Coordination with Instabilities

Sujit Nair (Caltech)

Collaborators: N. Leonard, E. Kanso, J. Marsden

Geometric Mechanics: Continuous and discrete, finite and infinite dimensional

August 12-17, 2007

Banff International Research Station for Mathematical Innovation and Discovery
Outline

- Background
- Motivation
- Controlled Lagrangian (CL), matching conditions
- Network of SMC, SO(3) and SE(3) systems
- Region of attraction for CL
- Kinetic shaping for collision avoidance
- Area surveillance using underactuated hovercrafts
Background

- Network of sensor equipped Lagrangian vehicles
- Nontrivial dynamics → integrate coordination with stabilization
  - Couple systems to yield multi-body Lagrangian system
- AUVs for sampling, satellite groups for interferometry, helicopters for area surveillance etc

Source: Naomi Leonard
Motivation

- Individuals have nontrivial/unstable dynamics
- Not reducible to single/double integrators
  - Underactuated systems
- Achieve “large” regions of attraction
- Treat coordination and stabilization equally
  - Naive coupling may not work
Example systems

- Simplified Matching Condition systems (SMCs)
- $SO(3)$ (satellite) systems
- $SE(3)$ (underwater vehicle) systems
- Hydrodynamically coupled rigid bodies (Eva’s talk)
- Hovercrafts for surveillance
Related Work

- Hansmann et al. [2006], McInnes C.R [1996]
- Jadabaie et al. [2003], Ogren et al. [2002], Olfati-Saber et al. [2004], Fax et al. [2002] (single/double integrators)
- Desai et al. [1997], Tanner et al. [2003] (nonholonomic case)
- Lawton and Beard [2002] (relative attitude stabilization)
- Nijmeijer and Rodriguez-Angeles [2001] (two robotic manipulators)
- Belta and Kumar [2002]
Notation

- Configuration variables: \((\mathbf{x}, \theta) = (x^\alpha, \theta^a) \in Q^m \times \mathbb{R}^n, \ \alpha = 1, \ldots, m, \ a = 1, \ldots, n, \ n \geq m\)

- \(\theta\) are the actuation directions

- \(L = K - V, \ K = \text{kinetic energy}, \ V = \text{potential energy}\)

- \(\mathcal{E}_L q = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q}, \ q\ \text{is generalized coordinate}\)
Controlled Lagrangian (CL)

• Choose controls $u$ such that

\[ E_L x = 0 \quad \implies \quad E_{L_c} x = 0 \]

\[ E_L \theta = u \quad \implies \quad E_{L_c} \theta = 0 \]

• After adding dissipation

  – Asymptotically stabilize relative equilibria
  – Asymptotically stabilize constant momentum
**SMCs and CL**

- \( L = \frac{1}{2}(\dot{x} \dot{\theta}) \begin{pmatrix} g_{\alpha\beta} & g_{\alpha a} \\ g_{a\alpha} & g_{ab} \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} - V(x, \theta) \)
  
  \[ \begin{align*} 
  [g_{\alpha\beta}] & : m \times m \\
  [g_{\alpha a}] & : m \times n \\
  [g_{ab}] & : n \times n 
  \end{align*} \]

- Kinetic energy is symmetric in \( \theta^a \)

- Simplified Matching Conditions (SMCs) are

  **SMC1** \( g_{ab} = \text{constant} \)

  **SMC2** \( \frac{\partial g_{\alpha a}}{\partial x^\beta} = \frac{\partial g_{\beta a}}{\partial x^\alpha} \)

  **SMC3** \( [g_{\alpha a}] [g_{ab}]^{-1} [V_{21}] \) is a symmetric \( m \times m \) matrix with \( \delta^2 V(x, \theta) = \)

  \[
  \begin{pmatrix}
  [V_{11}] & [V_{12}] \\
  [V_{21}] & [V_{22}] 
  \end{pmatrix}
  \]

  \[ \begin{align*} 
  [V_{11}] & : m \times m \\
  [V_{12}] & : m \times n \\
  [V_{21}] & : n \times m \\
  [V_{22}] & : n \times n 
  \end{align*} \]

- Bloch et al. [2000,2001]
Interpretation of SMCs

- SMC1 & SMC2 $\Rightarrow$ no kinetic coupling

$$\frac{d}{dt}(g_{\alpha\beta}\dot{x}^\beta) + g_{\alpha\beta}\ddot{x}^\beta - \frac{1}{2}\frac{\partial g_{\beta\gamma}}{\partial x^\alpha}\dot{x}^\beta\dot{x}^\gamma + \frac{\partial V}{\partial x^\alpha} = 0$$

- SMC1 & SMC2 $\Rightarrow$ no gyroscopic terms

$$E_{R^\mu}(x^\alpha) = \left(\frac{\partial(g^{ab}g_{\alpha a})}{\partial x^\beta} - \frac{\partial(g^{ab}g_{\beta a})}{\partial x^\alpha}\right)\mu_b\dot{x}^\beta = 0$$

- SMC3 necessary for potential matching (trivial if $V = V(x^\alpha)$)

- Inverted pendulum/cart (planar or spherical) are SMC systems

- Furuta pendulum, rigid body in $SO(3)$, ball and beam are not SMC systems
SMCs and CL

- $L_c = L_{\rho,\kappa,\epsilon} = K_{\rho,\kappa} - (V + V_\epsilon)$
- SMC1 & SMC2 $\implies$ existence of $K_{\rho,\kappa}$
- SMC3 $\implies$ existence of $V_\epsilon$
- Bloch et al [2001] give conditions on above parameters such that the closed-loop system is stable in full state space ( $K_{\rho,\kappa}$ and $V + V_\epsilon$ are made negative definite at origin )
- Add dissipative control to asymptotically stabilize
Coordination of SMCs

- Can couple SMC systems using controls such that the network is also SMC
- Can choose dissipation/Lyapunov function to asymptotically stabilize
  - Relative equilibria
  - Constant momentum solution
- For inverted pendulum cart systems, attach coupling springs carefully
Coordination of SMCs
Coordination of SMCs
Comparison with LQR

- Individual stabilized using LQR
- Systems coupled with set-points
  - \( u_{Li} = u_{Li}(x_i, \theta_i, \dot{x}_i, \dot{\theta}_i) \) for \( i = 1, 2 \)
  - \( u_{L1c} = u_{L1}(x_1, \theta_1 - \theta_2, \dot{x}_1, \dot{\theta}_1 - \dot{\theta}_2) \), \( u_{L2c} = u_{L2}(x_2, \theta_2 - \theta_1, \dot{x}_2, \dot{\theta}_2 - \dot{\theta}_1) \)
- LQR has better settling time and less overshoot
- CL gives large region of attraction
- CL technique uses non-trivial coupling terms
  - \( u_i = u_i(x_i, \theta_i, \dot{x}_i, \dot{\theta}_i) \) for \( i = 1, 2 \)
  - \( u_{1c} = u_1(x_1, \theta_1 - \theta_2, \dot{x}_1, \dot{\theta}_1 - \dot{\theta}_2) \), \( u_{2c} = u_2(x_2, \theta_2 - \theta_1, \dot{x}_2, \dot{\theta}_2 - \dot{\theta}_1) \) DOES NOT work
- Necessary to treat coordination and stabilization in an integral manner
SO(3) and SE(3) systems

- Goal for $SO(3)^n$: Align the systems with each rotating about its unstable, middle axis
- Goal for $SE(3)^n$: Align the systems with each rotating about its unstable, middle axis and translating along the same axis
- Control law consists of
  - Kinetic energy shaping term (stabilize each body)
  - Potential energy shaping term (couple all bodies)
- Stability proven using Energy-Momentum methods
SO(3) Spin Stabilization

- Bloch, Krishnaprasad, Marsden and Sanchez De Alvarez[1992]
  - Choose controls s.t. closed-loop also Hamiltonian
  - Single external torque about long axis to stabilize middle axis rotation
  - BUT open-loop & closed-loop evolve on different spaces (i.e., different Poisson brackets)

- Our approach
  - DO NOT deform Poisson structure
  - Closed-loop also evolves on \( so(3)^* \) (critical for integrating stabilization and coordination control laws)
  - Need two external torques
SO(3) Spin Stabilization

\[ I_1(Ω)_1 = (I_2 - I_3)(Ω)_2(Ω)_3 \]
\[ I_2(Ω)_2 = (I_3 - I_1)(Ω)_3(Ω)_1 + u_2^{ks} \quad I_1 > I_2 > I_3 \]
\[ I_3(Ω)_3 = (I_1 - I_2)(Ω)_1(Ω)_2 + u_3^{ks} \]

Controls could be about any of the two axis (not necessarily about the 2 and 3 axis). Choose

\[ u_2^{ks} = \left( I_1 \left( 1 - \frac{1}{ρ_2} \right) + I_3 \left( \frac{ρ_3}{ρ_2} - 1 \right) \right)(Ω)_3(Ω)_1 \]
\[ u_3^{ks} = \left( I_1 \left( \frac{1}{ρ_3} - 1 \right) + I_2 \left( 1 - \frac{ρ_2}{ρ_3} \right) \right)(Ω)_1(Ω)_2 \]

where \( ρ_3 > \frac{I_1}{I_3} > 1, \quad ρ_2 = (ρ_3 - 1)\frac{I_3}{I_2} + 1 > 1 \)

\[ \Rightarrow ρ_2I_2 > ρ_3I_3 > I_1 \]
SO(3) Spin Stabilization

Closed-loop equations are:

\[ I_1(\dot{\Omega})_1 = (\rho_2 I_2 - \rho_3 I_3)(\Omega)_2(\Omega)_3 \]
\[ \rho_2 I_2(\dot{\Omega})_2 = (\rho_3 I_3 - I_1)(\Omega)_3(\Omega)_1 \]
\[ \rho_3 I_3(\dot{\Omega})_3 = (I_1 - \rho_2 I_2)(\Omega)_1(\Omega)_2 \]

- Looks like Rigid Body with axis \( \rho_2 I_2, \rho_3 I_3, I_1 \)
- \( \rho_2 I_2 > \rho_3 I_3 > I_1 \)
- Controls still preserve the SO(3) symmetry

\[ \Rightarrow \text{Middle axis rotation has been stabilized} \]
**SO(3) Network**

- **Goal:** Align the systems with each rotating about its unstable, middle axis

- **Strategy:**
  - Use kinetic shaping to stabilize middle axis rotation
  - Merge with potential shaping ($\text{tr}(R_i^T R_j)$) to couple systems
  - $SO(3)$ symmetry for closed-loop network
  - Prove stability using Energy-Momentum method

- **For asymptotically stabilizing relative equilibrium**
  - Reduce symmetry to $S^1$
  - Add dissipative controls and use a result from Bullo[1998]
SE(3) network

- Analogous to SO(3) case
- Use kinetic shaping to stabilize individual middle axis translation/rotation
- Use potentials to couple the systems
- Stability proven using Energy-Momentum method
- Add dissipation to achieve asymptotically stable relative equilibria
Region Of Attraction (ROA)

- Question: How large a ROA can one achieve using CL?
- Motivation: Can one swing up and coordinate a network of inverted pendula? (Only one known publication by David Angeli)
- Result: One cannot achieve almost global stability using CL for an inverted pendulum on a cart system.
  - Not a topological obstruction
  - Impossibility arises because of loss of rank of $g_{\alpha a}$
- Solution: Combine CL with other swing up strategies
- Example: Use DMOC to swing up and then switch to CL controller
  - DMOC controller function of time
  - DMOC output is not feedback controller
Swing up of pendula
Swing up of pendula
Energy shaping for collision avoidance

• Collision avoidance important for multivehicle tasks
• Avoid vehicles within your sensor radius
• Current methods
  – Potential based local interaction (potential part of Lagrangian)
  – Gyroscopic force based local interaction (linear in velocity part of Lagrangian)
• New result: Kinetic shaping based local interaction (quadratic in velocity part of Lagrangian)
• Consider $L = \frac{1}{2}(\mathbf{x}_i - \mathbf{x}_j)^2(\dot{\mathbf{x}}_i^2 + \dot{\mathbf{x}}_j^2)$ as Lagrangian for two particle system
  – Can be extended to $n$ particle system
• For $L$, particles don’t collide $\iff \dot{\mathbf{x}}_i(0) + \dot{\mathbf{x}}_j(0) \neq 0$
• Ongoing: cost comparison for the above three collision avoidance schemes
Energy shaping for collision avoidance
Energy shaping for collision avoidance
Area surveillance using hovercrafts

- Multi (underactuated) vehicle (optimal) surveillance with collision avoidance
- Use potentials to design setpoint controllers
- Use gyroscopic forces for collision avoidance
- Probability distribution for events
- Decentralized inner/outer loop philosophy
Area surveillance using hovercrafts

Notation

Area $A$, Lattice $= \{q_{fj} \in L\}$

- Select lattice distribution $A \cap L$
- Design (optimal) set point controllers $u_i(q_{fj}, T, t)$
- Assign target $q_{fj}$ to vehicle $i$
- Once vehicles reach targets, assign new targets, delete used targets
- Continue until targets are exhausted or until threshold is reached
Area surveillance using hovercrafts
Area surveillance using hovercrafts
Area surveillance using hovercrafts