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The actual numerical value of $f''(x)$ will not be important in this course, we will only be interested in its sign.

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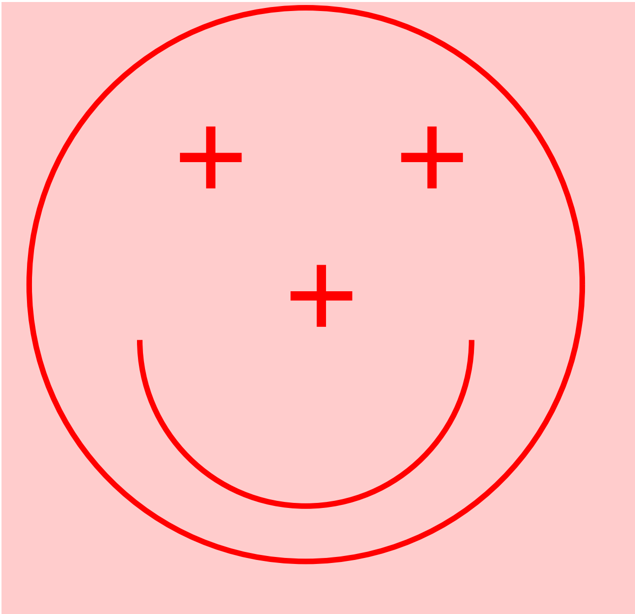

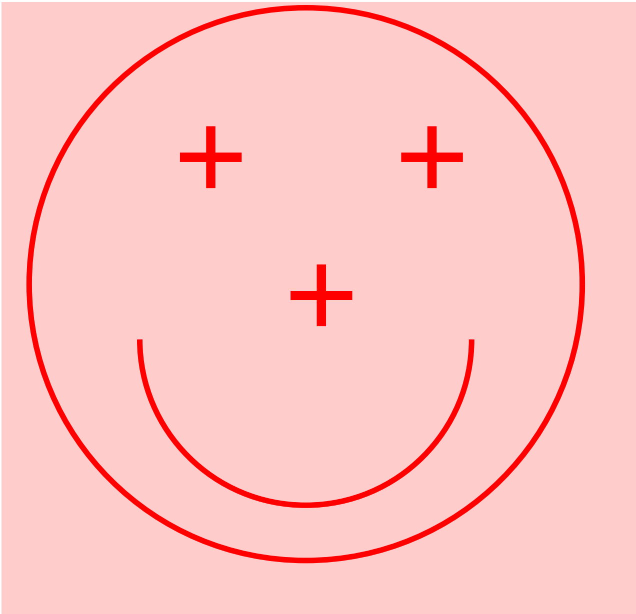

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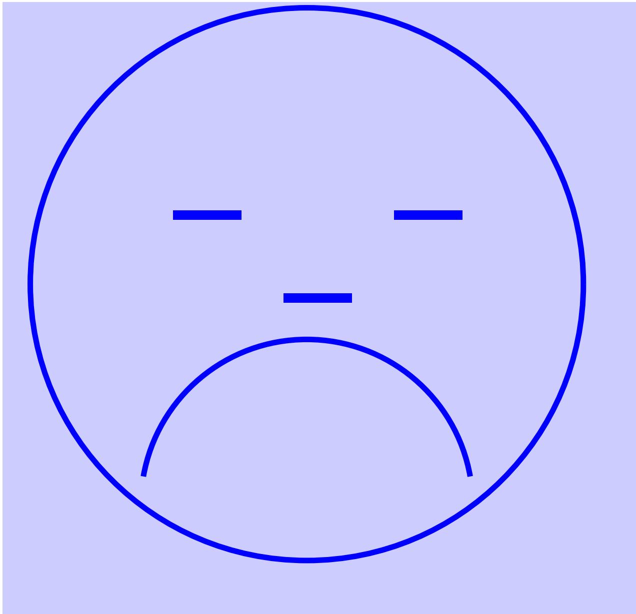

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It is good policy to label all extrema and inflection points when sketching the graph of a function.

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Step 3: Put together as good a table as you can showing the signs of $f'(x)$ and $f''(x)$ on the intervals into which the interesting values divide the domain of f.

Step 4: Plot the “interesting points” and connect them with curves which are either left or right half-smiles or half-frowns.