UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Practice Final Examination

December, 1999 Time:3 hours Instructors: *MacLean, Marshall*

CLOSED BOOK — GRAPHING CALCULATORS NOT PERMITTED

PART I

The possible answers to all questions in Part I are in **ANSWER SET I:**

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

If $\frac{8}{13} - \frac{3}{5}$ is written in its simplest form as $\frac{a}{10b+c}$, where a, b, and c are digits, then

(1) $a =$ **(2)** $b =$ **(3)** $c =$

$x^2 - 6x + 12$ is to be written in the form $a(x-b)^2 + c$ by completing squares. We must have:

(4) $a =$ **(5)** $b =$ **(6)** $c =$

$6x^2 + 96x + 375$ is to be written in the form $a(x+b)^2 - c$ by completing squares. We must have:

(7) $a =$ **(8)** $b =$ **(9)** $c =$

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(10) If a is the slope of the line through the points $(-1, -9)$ and $(1, 9)$, then a is:

. . . . 2

The roots of $x^2 + x - 5 = 0$ in their simplest form are $\dfrac{-A \pm \sqrt{10B + C}}{D}$. We must have:

(12) $A =$ **(13)** $B =$ **(14)** $C =$ **(15)** $D =$

The polynomial $p(x) = 6x^4 + x^3 + 5x^2 + x - 1$ can be factored in the form $(x-a)(x+b)(cx+1)(dx-1)$, where a, b, c, and d are digits. Their values are:

(16) $a =$ **(17)** $b =$ **(18)** $c =$ **(19)** $d =$

$3\dfrac{2\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$ can be simplified to the expression $a - b\sqrt{10c}$, where a, b, and c are digits. Their values are:

(20) $a =$ **(21)** $b =$ **(22)** $c =$

If we solve the inequality $\left|\dfrac{2 - x}{3}\right| \geq 1$, the solution is an interval of the form $(-\infty, -a] \cup [b, \infty)$. The values of a and b are:

(23) $a =$ **(24)** $b =$

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(25) The y-intercept of the line perpendicular to the line $y = -\frac{x}{5}$ which passes through the point $(-1, 0)$ is:

Evaluate the limits:

(26) $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ if $f(x) = \dfrac{45}{x+1}$ and $x = 2$

(27) $\lim_{x \to 3} \dfrac{3x^2 - 6x - 21}{3x - 21}$

(28) $\lim_{x \to -6} \dfrac{-36 - 12x - x^2}{x + 6}$

…3

(29) The natural domain of the function $f(x) = \sqrt[4]{5x^2 - 1}$ is of the form $\left(-\infty, -\frac{1}{\sqrt{a}}\right] \cup \left[\frac{1}{\sqrt{a}}, \infty\right)$. a =?

(30) The $y-$intercept of the line perpendicular to the line $\frac{x}{6} = y$ which passes through the point $(1,3)$ is:

(31) Let $f(x) = \frac{x^3}{(x+4)^3}$. Find $\frac{f'(-5)}{50}$

(32) The minimum value of $f(x) = x^2 + 8x + 20$ is:

(33) Find the positive solution of the equation $2^{x^2-1} = 8$:

(34) The largest solution of $\ln(x+3) + \ln(x+7) = \ln(20x)$ is:

(35) Find the absolute maximum value of the function $f(x) = 4 + xe^{1-x}$.

(36) A rectangular flower garden is to be fenced in against a long building using 8 metres of fencing. What is the maximum area (in square metres) possible? Note that no fencing needs to be used against the building.

PART II

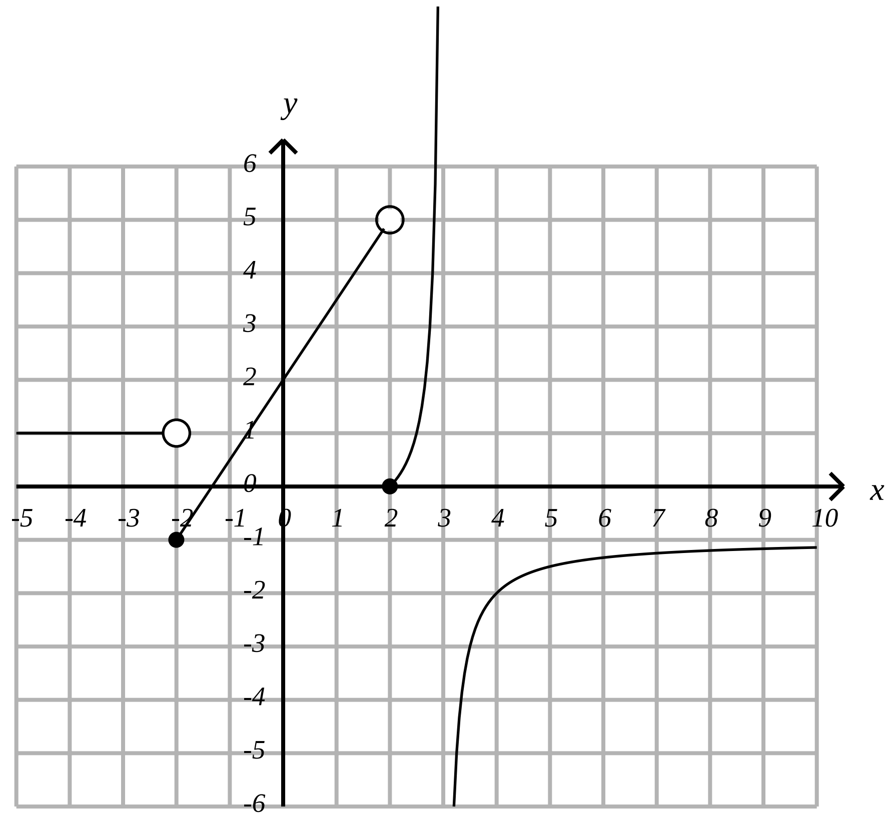
The possible answers to all questions in Part II are in the **ANSWER SET:**

(A) $-\infty$ (B) -4 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 4 (I) 8 (J) ∞

(37) $\lim_{x\to -6^-} \frac{5x-9}{x+6}$ =?

(38) $\lim_{x\to -6^+} \frac{5x-9}{x+6}$ =?

...4



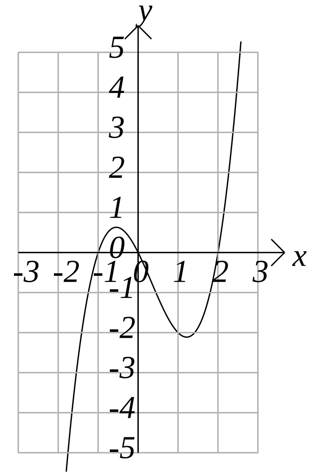
Part of the graph of $y = f(x) = \begin{cases} 1 & \text{if } x < -2 \\ \frac{3}{2}x + 2 & \text{if } -2 \le x < 2 \\ \dfrac{1}{3-x} - 1 & \text{if } 2 \le x \text{ and } x \ne 3 \end{cases}$ is shown above. Find:

(39) $\lim_{x\to-\infty} f(x)$ **(40)** $\lim_{x\to-2^-} f(x)$ **(41)** $\lim_{x\to-2^+} f(x)$ **(42)** $\lim_{x\to2^-} f(x)$

(43) $\lim_{x\to2^+} f(x)$ **(44)** $\lim_{x\to3^-} f(x)$ **(45)** $\lim_{x\to3^+} f(x)$ **(46)** $\lim_{x\to\infty} f(x)$

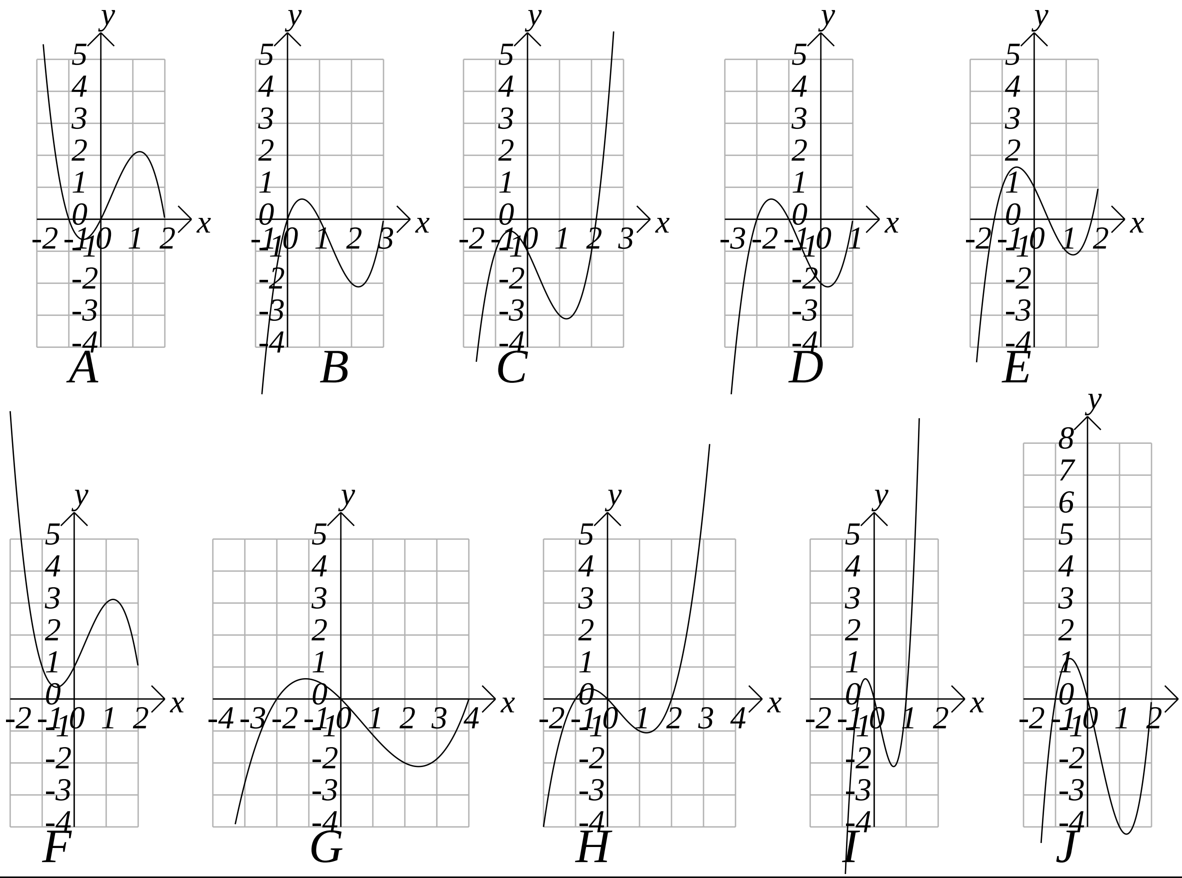
…5

PART III



The graph of $y = f(x) = x(x+1)(x-2)$ is shown to the right. Parts of the graphs of

(47) $y = f(x+1)$,
(48) $y = f(x) - 1$,
(49) $y = f(x-1)$,
(50) $y = -f(x)$,
(51) $y = 2f(x)$,
(52) $y = f(2x)$,
(53) $y = f(x)/2$,
(54) $y = f(x/2)$, and
(55) $y = -f(x) + 1$, are shown below.
Match them.



Evaluate the logarithms:

If $\log_2 8192 = 10a + b$ then **(56)** a = and **(57)** b =

If $\log_{27} 81 = \dfrac{c}{d}$ then **(58)** c = and **(59)** d =

If $\log_4 9343 = \dfrac{e}{f}$ then **(60)** e = and **(61)** f =

…6

The graphs of the functions in questions 17 to 25 are shown below. Match them.

(62) $f(x) = 3x^2 - 4x + 1$

(63) $f(x) = 2x^3 - x^4$

(64) $f(x) = 4x - x^2 + 1$

(65) $f(x) = \dfrac{2x+3}{2x-3}$

(66) $f(x) = \dfrac{1}{2}\left(3x - x^3\right)$

(67) $f(x) = \ln(\sqrt{3x+5})$

(68) $f(x) = \dfrac{1}{10}\left(2x^3 - 6x^2 + 3\right)$

(69) $f(x) = xe^{-x+1}$

(70) $f(x) = \dfrac{1}{20}\left(x^4 - 4x^2\right)$