

SMOOTH DOUBLE SUBVARIETIES ON SINGULAR VARIETIES

BY M. R. GONZALEZ-DORREGO

ABSTRACT

Let k an algebraically closed field. Let $\text{char } k = 0$.

Let S be a reduced irreducible nonsingular surface such that $2S = X \cap F$, where X is a threefold and F is a $(N - 1)$ -dimensional variety in \mathbb{P}^N , $N \geq 4$, with X normal with canonical nonisolated singularities. Assume that $S \cap \text{Sing}(X) \neq \emptyset$. It is known that, except for a finite number of points, every point of a normal threefold with canonical singularities has an analytic neighbourhood which is (nonsingular) or isomorphic to $P \times \mathbb{A}^1$, where P is a Du Val surface singularity. We prove that S has empty intersection with canonical singularities of X which have analytical neighbourhoods isomorphic to $P \times \mathbb{A}^1$, where P is a rational surface singularity of types \mathbb{A}_{2k} , $k \in \mathbb{N}$, \mathbb{E}_6 and \mathbb{E}_8 . We give conditions for S to have non-empty intersection with canonical singularities of X which have analytical neighbourhoods isomorphic to $P \times \mathbb{A}^1$, where P is a rational surface singularity of types \mathbb{A}_{2k+1} , $k \in \mathbb{N}$, D_n , $n \geq 4$, and \mathbb{E}_7 .

We generalize these results to X , an n -dimensional variety: let Z be a reduced irreducible nonsingular $(n - 1)$ -dimensional variety such that $2Z = X \cap F$, where X is an n -fold and F is a $(N - 1)$ -fold in \mathbb{P}^N , X normal with canonical singularities and such that $Z \cap \text{Sing}(X) \neq \emptyset$.

Let S be a reduced irreducible nonsingular surface such that $2S = X \cap F$, where X is a normal threefold with only terminal singularities of index 1 and F is a $(N - 1)$ -fold in \mathbb{P}^N , $N \geq 4$. Assume that $S \cap \text{Sing}(X) \neq \emptyset$. Terminal singularities of index 1 of X are locally analytically isomorphic to hypersurface singularities given by $f + tg$, where $g \in k[x, y, z, t]$ is arbitrary and $f \in k[x, y, z]$ represents a Du Val singularity. Then S has empty intersection with singularities of X , where f represents a surface singularity either of type \mathbb{A}_{2r} , $r \in \mathbb{N}$, or \mathbb{E}_6 or \mathbb{E}_8 . We give conditions for S to have non-empty intersection with singularities of X which have analytical neighbourhoods isomorphic to $P \times \mathbb{A}^1$, where P is a rational surface singularity of types \mathbb{A}_{2k+1} , $k \in \mathbb{N}$, D_n , $n \geq 4$, and \mathbb{E}_7 .

We also study the case of a reduced irreducible nonsingular surface S such that $2S = X \cap F$, where X is a threefold and F is a $(N - 1)$ -dimensional variety in \mathbb{P}^N , $N \geq 4$, X normal with an isolated Gorenstein threefold singularity P .

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