

UNIVERSITY OF SASKATCHEWAN
Department of Mathematics
and Statistics

Mathematics 211.3 Final Examination

December 18, 1996

Time: 3 hours

Instructor:

M.J. Miket

Note:

1. This is an open book exam.
 2. Your name and student ID number should be on all examination material handed in.
 3. Show all your work and clearly identify your answers.
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1. Using a (hypothetical) computer based on the decimal system of numbers that carries three digits and performs arithmetic with chopping

(a) compute $-321+32.1$

(b) compute the relative error of the above result.

(2 marks)

2. How would you compute the function $f(x)$, defined by $f(x) = \sqrt{x+4} - 2$ for values of x near 0 in such a way that loss of accuracy due to subtraction of almost equal numbers is avoided?

(2 marks)

3. For the function $f(x) = e^x$

(a) find the first three terms of the Taylor series around $x_0 = 1$

(b) write down the appropriate remainder.

(4 marks)

4. How many bisections will be needed to compute the root of

$$f(x) = x^3 + x^2 - 3x - 3$$

contained in $[1,2]$ with (absolute) error not exceeding 10^{-4} ?

(3 marks)

5. Use the secant method to approximately compute the root from the previous problem.
(4 marks)
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6. The following table of values is known about the function $f(x)$:

x	1.01	2.22	3.20	4.16
$f(x)$	0.17537	0.37784	0.52992	0.66393

Find an approximation for $f(2.75)$ using

- (a) linear interpolation
(b) all points.
- (6 marks)
7. If $|f^i(x)| \leq i$ for all i and all x , what is an error bound for the interpolation in the previous problem.
(4 marks)
8. If a natural cubic spline, $s(x)$, is used to interpolate the data given in the preceding problem, it would be, first, necessary to find the values of the second derivative, i.e. $s''(x)$, at the given knots.
Write down the linear system of equations for the values of the second derivative. Do not solve the system.
(5 marks)
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x	$F(h)$
0.1	-0.12819
0.2	-0.12824
0.4	-0.12836

9. If the quantity Q is approximated by the expression $F(h)$ so that

$$Q = F(h) + c_1 h^2 + c_2 h^4 + c_3 h^6 + \dots,$$

what is the best approximation of Q based on the above numerical results and using Richardson's extrapolation.
(4 marks)

10. The following information about the function $f(x)$ is known

x	2.0	2.2	2.4	2.6	2.8	3.0	3.2
$f(x)$	7.389	9.025	11.023	13.464	16.445	20.086	24.533

It is necessary to estimate $\int_2^{3.2} f(x) dx$ using these data.

- (a) Estimate this integral using the composite trapezoidal rule.
(b) Estimate this integral using the composite Simpson's rule.

(4 marks)

11. Set up the Gaussian quadrature for $\int_2^{3.2} f(x) dx$ using three points (i.e. what are the weights and the nodes of the quadrature).

(4 marks)

12. Use Gaussian elimination with partial pivoting to reduce the first column of $Ax = b$, where the augmented coefficient matrix is

$$\begin{bmatrix} 0 & 2 & 0 & 1 & 0 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{bmatrix}.$$

(3 marks)

13. Using Gaussian elimination solve the system $Ax = b$, where

$$A = \begin{bmatrix} 4 & -2 & 1 \\ -3 & -1 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

and

$$b = \begin{bmatrix} 15 \\ 8 \\ 13 \end{bmatrix}.$$

(3 marks)

14. Find $|A|$, where A is given in problem 13.

(2 marks)

THE END