Mathematics 101.3 Practice Quiz #3 Solutions

The possible answers to all questions are the digits in the **ANSWER SET:**

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

Evaluate the logarithms:

If $\log_2 2048 = 10a + b$ then **(1)** $a =$ and **(2)** $b =$

Solution: $\log_2 2048 = \log_2 2^{11} = 11 = 10(1) + 1$ so **(1)** $a = 1$ **B** and **(2)** $b = 1$ **B**

If $\log_9 243 = \dfrac{c}{d}$ then **(3)** $c =$ and **(4)** $d =$

Solution: $\log_9 243 = \log_9 3^5 = \dfrac{\log_3 3^5}{\log_3 9} = \dfrac{\log_3 3^5}{\log_3 3^2} = \dfrac{5}{2}$ so **(3)** $c = 5$ **F** and **(4)** $d = 2$ **C**

If $\log_8 128 = \dfrac{e}{f}$ then **(5)** $e =$ and **(6)** $f =$

Solution: $\log_8 128 = \dfrac{\log_2 2^7}{\log_2 8} = \dfrac{\log_2 2^7}{\log_2 2^3} = \dfrac{7}{3}$ so **(5)** $e = 7$ **H** and **(6)** $f = 3$ **D**

Find $f'(1)$ if:

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(7) $f(x) = (2x-1)^4$ **Solution:** $f'(x) = 4(2x-1)^3(2x-1)' = 4(2x-1)^3(2)$, so $f'(1) = 8$ **I**

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(8) $f(x) = 16\sqrt{x^2+3}$ **Solution:** $f(x) = 16(x^2+3)^{\frac{1}{2}}$, so

$f'(x) = 16\dfrac{1}{2}(x^2+3)^{-\frac{1}{2}}(x^2+3)' = 16\dfrac{1}{2}(x^2+3)^{-\frac{1}{2}}(2x)$, and

$f'(1) = 16\dfrac{1}{2}(1^2+3)^{-\frac{1}{2}}(2(1)) = 8(4)^{-\frac{1}{2}}2 = 8\dfrac{1}{\sqrt{4}} = 8\dfrac{1}{2}2 = 8$ **I**

(9) $f(x) = 8\dfrac{x+1}{x-1}$ **Solution:** **Undefined**

(9) Corrected $f(x) = 8\dfrac{x-1}{x+1}$ **Solution:** $f'(x) = 8\dfrac{(x+1)(1)-(1)(x-1)}{(x+1)^2} = 8\dfrac{2}{(x+1)^2} = \dfrac{16}{(x+1)^2}$, so $f'(1) = \dfrac{16}{(1+1)^2} = 4$ **E**

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(10) $f(x) = 8\ln(3x+5)$ **Solution:** $f'(x) = 8\dfrac{(3x+5)'}{3x+5} = 8\dfrac{3}{3x+5} = \dfrac{24}{3x+5}$, so $f'(1) = \dfrac{24}{8} = 3$ **D**

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(11) $f(x) = 8e^{\left(\frac{x-1}{2}\right)}$ **Solution:** $f'(x) = 8e^{\left(\frac{x-1}{2}\right)}\left(\dfrac{x-1}{2}\right)' = 8e^{\left(\frac{x-1}{2}\right)}\dfrac{1}{2}$, so $f'(1) = 8e^0\dfrac{1}{2} = 4$
E

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(12) $f(x) = \dfrac{1}{20}(x^2+1)^5$ **Solution:** $f'(x) = \dfrac{1}{20}5(x^2+1)^4(x^2+1)' = \dfrac{1}{20}5(x^2+1)^4(2x) = \dfrac{x}{2}(x^2+1)^4$, so $f'(1) = \dfrac{1}{2}(1^2+1)^4 = \dfrac{1}{2}2^4 = 8$ **I**

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(13) $f(x) = 60\ln\left(\dfrac{x+2}{x+3}\right)$ **Solution:** $f(x) = 60[\ln(x+2) - \ln(x+3)]$, so

$f'(x) = 60\left[\dfrac{(x+2)'}{x+2} - \dfrac{(x+3)'}{x+3}\right] = 60\left[\dfrac{1}{x+2} - \dfrac{1}{x+3}\right]$, and $f'(1) = 60\left[\dfrac{1}{1+2} - \dfrac{1}{1+3}\right] = 60\dfrac{1}{12} = 5$
F

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(14) $f(x) = \dfrac{256x^3}{(x+3)^3}$ **Solution:** $f(x) = 256\left(\dfrac{x}{x+3}\right)^3$, so $f'(x) = 256(3)\left(\dfrac{x}{x+3}\right)^2\left(\dfrac{x}{x+3}\right)' =$

$256(3)\left(\dfrac{x}{x+3}\right)^2\left(\dfrac{(x+3)(1)-x(1)}{(x+3)^2}\right) = 256(3)\left(\dfrac{x}{x+3}\right)^2\left(\dfrac{3}{(x+3)^2}\right) = 256(3)\dfrac{3x^2}{(x+3)^4}$, so

$f'(1) = 256(3)\dfrac{3(1)^2}{(1+3)^4} = 256(3)\dfrac{3}{4^4} = 9$ **J**

(15) **Corrected** $f(x) = \frac{6}{11}\ln(x+1)^4(x+2)^5$ **Solution:** $f(x) = \frac{6}{11}[4\ln(x+1)+5\ln(x+2)]$, so $f'(x) = \frac{6}{11}\left[4\frac{(x+1)'}{x+1} + 5\frac{(x+2)'}{x+2}\right] = \frac{6}{11}\left[\frac{4}{x+1} + \frac{5}{x+2}\right]$, and

$$f'(1) = \frac{6}{11}\left[\frac{4}{1+1} + \frac{5}{1+2}\right] = \frac{6}{11}\left[2 + \frac{5}{3}\right] = \frac{6}{11}\left[\frac{6}{3} + \frac{5}{3}\right] = \frac{6}{11}\left[\frac{11}{3}\right] = 2 \text{ C}$$

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(16) $f(x) = x^3e^{-x+1}$ **Solution:** $f'(x) = (x^3)'e^{-x+1} + x^3e^{-x+1}(-x+1)' =$

$3x^2e^{-x+1} + x^3e^{-x+1}(-x+1)' = 3x^2e^{-x+1} + x^3e^{-x+1}(-1)$, so

$$f'(1) = 3(1)^2e^{-1+1} + (1)^3e^{-1+1}(-1) = 3(1) - 1 = 2 \text{ C}$$

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(17) $f(x) = \frac{x^3e}{e^x}$ **Solution:** $f(x) = e\frac{x^3}{e^x}$, so $f'(x) = e\frac{e^x(x^3)' - x^3(e^x)'}{(e^x)^2} = e\frac{e^x(3x^2) - x^3e^x}{e^{2x}}$, and

$$f'(1) = e\frac{e^1(3(1)^2) - (1)^3e^1}{e^{2(1)}} = e\frac{e(3) - e}{e^2} = 2 \text{ C}$$

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(18) $f(x) = -\frac{e^{x-1}}{x^3}$ **Solution:**

$f'(x) = -\frac{x^3\left(e^{x-1}\right)' - \left(e^{x-1}\right)(x^3)'}{(x^3)^2} = -\frac{x^3e^{x-1}(1) - e^{x-1}(3x^2)}{x^6} = -e^{x-1}\frac{x^3 - 3x^2}{x^6}$, so

$$f'(1) = -e^{1-1}\frac{(1)^3 - 3(1)^2}{(1)^6} = -(-2) = 2. \text{ C}$$

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(19) $f(x) = \frac{1}{\ln 2}2^{x^2-1}$ **Solution:** $f'(x) = \frac{1}{\ln 2}(\ln 2)2^{x^2-1}(x^2-1)' = 2^{x^2-1}(2x)$, so $f'(1) = 2^{(1)^2-1}(2(1)) = 2^0(2) = 2$ **C**

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(20) $f(x) = \frac{3^x}{\ln 3}$ **Solution:** $f'(x) = \frac{1}{\ln 3}(3^x)' = \frac{1}{\ln 3}(\ln 3)3^x = 3^x$, so $f'(1) = 3^1 = 3$ **D**

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(21) $f(x) = 2\pi^3 + 9x$ **Solution:** 9 **J**

Solve for x:

(22) $\log_5 5^x = 7$ **Solution:** $\log_5 5^x = x \log_5 5 = x(1) = 7$ **H**

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(23) $\log_3 x^2 + \log_3 x = 3$ **Solution:** $\log_3 x^2 + \log_3 x = \log_3 x^3 = 3\log_3 x = 3$, so

$\log_3 x = 1$, $3^{(\log_3 x)} = x = 3^1 = 3$, and $x = 3$ **D**

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(24) $\log_{10} x^{\frac{5}{2}} - \log_{10} \sqrt{x} = \log_{10} 25$

Solution: $\log_{10} x^{\frac{5}{2}} - \log_{10} \sqrt{x} = \log_{10} \frac{x^{\frac{5}{2}}}{\sqrt{x}} = \log_{10} \frac{x^{\frac{5}{2}}}{x^{\frac{1}{2}}} = \log_{10} x^{\frac{5}{2}-\frac{1}{2}} = \log_{10} x^2$, and

$\log_{10} x^2 = \log_{10} 25$ implies $x^2 = 25$, so $x = 5$. **F**

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(25) $\log_7 7^{2x} = 8$ **Solution:** $\log_7 7^{2x} = 2x \log_7 7 = 2x(1) = 2x = 8$, so $x = 4$ **E**
