Mathematics 101.3 Practice Quiz #2— Solutions

PART I

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

Evaluate the limits:

(1) $\lim_{x\to -5} \frac{10-3x-x^2}{x+5} = \lim_{x\to -5} \frac{(x+5)(2-x)}{x+5} = \lim_{x\to -5} 2-x = 2-(-5) = 7$ **H**

(2) $\lim_{x\to 7} \frac{3x^2-28x+49}{2x-14} = \lim_{x\to 7} \frac{(3x-7)(x-7)}{2(x-7)} = \lim_{x\to 7} \frac{3x-7}{2} = \frac{3(7)-7}{2} = 7$ **H**

(3) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ if $f(x) = -\frac{3}{x}$ **Correction: and $x = 1$** **D**

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-\frac{3}{x+h} - \left(-\frac{3}{x}\right)}{h} = \lim_{h\to 0} \frac{\frac{-3}{x+h} + \frac{3}{x}}{h} = \lim_{h\to 0} \frac{\frac{-3}{x+h}\left(\frac{x}{x}\right) + \frac{3}{x}\left(\frac{x+h}{x+h}\right)}{h} =$$

$$\lim_{h\to 0} \frac{\frac{-3x}{x(x+h)} + \frac{3(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{-3x+3x+3h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{3h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{3}{x(x+h)} = \frac{3}{x(x+0)} = \frac{3}{x^2}$$

When $x = 1$, $\frac{3}{x^2} = \frac{3}{1^2} = 3$

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(4) The natural domain of the function $f(x) = \sqrt{9x+1}$ is of the form $[-\frac{1}{a}, \infty)$. $a = ?$ **J**

We need $9x + 1 \geq 0$, so we must have $9x \geq -1$ and thus $x \geq -\frac{1}{9}$, so $a = 9$.

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(5) If $f(x) = 4x^2 - 6x + 9$ then $f'(1) = ?$ $f'(x) = 8x - 6$, so $f'(1) = 8(1) - 6 = 2$ **C**

(6) If $f(x) = 5x^2 - 3x + 20$ then $f'(1) = ?$ $f'(x) = 10x - 3$, so $f'(1) = 10(1) - 3 = 7$ **H**

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(7) The y-intercept of the line tangent to the graph $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$ is: B

Since $y = x^{-1}$, we have $y' = (-1)x^{-1-1} = -x^{-2}$. When $x = 2$ we have $y' = -2^{-2} = -\frac{1}{4}$, the slope of the tangent line at the point $\left(2, \frac{1}{2}\right)$. The Point-Slope Form of the equation of the tangent line is thus $y - \frac{1}{2} = -\frac{1}{4}(x-2)$. We set set $x = 0$ and solve for y to get the y-intercept:
$y - \frac{1}{2} = -\frac{1}{4}0 - 2) = \frac{1}{2}$, $y = \frac{1}{2} + \frac{1}{2} = 1$.

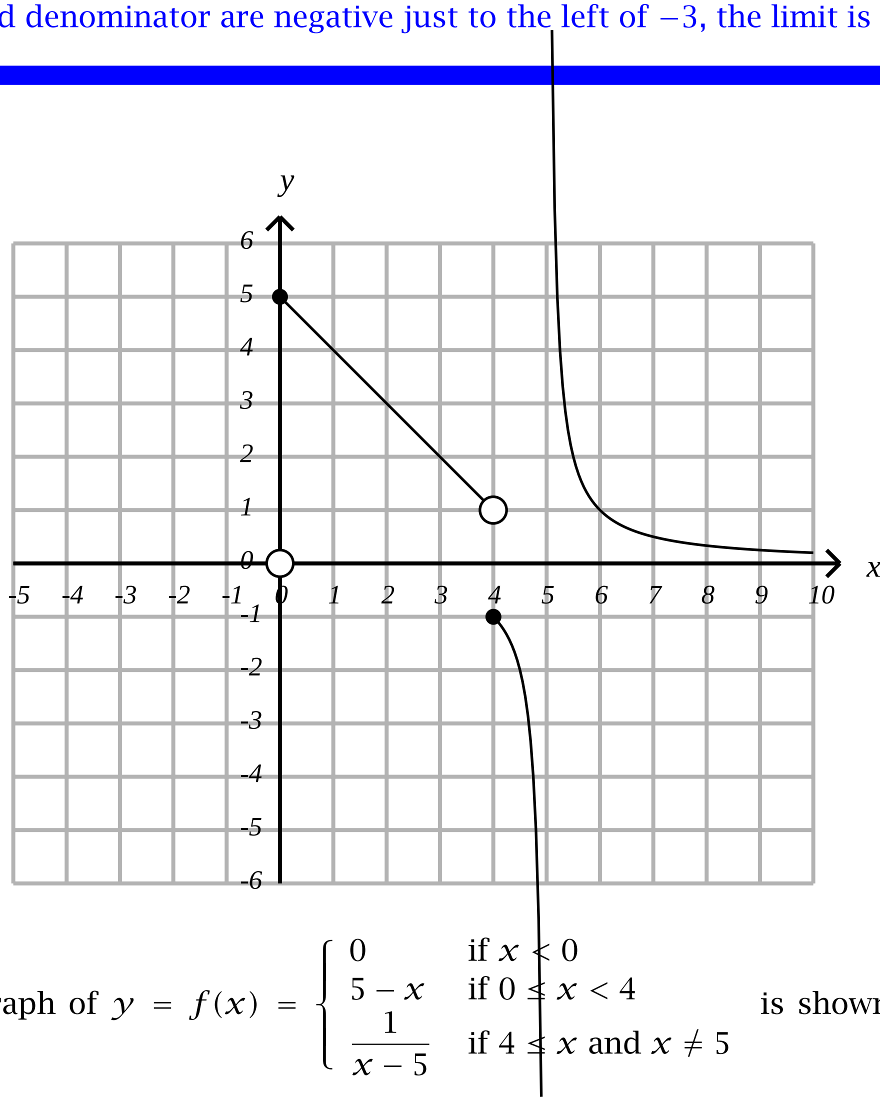
PART II

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) $-\infty$ (B) -5 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 5 (I) 8 (J) ∞

(8) $\lim_{x \to -3^-} \frac{4x+7}{x+3} = ?$ J

Since $\lim_{x \to -3^-} 4x + 7 = 4(-3) + 7 = -5$ and $\lim_{x \to -3^-} x + 3 = -3 + 3 = 0$ the limit is not finite. Since both numerator and denominator are negative just to the left of -3, the limit is $+\infty$.



Part of the graph of $y = f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 5 - x & \text{if } 0 \leq x < 4 \\ \dfrac{1}{x-5} & \text{if } 4 \leq x \text{ and } x \neq 5 \end{cases}$ is shown above. Find: **Note the**

renumbering of the questions!

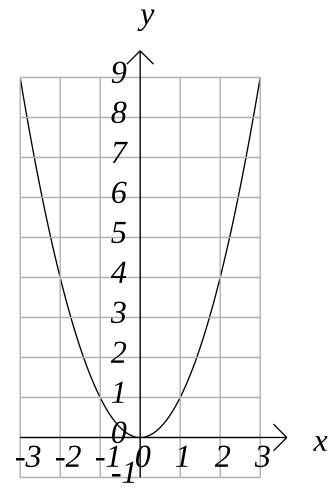
The answers are simply read off from the graph.

(9) $\lim_{x\to-\infty} f(x)$ E **(10)** $\lim_{x\to0^-} f(x)$ E **(11)** $\lim_{x\to0^+} f(x)$ H **(12)** $\lim_{x\to4^-} f(x)$ F

(13) $\lim_{x\to4^+} f(x)$ D **(14)** $\lim_{x\to5^-} f(x)$ A **(15)** $\lim_{x\to5^+} f(x)$ J **(16)** $\lim_{x\to\infty} f(x)$ E

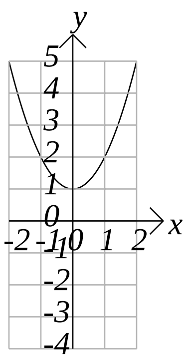
PART III

The graph of $y = f(x) = x^2$ with domain $[-2, 2]$ is shown to the right. Parts of the graphs of

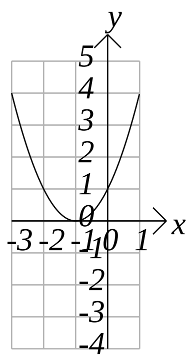


(16) $y = f(x) + 1$, A
(17) $y = f(x + 1)$, B
(18) $y = f(x) - 1$, C
(19) $y = f(x - 1)$, D
(20) $y = -f(x)$, E
(21) $y = 2f(x)$, F
(22) $y = f(2x)$, G
(23) $y = f(x)/2$, H
(24) $y = f(x/2)$, I

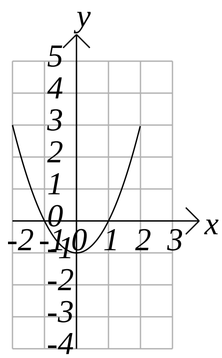
and **(25)** $y = -f(x) + 1$, J are shown below.
Match them.



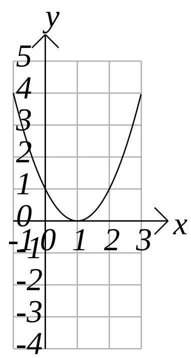
A



B



C



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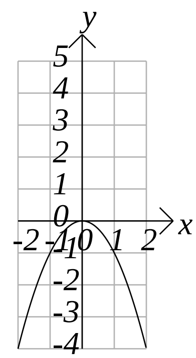
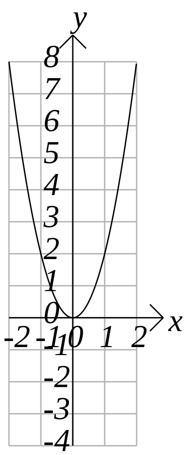
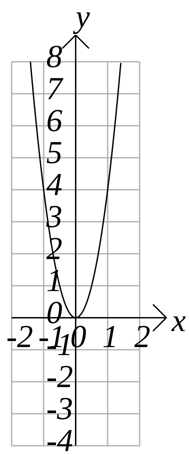


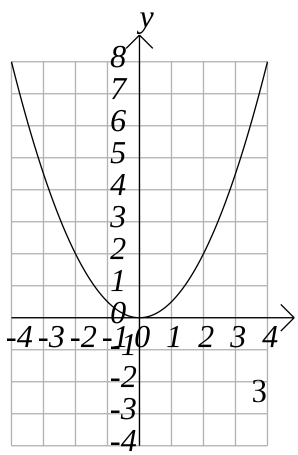
E



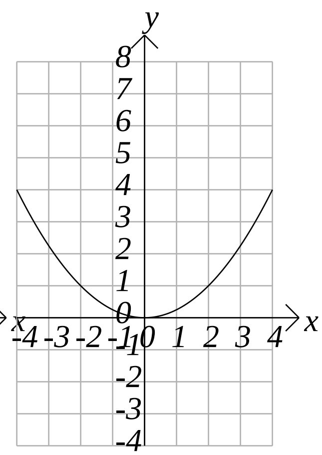
F



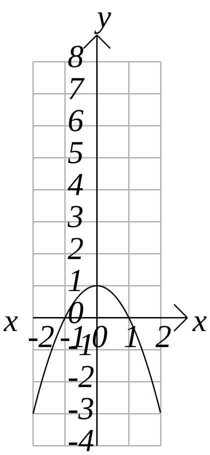
G



H



I



J