Mathematics 211.3 Final Examination

Solutions

1. (a) -321+32.1 = -288.9 so it would be -288

   (b) $\frac{288.9-288}{288} = 0.003$

2. $f(x) = \sqrt{x+4} - 2 = \frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)} = \frac{x+4-4}{\sqrt{x+4}+2} = \frac{x}{\sqrt{x+4}+2}$

3. $f(x) = e^x$

   (a) $e^x = e + e(x-1) + e\frac{1}{2!}(x-1)^2 + e\frac{1}{3!}(x-1)^3 + \cdots$

   (b) $R_2(x) = \frac{e^{\xi}}{3!}(x-1)^3$ where ξ is between x and 1

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4. $\frac{1}{2^n}(2-1) \leq 10^{-4}$ implies that $2^n \geq 10^4$
   i.e. $n \geq \frac{4ln10}{ln2} = 13.3$ and so 14 bisections

5. Setting $x_0 = 1, x_1 = 2$ it follows that
   $x_3 = 2 - \frac{f(2)(2-1)}{f(2)-f(1)} = 1.571$
   $x_4 = 1.571 - \frac{f(1.571)(1.571-2)}{f(1.571)-f(2)} = 1.705$
   (the exact root is $\sqrt{3}$.

6.

1.01	0.17537			
		0.16733		
2.22	0.37784		-0.00555	
		0.15518		-0.00079
3.20	0.52992		-0.00804	
		0.13959		
4.16	0.66393			

   (a) 0.37784+0.15518(2.75-2.22)=0.46009

   (b) 0.17537+0.16733(2.75-1.01)-0.00555(2.75-1.01)(2.75-2.22)-0.00079(2.75-1.01)
   (2.75-2.22)(2.75-3.20)=0.46170

7. (a) $\frac{1}{4\times 2}2(3.20-2.22) = \frac{0.98}{4} = 0.245$
   (b) $\frac{1}{4\times 4}4 \times 1.21^4 = \frac{2.14}{4} = 0.535$

8. the system is $Az = b$ where

$$A = \begin{bmatrix} 2(1.91+0.98) & 1.21 \\ 1.21 & 2(0.98+0.96) \end{bmatrix}$$

and

$$b = \begin{bmatrix} -0.07288 \\ -0.09354 \end{bmatrix}$$

and

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

9.

$$\begin{bmatrix} 0.4 & -0.12836 & & \\ 0.2 & -0.12824 & -0.12820 & \\ 0.1 & -0.12819 & -0.12817 & -0.12817 \end{bmatrix}$$

and the three entrees are computed as follows

$\frac{2^2(-0.12819)+0.12824}{2^2-1}$

$\frac{2^2(-0.12824)+0.12836}{2^2-1}$

$\frac{2^4(-0.12817)+0.12820}{2^4-1}$

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10. (a) $\frac{0.2}{2}(7.389 + 2x9.025 + 2x11.023 + 2x13.464 + 2x16.445 + 2x20.086 + 24.533) = 17.201$
(b) $\frac{0.2}{3}(7.389 + 4x9.025 + 2x11.023 + 4x13.464 + 2x16.445 + 4x20.086 + 24.533) = 17.144$

11. Setting $x = 2.6 + 0.6t$ it follows that
$\int_2^{3.2} f(x)\,dx = 0.6\int_{-1}^{1} f(2.6+0.6)\,dt \doteq 0.6[\frac{5}{9}f(2.6-0.6\sqrt{\frac{3}{5}}) + \frac{8}{9}f(2.6) + \frac{5}{9}f(2.6+0.6\sqrt{\frac{3}{5}})]$

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12. The augmented matrix

$$\left[\begin{array}{rrrr|r} 0 & 2 & 0 & 1 & 0 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{array}\right].$$

becomes after pivoting

$$\left[\begin{array}{rrrr|r} 6 & 1 & -6 & -5 & 6 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 0 & 2 & 0 & 1 & 0 \end{array}\right]$$

and then

$$\left[\begin{array}{rrrr|r} 6 & 1 & -6 & -5 & 6 \\ & \frac{5}{3} & 5 & \frac{11}{3} & -4 \\ & -\frac{11}{3} & 4 & \frac{13}{3} & -11 \\ & 2 & 0 & 1 & 0 \end{array}\right].$$

13.

$$\left[\begin{array}{ccc|c} 4 & -2 & 1 & 15 \\ -3 & -1 & 4 & 8 \\ 1 & -1 & 3 & 13 \end{array}\right]$$

becomes after elimination

$$\left[\begin{array}{ccc|c} 4 & -2 & 1 & 15 \\ & -\frac{5}{2} & \frac{19}{4} & \frac{77}{4} \\ & & \frac{36}{20} & \frac{108}{20} \end{array}\right]$$

and so $x_3 = 3, x_2 = -2, x_1 = 2$

14. $||A|| = 4 \times -\frac{5}{2} \times \frac{36}{20} = -18$