

UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Final Examination

December 16, 1999

Time: 3 hours

Instructors: *Lorer, MacLean, Marshall*

CLOSED BOOK — CALCULATORS ARE NOT PERMITTED

PART I: The answers to all questions are the digits in **ANSWER SET I:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

If $\frac{8}{9} - \frac{6}{7}$ is written in its simplest form as $\frac{a}{10b+c}$, where $a, b,$ and c are digits, then

(1) $a =$

(2) $b =$

(3) $c =$

$x^2 + 10x + 33$ is to be written in the form $(x + b)^2 + c$ by completing squares.

We must have:

(4) $b =$

(5) $c =$

$7x^2 + 112x + 490$ is to be written in the form $7[(x + b)^2 + c]$ by completing squares.

We must have:

(6) $b =$

(7) $c =$

(8) If m is the slope of the line through the points $(10, 15)$ and $(20, 75)$, then m is:

The roots of $4x^2 + 3x - 3 = 0$ in their simplest form are $\frac{-a \pm \sqrt{10b+c}}{d}$. We must have:

(9) $a =$

(10) $b =$

(11) $c =$

(12) $d =$

$\frac{3\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$ can be simplified to the expression $a + \sqrt{10b+c}$, where $a, b,$ and c are digits.

Their values are:

(13) $a =$

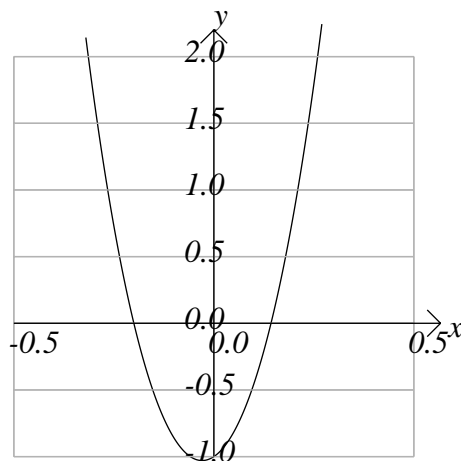
(14) $b =$

(15) $c =$

...2

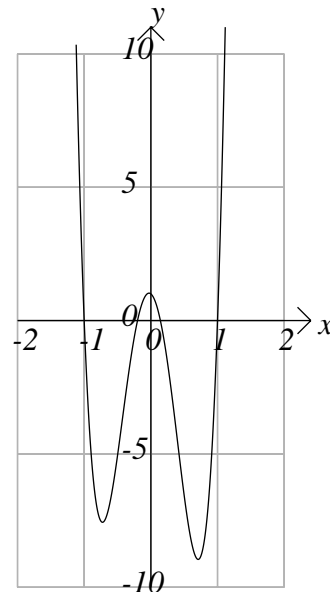
The polynomial $p(x) = 35x^4 + 2x^3 + 34x^2 + 2x - 1$ can be factored in the form $(x^2 + a^2)(bx + 1)(cx - 1)$, where a , b , and c are digits. Its graph is shown to the right.

We must have: (16) $a =$ (17) $b =$ (18) $c =$



The polynomial $p(x) = 35x^4 + 2x^3 - 36x^2 - 2x + 1$ can be factored in the form $(x^2 - a^2)(bx + 1)(cx - 1)$, where a , b , and c are digits. Its graph is shown to the right.

We must have: (19) $a =$ (20) $b =$ (21) $c =$



If we solve the inequality $\left| \frac{1-x}{17} \right| < 1$, the solution is an interval of the form $(-(10+a), 10+b)$.

The values of a and b are:

(22) $a =$

(23) $b =$

Consider the line perpendicular to the line $y = \frac{8x}{5} + 1$ which passes through the point $\left(\frac{40}{13}, \frac{40}{13}\right)$

(24) What is its x -intercept?

(25) What is its y -intercept?

Evaluate the limits:

(26) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = \frac{16}{(x+2)^2}$ and $x = -4$

$$(27) \lim_{x \rightarrow 2} \frac{5x^2 - 5x - 210}{5x - 35}$$

$$(28) \lim_{x \rightarrow 6} \frac{x^2 - 8x + 12}{x - 6}$$

(29) The natural domain of the function $f(x) = \ln(1 - 7x^2)$ is of the form $\left(-\frac{1}{\sqrt{a}}, \frac{1}{\sqrt{a}}\right)$. $a = ?$

(30) Let $f(x) = \frac{(x+1)^3}{(x+5)^3}$. Find $\frac{f'(-6)}{100}$

(31) The minimum value of $f(x) = 3x^2 - 18x + 30$ is:

(32) Find the solution of the equation $2^{3x-1} = 256$:

(33) The largest solution of $\ln(x+4) + \ln(x+5) = \ln(18x)$ is:

(34) Find the absolute maximum value of the function $f(x) = 2 + x^2e^{2-x}$ over the interval $[0, 3]$.

(35) Two non-negative numbers must add up to 16. If they are chosen so as to make their product as large as possible, then one-sixteenth of the maximum value of their product is:

Evaluate the logarithms:

If $\log_2 4096 = 10a + b$ then (36) $a =$ and (37) $b =$

If $\log_{125} 625 = \frac{c}{d}$ then (38) $c =$ and (39) $d =$

If $\log_9 243 = \frac{e}{f}$ then (40) $e =$ and (41) $f =$

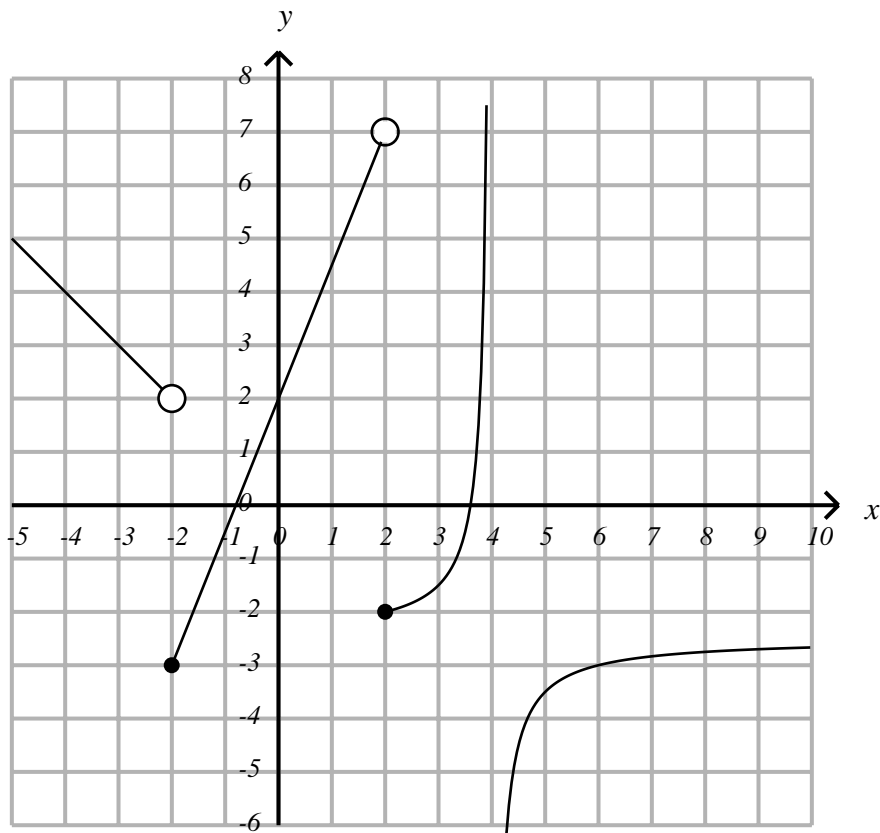
...4

PART II: The answers to all questions are the digits in **ANSWER SET II:**

(A) $-\infty$ (B) -3 (C) -2.5 (D) -2 (E) 0 (F) 1 (G) 2 (H) 4 (I) 7 (J) ∞

(42) $\lim_{x \rightarrow 5^-} \frac{6x - 19}{x - 5} = ?$

(43) $\lim_{x \rightarrow 5^+} \frac{6x - 19}{x - 5} = ?$



Part of the graph of $y = f(x) = \begin{cases} -x & \text{if } x < -2 \\ \frac{5}{2}x + 2 & \text{if } -2 \leq x < 2 \\ \frac{1}{4-x} - 2.5 & \text{if } 2 \leq x \text{ and } x \neq 4 \end{cases}$ is shown above.

Find:

(44) $\lim_{x \rightarrow -\infty} f(x)$

(45) $\lim_{x \rightarrow -2^-} f(x)$

(46) $\lim_{x \rightarrow -2^+} f(x)$

(47) $\lim_{x \rightarrow 2^-} f(x)$

(48) $\lim_{x \rightarrow 2^+} f(x)$

(49) $\lim_{x \rightarrow 4^-} f(x)$

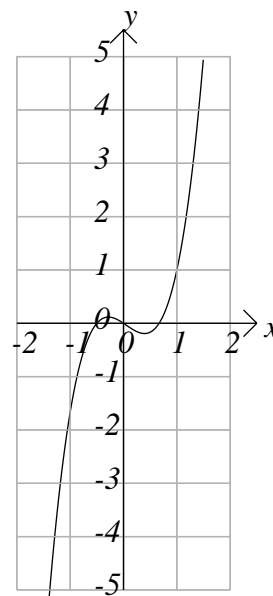
(50) $\lim_{x \rightarrow 4^+} f(x)$

(51) $\lim_{x \rightarrow \infty} f(x)$

...5

PART III

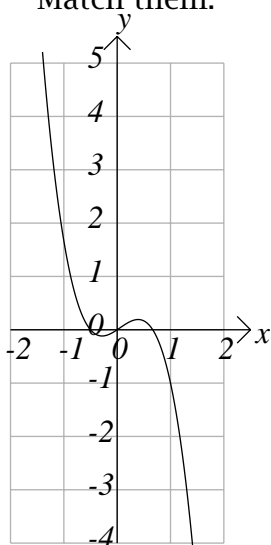
The graph of
 $y = f(x) = x(2x + 1)(3x - 2)/3$
 is shown to the right.



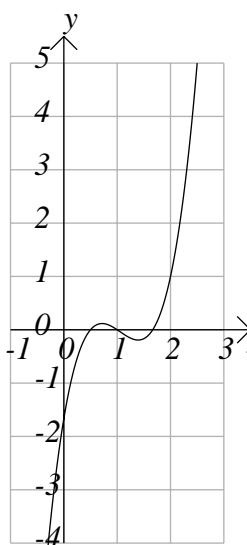
Parts of the graphs of

- (52) $y = f(x + 1)$,
- (53) $y = f(x) - 1$,
- (54) $y = f(x - 1)$,
- (55) $y = -f(x)$,
- (56) $y = 2f(x)$,
- (57) $y = f(2x)$,
- (58) $y = f(x)/2$,
- (59) $y = f(x/2)$, and
- (60) $y = -f(x) + 1$, are shown below.

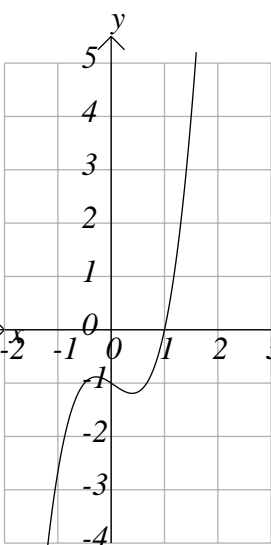
Match them.



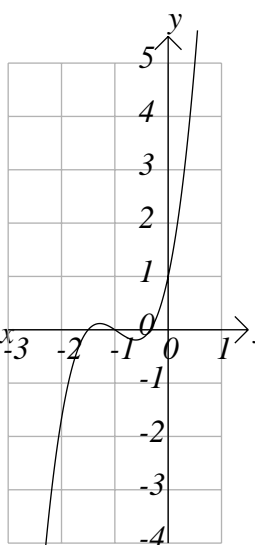
A



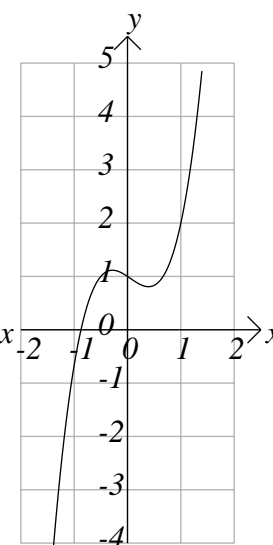
B



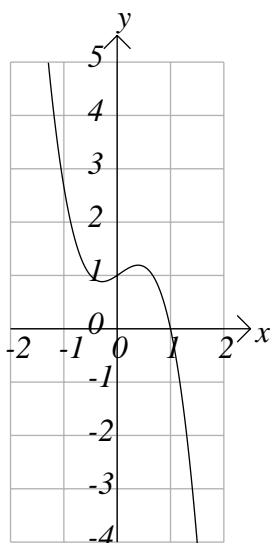
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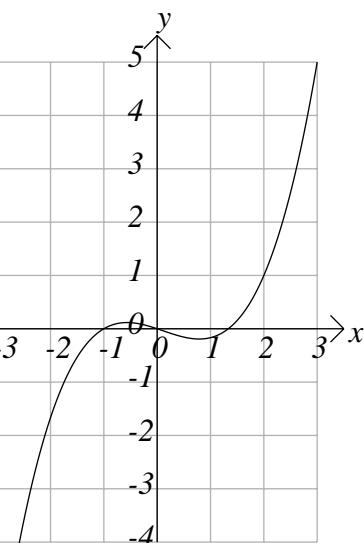
D



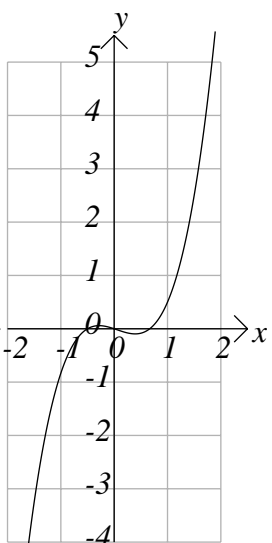
E



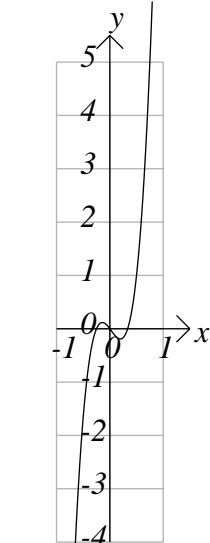
F



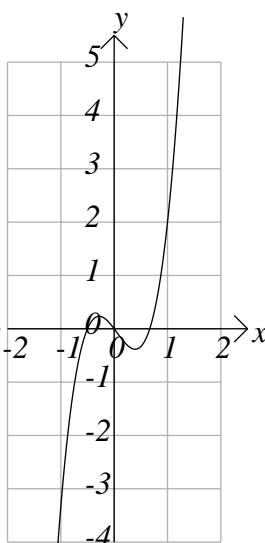
G



H



I



J

...6

The graphs of the functions in questions 61 to 70 are shown below. Match them.

(61) $f(x) = 4x^2 - 3x - 4$

(62) $f(x) = 2x^3 - x^4 + 1$

(63) $f(x) = 3x - 4x^2 - 1$

(64) $f(x) = \frac{12x + 3}{2x - 3}$

(65) $f(x) = \frac{1}{2}(3x - x^3)$

(66) $f(x) = \ln(\sqrt{5x + 3})$

(67) $f(x) = \frac{1}{10}(2x^3 - 3x^2 + 3)$

(68) $f(x) = xe^{-x+1}$

(69) $f(x) = \frac{1}{20}(x^4 - 4x^2) - 1$

(70) $f(x) = \frac{3x + 2}{3x - 2}$

