4} = 2$ and $m_{AC} = \dfrac{15-3}{5-(-1)} = \dfrac{12}{6} = 2$. Since the segments AB and AC have the same slope, A, B and C must be collinear.

15. The slope of the line segment AB is $\dfrac{4-1}{7-1} = \dfrac{1}{2}$, the slope of CD is $\dfrac{7-10}{-1-5} = \dfrac{1}{2}$, the slope of BC is $\dfrac{10-4}{5-7} = -3$, and the slope of DA is $\dfrac{1-7}{1-(-1)} = -3$. So AB is parallel to CD and BC is parallel to DA. Hence $ABCD$ is a parallelogram.

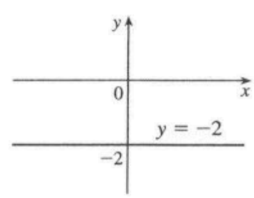
16. The slopes of the four sides are $m_{AB} = \dfrac{3-1}{11-1} = \dfrac{1}{5}$, $m_{BC} = \dfrac{8-3}{10-11} = -5$, $m_{CD} = \dfrac{6-8}{0-10} = \dfrac{1}{5}$, and

$m_{DA} = \dfrac{1-6}{1-0} = -5$. Hence $AB \parallel CD$, $BC \parallel DA$, $AB \perp BC$, $BC \perp CD$, $CD \perp DA$, and $DA \perp AB$, and so $ABCD$ is a rectangle.

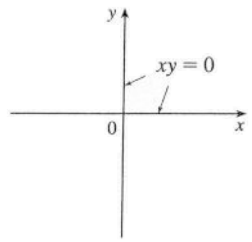
17. $x = 3$



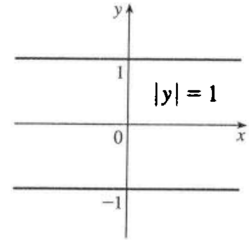
18. $y = -2$



19. $xy = 0 \Leftrightarrow x = 0$ or $y = 0$. The graph consists of the coordinate axes.



20. $|y| = 1 \Leftrightarrow y = 1$ or $y = -1$



21. By the point-slope form of the equation of a line, an equation of the line through $(2, -3)$ with slope 6 is $y - (-3) = 6(x - 2)$ or $y = 6x - 15$.

22. $y - 4 = -3\,[x - (-1)]$ or $y = -3x + 1$

23. $y - 7 = \frac{2}{3}\,(x - 1)$ or $y = \frac{2}{3}x + \frac{19}{3}$

24. $y - (-5) = -\frac{7}{2}\,[x - (-3)]$ or $y = -\frac{7}{2}x - \frac{31}{2}$

25. The slope of the line through $(2, 1)$ and $(1, 6)$ is $m = \dfrac{6 - 1}{1 - 2} = -5$, so an equation of the line is $y - 1 = -5(x - 2)$ or $y = -5x + 11$.

26. For $(-1, -2)$ and $(4, 3)$, $m = \dfrac{3 - (-2)}{4 - (-1)} = 1$. So $y - 3 = 1(x - 4)$ or $y = x - 1$.

27. By the slope-intercept form of the equation of a line, an equation of the line is $y = 3x - 2$.

28. By the slope-intercept form of the equation of a line, an equation of the line is $y = \frac{2}{5}x + 4$.

29. Since the line passes through $(1, 0)$ and $(0, -3)$, its slope is $m = \dfrac{-3 - 0}{0 - 1} = 3$, so an equation is $y = 3x - 3$.

Another method: From Exercise 61, $\dfrac{x}{1} + \dfrac{y}{-3} = 1 \Rightarrow -3x + y = -3 \Rightarrow y = 3x - 3$.

30. For $(-8, 0)$ and $(0, 6)$, $m = \dfrac{6 - 0}{0 - (-8)} = \dfrac{3}{4}$. So an equation is $y = \frac{3}{4}x + 6$.

Another method: From Exercise 61, $\dfrac{x}{-8} + \dfrac{y}{6} = 1 \Rightarrow -3x + 4y = 24 \Rightarrow y = \frac{3}{4}x + 6$.

31. Since $m = 0$, $y - 5 = 0(x - 4)$ or $y = 5$.

32. Since m is undefined, we have the vertical line $x = 4$.

33. Putting the line $x + 2y = 6$ into its slope-intercept form gives us $y = -\frac{1}{2}x + 3$, so we see that this line has slope $-\frac{1}{2}$. Thus, we want the line of slope $-\frac{1}{2}$ that passes through the point $(1, -6)$: $y - (-6) = -\frac{1}{2}(x - 1) \Leftrightarrow y = -\frac{1}{2}x - \frac{11}{2}$.

34. $2x + 3y + 4 = 0 \Leftrightarrow y = -\frac{2}{3}x - \frac{4}{3}$, so $m = -\frac{2}{3}$ and the required line is $y = -\frac{2}{3}x + 6$.

35. $2x + 5y + 8 = 0 \Leftrightarrow y = -\frac{2}{5}x - \frac{8}{5}$. Since this line has slope $-\frac{2}{5}$, a line perpendicular to it would have slope $\frac{5}{2}$, so the required line is $y - (-2) = \frac{5}{2}\,[x - (-1)] \Leftrightarrow y = \frac{5}{2}x + \frac{1}{2}$.

36. $4x - 8y = 1 \Leftrightarrow y = \frac{1}{2}x - \frac{1}{8}$. Since this line has slope $\frac{1}{2}$, a line perpendicular to it would have slope -2, so the required line is $y - \left(-\frac{2}{3}\right) = -2\left(x - \frac{1}{2}\right) \Leftrightarrow y = -2x + \frac{1}{3}$.