Mathematics 101.3 Quiz #3—Solutions

The possible answers to all questions are the digits in the **ANSWER SET:**

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

Evaluate the logarithms:

If $\log_2 4096 = 10a + b$ then **(1)** $a =$ and **(2)** $b =$

Solution: $\log_2 4096 = \log_2 2^{12} = 12 = 10(1) + 2$ so **(1)** $a = 1$ **B** and **(2)** $b = 2$ **C**

If $\log_{25} 125 = \dfrac{c}{d}$ then **(3)** $c =$ and **(4)** $d =$

Solution: $\log_{25} 125 = \log_{25} 5^3 = \dfrac{\log_5 5^3}{\log_5 25} = \dfrac{\log_5 5^3}{\log_5 5^2} = \dfrac{3}{2}$ so **(3)** $c = 3$ **D** and **(4)** $d = 2$ **C**

If $\log_4 512 = \dfrac{e}{f}$ then **(5)** $e =$ and **(6)** $f =$

Solution: $\log_4 512 = \dfrac{\log_2 2^9}{\log_2 4} = \dfrac{\log_2 2^9}{\log_2 2^2} = \dfrac{9}{2}$ so **(5)** $e = 9$ **J** and **(6)** $f = 2$ **C**

Find $f'(1)$ if:

(7) $f(x) = \dfrac{1}{12}(3x-1)^4$ **Solution:** $f'(x) = \dfrac{1}{12}4(3x-1)^3(3x-1)' = \dfrac{1}{12}4(3x-1)^3 3$, so

$f'(1) = \dfrac{1}{12}4(3(1)-1)^3 3 = 2^3 = 8$ **I**

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(8) $f(x) = 8\sqrt{x^3+3}$ **Solution:** $f(x) = 8(x^3+3)^{\frac{1}{2}}$, so

$f'(x) = 8\dfrac{1}{2}(x^3+3)^{-\frac{1}{2}}(x^3+3)' = 4(x^3+3)^{-\frac{1}{2}}(3x^2)$, and

$f'(1) = 4(1^3+3)^{-\frac{1}{2}}(3(1)^2) = 4(4)^{-\frac{1}{2}}3 = 6$ **G**

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(9) $f(x) = 54\dfrac{x+1}{x+2}$ **Solution:** $f'(x) = 54\dfrac{(x+2)(1)-(1)(x+1)}{(x+2)^2} = 54\dfrac{1}{(x+2)^2} = \dfrac{54}{(x+2)^2}$, so

$f'(1) = \dfrac{54}{(1+2)^2} = \dfrac{54}{9} = 6$ **G**

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(10) $f(x) = \dfrac{64}{5}\ln(5x+3)$ **Solution:** $f'(x) = \dfrac{64}{5}\dfrac{(5x+3)'}{5x+3} = \dfrac{64}{5}\dfrac{5}{5x+3} = \dfrac{64}{5x+3}$, so

$f'(1) = \dfrac{64}{8} = 8$ **I**

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(11) $f(x) = \dfrac{15}{e}e^{\frac{x+2}{3}}$ **Solution:** $f'(x) = \dfrac{15}{e}e^{\frac{x+2}{3}}\left(\dfrac{x+2}{3}\right)' = \dfrac{15}{e}e^{\frac{x+2}{3}}\left(\dfrac{1}{3}\right) = \dfrac{5}{e}e^{\frac{x+2}{3}}$, so

$f'(1) = \dfrac{5}{e}e^{\frac{1+2}{3}} = 5$ **F**

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(12) $f(x) = \dfrac{1}{9}(x^3+1)^3$ **Solution:** $f'(x) = \dfrac{1}{9}3(x^3+1)^2(x^3+1)' = \dfrac{1}{3}(x^3+1)^2(3x^2) =$, so

$f'(1) = \dfrac{1}{3}(1^3+1)^2(3(1)^2) = 4$ **E**

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(13) $f(x) = 100\ln\left(\dfrac{x+3}{x+4}\right)$ **Solution:** $f(x) = 100[\ln(x+3) - \ln(x+4)]$, so

$f'(x) = 100\left[\dfrac{(x+3)'}{x+3} - \dfrac{(x+4)'}{x+4}\right] = 100\left[\dfrac{1}{x+3} - \dfrac{1}{x+4}\right]$, and

$f'(1) = 100\left[\dfrac{1}{1+3} - \dfrac{1}{1+4}\right] = 100\dfrac{1}{20} = 5$ **F**

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(14) $f(x) = \dfrac{192x^5}{5(x+1)^5}$ **Solution:** $f(x) = \dfrac{192}{5}\left(\dfrac{x}{x+1}\right)^5$, so $f'(x) = \dfrac{192}{5}(5)\left(\dfrac{x}{x+1}\right)^4\left(\dfrac{x}{x+1}\right)' =$

$192\left(\dfrac{x}{x+1}\right)^4\left(\dfrac{(x+1)(1)-x(1)}{(x+1)^2}\right) = 192\left(\dfrac{x}{x+1}\right)^4\left(\dfrac{1}{(x+1)^2}\right) = 192\dfrac{x^4}{(x+1)^6}$, so

$f'(1) = 192\dfrac{1^4}{(1+1)^6} = \dfrac{194}{64} = 3$ **D**

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(15) $f(x) = 36 \ln\left(\frac{(x+3)^7}{(x+2)^5}\right)$ **Solution:** $f(x) = 36\left[7\ln(x+3) - 5\ln(x+2)\right]$, so

$$f'(x) = 36\left[7\frac{(x+3)'}{x+3} - 5\frac{(x+2)'}{x+2}\right] = 36\left[\frac{7}{x+3} - \frac{5}{x+2}\right], \text{ and}$$

$$f'(1) = 36\left[\frac{7}{1+3} - \frac{5}{1+2}\right] = 36\left[\frac{7}{4} - \frac{5}{3}\right] = 36\left[\frac{21-20}{12}\right] = 36\left[\frac{1}{12}\right] = 3$$ **D**

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(16) $f(x) = 2x^5e^{-x+1}$ **Solution:** $f'(x) = 2\left[(x^5)'e^{-x+1} + x^5\left(e^{-x+1}\right)'\right] =$

$$2\left[5x^4e^{-x+1} + x^5e^{-x+1}(-x+1)'\right] = 2\left[5x^4e^{-x+1} + x^5e^{-x+1}(-1)\right] = 2x^4e^{-x+1}[5-x], \text{ so}$$

$$f'(1) = 2(1)^4e^{-1+1}[5-1] = 2[4] = 8$$ **I**

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(17) $f(x) = \frac{x^5e}{e^x}$ **Solution:** $f(x) = e\frac{x^5}{e^x}$, so $f'(x) = e\frac{e^x(x^5)' - x^5(e^x)'}{(e^x)^2} = e\frac{e^x(5x^4) - x^5e^x}{e^{2x}}$, and

$$f'(1) = e\frac{e^1(5(1)^4) - (1)^5e^1}{e^{2(1)}} = e\frac{e(5)-e}{e^2} = 4$$ **E**

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(18) $f(x) = -2\frac{e^{x-1}}{x^5}$ **Solution:**

$$f'(x) = -2\frac{x^5\left(e^{x-1}\right)' - \left(e^{x-1}\right)(x^5)'}{(x^5)^2} = -2\frac{x^5e^{x-1}(1) - e^{x-1}(5x^4)}{x^{10}} = -2e^{x-1}\frac{x^5-5x^4}{x^{10}}, \text{ so}$$

$$f'(1) = -2e^{1-1}\frac{1^5 - 5(1)^4}{1^{10}} = -2\frac{-4}{1^{10}} = 8.$$ **I**

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(19) $f(x) = \frac{5}{3\ln 5}5^{x^3-1}$ **Solution:** $f'(x) = \frac{5}{3\ln 5}(\ln 5)5^{x^3-1}(x^3-1)' = \frac{5}{3}5^{x^3-1}3x^2$, so
$f'(1) = \frac{5}{3}5^{1^3-1}3(1)^2 = 5$ **F**

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(20) $f(x) = \frac{9^x}{\ln 9}$ **Solution:** $f'(x) = \frac{1}{\ln 9}(9^x)' = \frac{1}{\ln 9}(\ln 9)9^x = 9^x$, so $f'(1) = 9^1 = 9$ **J**

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(21) $f(x) = 2\left(e^{\pi}\right)^3 + 3x^2$ **Solution:** 6 **G**

Solve for x:

(22) $\log_7 7^x = 5$ **Solution:** $\log_7 7^x = x \log_7 7 = x(1) = 5$ **F**

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(23) $\log_5 x^2 + \log_5 x = 3$ **Solution:** $\log_5 x^2 + \log_5 x = \log_5 x^3 = 3\log_5 x = 3$, so

$\log_5 x = 1$, $5^{(\log_5 x)} = x = 5^1 = 5$, and $x = 5$ **F**

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(24) $\log_{10} x^{\frac{7}{2}} - \log_{10} \sqrt{x^3} = \log_{10} 25$

Solution: $\log_{10} x^{\frac{7}{2}} - \log_{10} x^{\frac{3}{2}} = \log_{10} \dfrac{x^{\frac{7}{2}}}{x^{\frac{3}{2}}} = \log_{10} x^2$, and

$\log_{10} x^2 = \log_{10} 25$ implies $x^2 = 25$, so $x = 5$. **F**

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(25) $\log_3 3^{3x} = 18$ **Solution:** $\log_3 3^{3x} = 3x \log_3 3 = 3x(1) = 3x = 18$, so $x = 6$ **G**
