

UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Practice Final Examination

December, 2000

Time: 3 hours

Instructor: Doug MacLean



CLOSED BOOK — CALCULATORS NOT PERMITTED

PART I

The possible answers to all questions in Part I are the digits in the ANSWER SET:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

If $\frac{8}{11} - \frac{3}{5}$ is written in its simplest form as $\frac{a}{10b+c}$, where a , b , and c are digits, then

(1) $a =$

(2) $b =$

(3) $c =$

$x^2 - 8x + 20$ is to be written in the form $a(x - b)^2 + c$ by completing squares. We must have:

(4) $a =$

(5) $b =$

(6) $c =$

$5x^2 + 50x + 120$ is to be written in the form $a(x + b)^2 - c$ by completing squares. We must have:

(7) $a =$

(8) $b =$

(9) $c =$

(10) If a is the slope of the line through the points $(-2, -8)$ and $(2, 8)$, then a is:

The roots of $2x^2 + x - 5 = 0$ in their simplest form are $\frac{-A \pm \sqrt{10B + C}}{D}$. We must have:

(11) $A =$

(12) $B =$

(13) $C =$

(14) $D =$

The polynomial $p(x) = 6x^4 + x^3 - 7x^2 - x + 1$ can be factored in the form $(x - a)(x + b)(cx + 1)(dx - 1)$, where a , b , c , and d are digits. Their values are:

(15) $a =$

(16) $b =$

(17) $c =$

(18) $d =$

...2



$\frac{2\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ can be simplified to the expression $a - b\sqrt{c}$, where a , b , and c are digits. Their values are:

(19) $a =$

(20) $b =$

(21) $c =$

If we solve the inequality $\left| \frac{3-x}{2} \right| \geq 1$, the solution is an interval of the form $(-\infty, -a] \cup [b, \infty)$. The values of a and b are:

(22) $a =$

(23) $b =$

(24) The x -intercept of the line perpendicular to the line $y = -\frac{x}{4}$ which passes through the point $(1, 0)$ is:

Evaluate the limits:

(25) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = -\frac{5}{x+2}$ and $x = -3$

(26) $\lim_{x \rightarrow -3} \frac{4x^2 + 20x - 56}{2x - 4}$

(27) $\lim_{x \rightarrow -5} \frac{-15 - 8x - x^2}{x + 5}$

(28) The natural domain of the function $f(x) = \sqrt[6]{49x^2 - 1}$ is of the form $(-\infty, -\frac{1}{a}] \cup [\frac{1}{a}, \infty)$. What is the value of a =?

(29) The y -intercept of the line perpendicular to the line $\frac{x}{6} = y$ which passes through the point $(1, 3)$ is:

(30) Let $f(x) = \frac{x^3}{(x+4)^3}$. Find $\frac{f'(-5)}{50}$

(31) The minimum value of $f(x) = x^2 + 8x + 18$ is:

(32) Find the positive solution of the equation $2^{x^2-1} = 8$:

(33) The largest solution of $\ln(x+3) + \ln(x+7) = \ln(20x)$ is:

...3



If $\log_2 8192 = 10a + b$ then (34) $a =$ and (35) $b =$

If $\log_{27} 81 = \frac{c}{d}$ then (36) $c =$ and (37) $d =$

If $\log_{49} 343 = \frac{e}{f}$ then (38) $e =$ and (39) $f =$

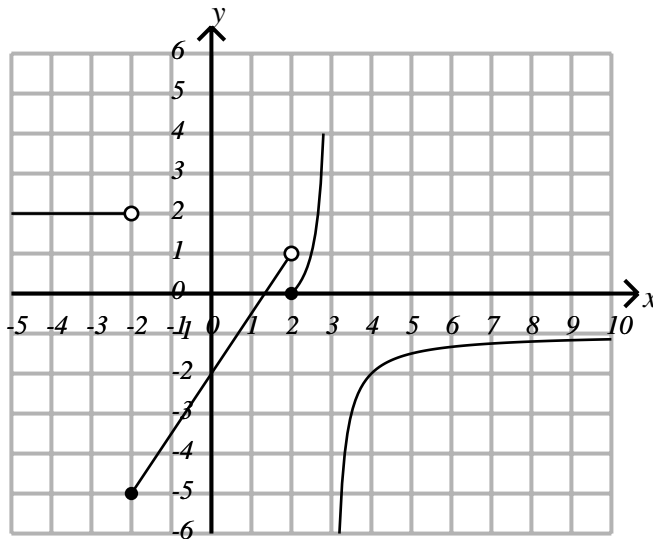
PART II

The possible answers to all questions in Part II are in the ANSWER SET:

(A) $-\infty$ (B) -5 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 4 (I) 5 (J) ∞

(40) $\lim_{x \rightarrow -5^-} \frac{6x - 1}{x + 5} = ?$

(41) $\lim_{x \rightarrow -5^+} \frac{6x - 1}{x + 5} = ?$



Part of the graph of $y = f(x) = \begin{cases} 2 & \text{if } x < -2 \\ \frac{3}{2}x - 2 & \text{if } -2 \leq x < 2 \\ \frac{1}{3-x} - 1 & \text{if } 2 \leq x \text{ and } x \neq 3 \end{cases}$ is shown above. Find:

(42) $\lim_{x \rightarrow -\infty} f(x)$

(43) $\lim_{x \rightarrow -2^-} f(x)$

(44) $\lim_{x \rightarrow -2^+} f(x)$

(45) $\lim_{x \rightarrow 2^-} f(x)$

(46) $\lim_{x \rightarrow 2^+} f(x)$

(47) $\lim_{x \rightarrow 3^-} f(x)$

(48) $\lim_{x \rightarrow 3^+} f(x)$

(49) $\lim_{x \rightarrow \infty} f(x)$

...5

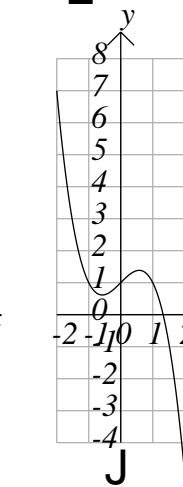
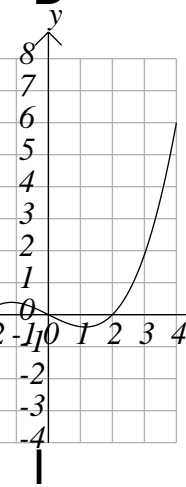
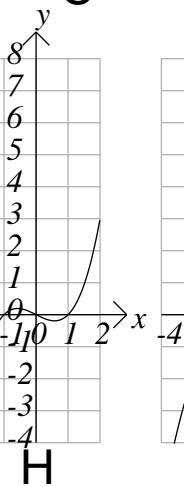
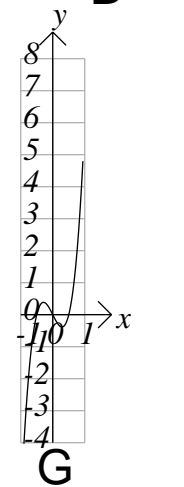
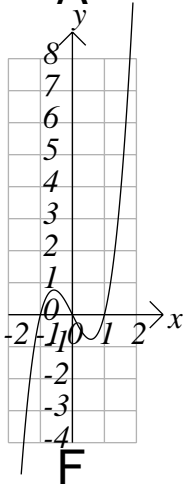
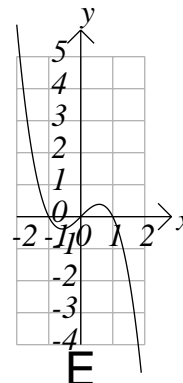
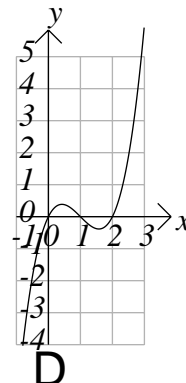
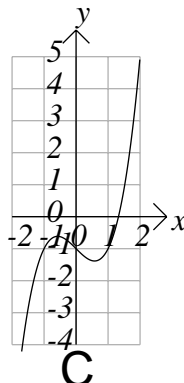
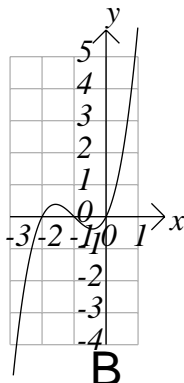
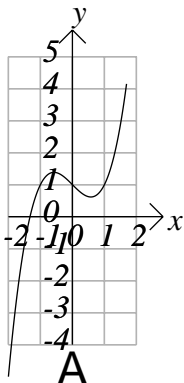
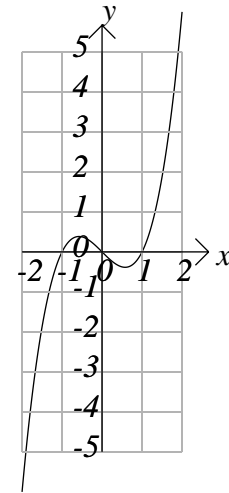


PART III

The graph of $y = f(x) = x(x + 1)(x - 2)$ is shown to the right.

Parts of the graphs of

- (50) $y = f(x + 1)$,
 - (51) $y = f(x) - 1$,
 - (52) $y = f(x - 1)$,
 - (53) $y = -f(x)$,
 - (54) $y = 2f(x)$,
 - (55) $y = f(2x)$,
 - (56) $y = f(x)/2$,
 - (57) $y = f(x/2)$, and
 - (58) $y = -f(x) + 1$, are shown below.
- Match them.



...6



PART IV

The graphs of the functions in questions 59 to 65 are shown below. Match them.

(59) $f(x) = 3x^2 - 4x + 1$

(60) $f(x) = 2x^3 - x^4$

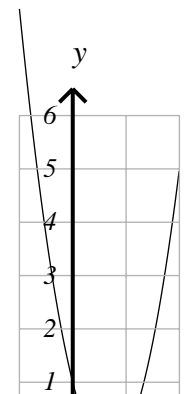
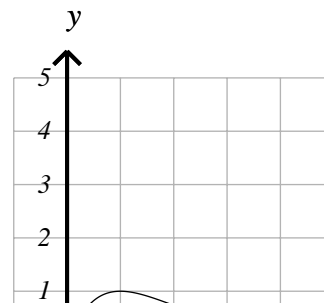
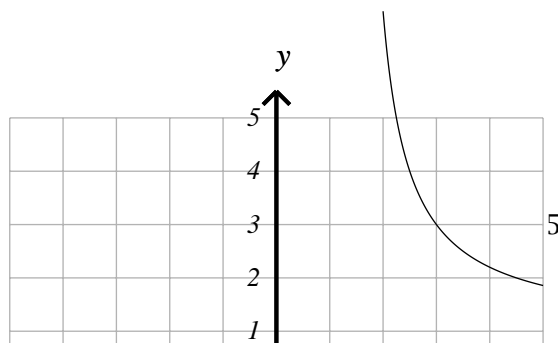
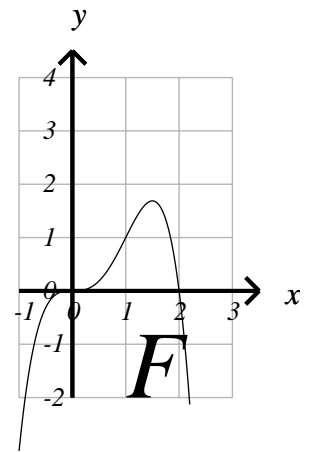
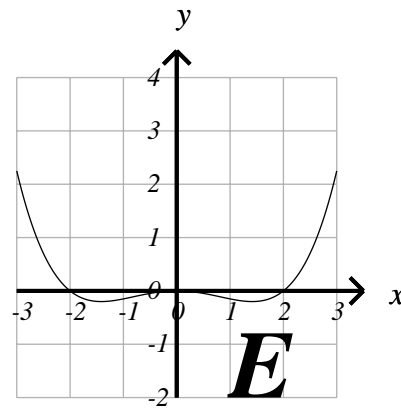
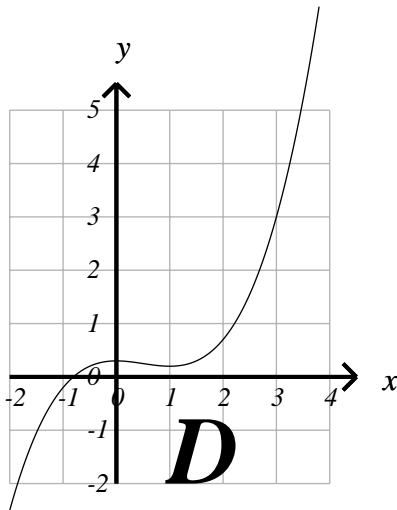
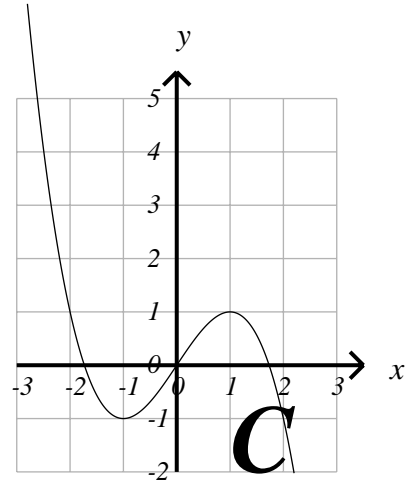
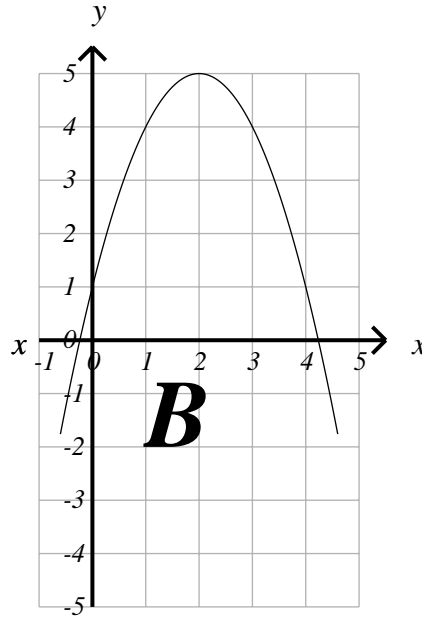
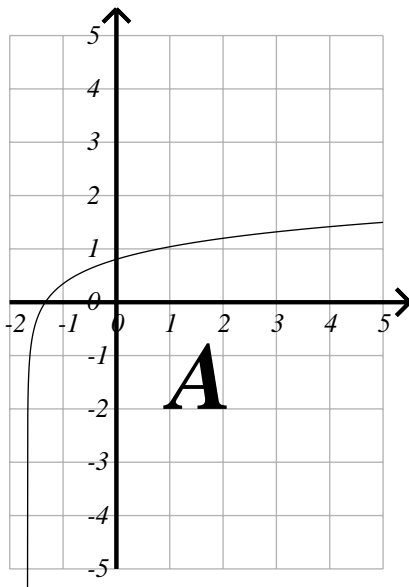
(61) $f(x) = 4x - x^2 + 1$

(62) $f(x) = \frac{2x + 3}{2x - 3}$

(63) $f(x) = \frac{1}{2}(3x - x^3)$

(64) $f(x) = \frac{1}{10}(2x^3 - 6x^2 + 3)$

(65) $f(x) = \frac{1}{20}(x^4 - 4x^2)$





PART V

The graphs of the functions in questions 66 to 73 are shown below. Match them.

(66) $f(x) = e^x$

(67) $f(x) = e^{-x}$

(68) $f(x) = 5e^{-\frac{1}{x^2}}$

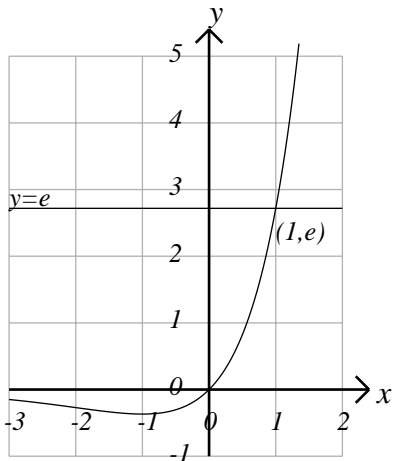
(69) $f(x) = xe^x$

(70) $f(x) = 5xe^{-2x}$

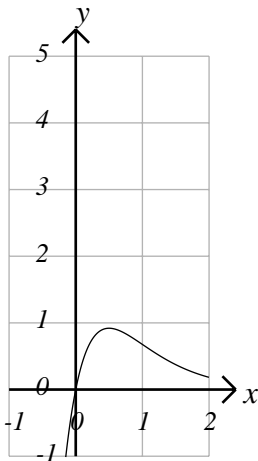
(71) $f(x) = 0.5 * xe^{x^2}$

(72) $f(x) = 5x^2e^{-2x}$

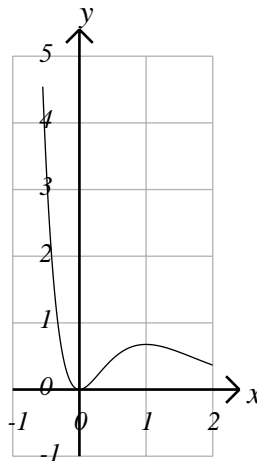
(73) $f(x) = 5x^2e^{2x}$



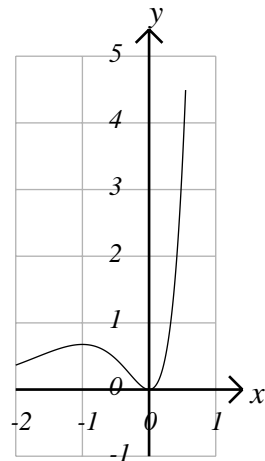
A



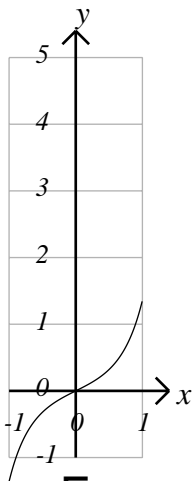
B



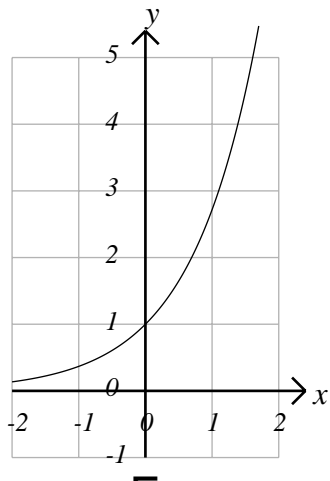
C



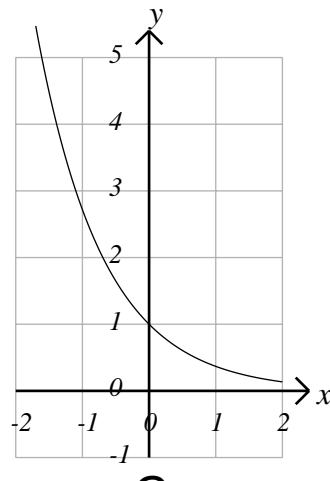
D



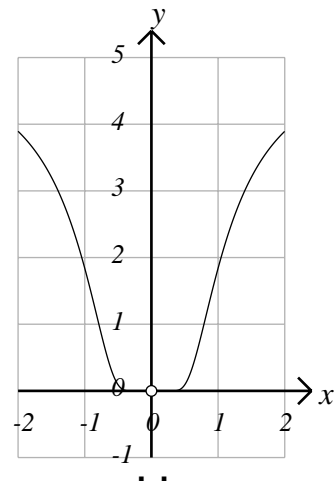
E



F



G



H



PART VI

The graphs of the functions in questions 74 to 80 are shown below. Match them.

(74) $f(x) = \ln x$

(75) $f(x) = x \ln x$

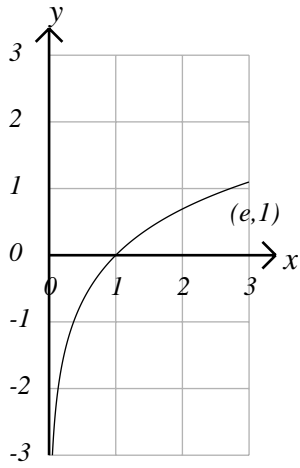
(76) $f(x) = x^2 \ln x$

(77) $f(x) = \ln x^2$

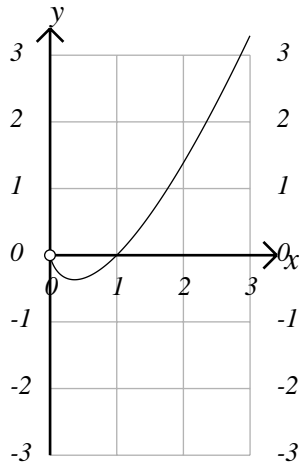
(78) $f(x) = \ln ex$

(79) $f(x) = \ln \sqrt{x}$

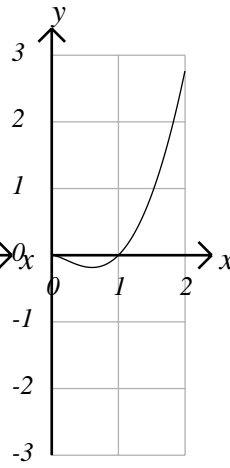
(80) $f(x) = \ln x^x$



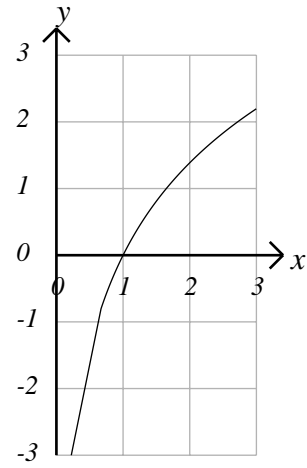
A



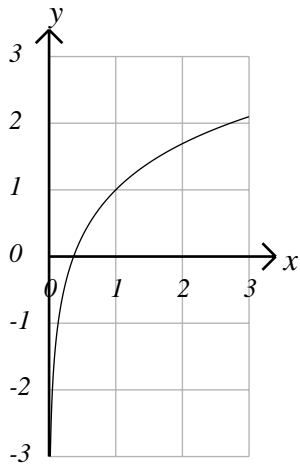
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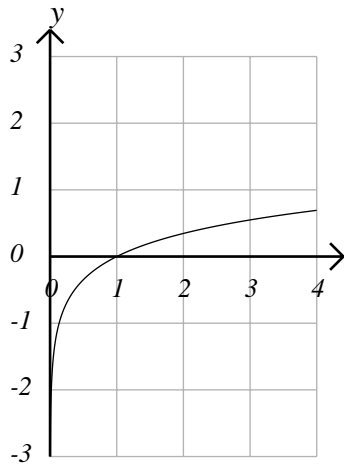
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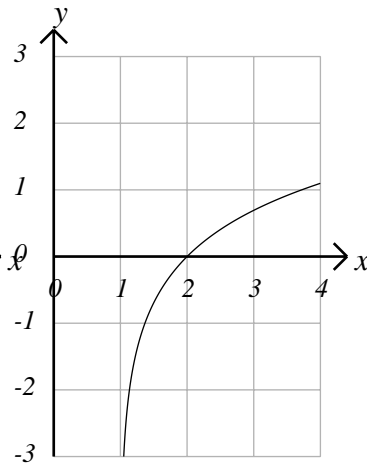
D



E



F



G

PART VII



Each question in this section is worth 5 marks.

The possible answers to all questions in Part VII are the digits in the **ANSWER SET**:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

(81) Find the absolute maximum value of the function $f(x) = 4 + xe^{1-x}$.

(82) A rectangular flower garden is to be fenced in against a long building using 8 metres of fencing. What is the maximum area (in square metres) possible? Note that no fencing needs to be used against the building.

(83) A spoonful of boiling water is placed in a room whose temperature is 20°C . In one minute its temperature drops to 60°C . How many more minutes will it take for its temperature to reach 25°C ?

Hint: The formula $T = T_\infty + (T_0 - T_\infty)e^{kt}$ applies.

(84) What is the area between the graph of $y = 3x^2 + 2x$ and the x -axis from $x = 1$ to $x = 2$?
