Math 379 H2

Question 1. Consider a slightly differently defined Riemann sphere: $\Sigma = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$. Compute the formula for z (projection) and also obtain the formula for the inverse z'. The formulas will look very similar to what we had in class. Use the same definition of the stereographic projection as in my lectures (projection from the North Pole). Consider rotations of the space $\mathbb{R}^3$ about x_1, x_2, x_3 axis. I did in class rotations about x_3. For each type of rotation compute the corresponding Möbius transformation.

Question 2. p.26, problem 2.2

Question 3. p.27, problem 2.5

Question 4. p.27, problem 2.11

Question 5. p.28, problem 2.14

Question 6. p.29, problem 2.17

Question 7. Using the notation of the first problem, determine the rotation matrix corresponding to the Möbius transformation

$$f(z) = \frac{az + b}{-\bar{b}z + \bar{a}}, \quad |a|^2 + |b|^2 = 1$$

How do we know these are rotations and not some other transformations? Identify special cases considered in the first problem. Hint: Please start working on the first and last problem right away if you can. This coming Monday I will give you a hint as to how to proceed if you are stuck.