Exercises for Sketching-1

Sketch the graph of $y = f(x)$ if:

(1) $f(x) = \dfrac{2x-3}{x+1}$ Solution

(2) $f(x) = \dfrac{x+1}{3x-1}$ Solution

(3) $f(x) = \dfrac{ax+b}{cx+d}$ Solution

(4) $f(x) = \dfrac{2x-5}{x^2+1}$ Solution

(5) $f(x) = \dfrac{2x-5}{x^2-5x+4}$ Solution

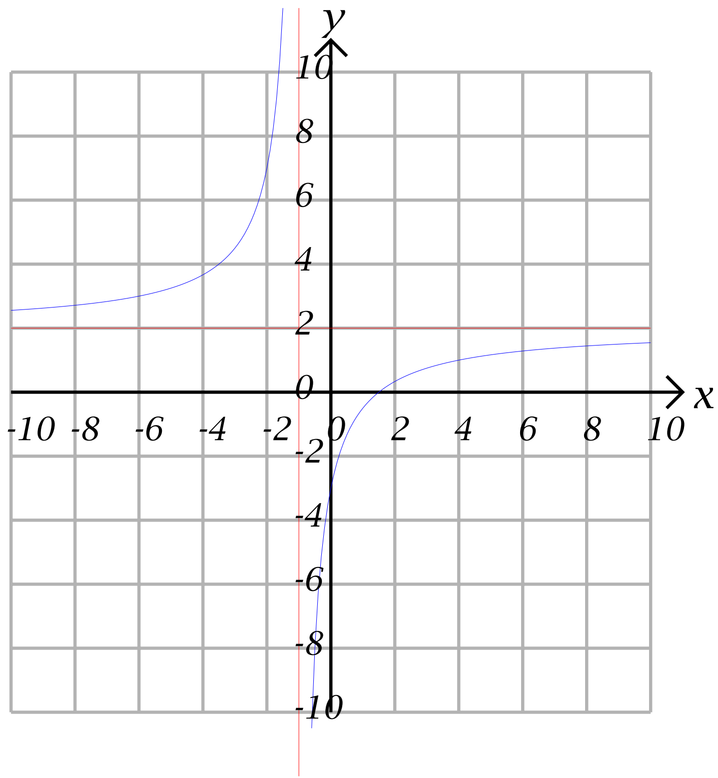
Solutions

(1) $f(x) = \dfrac{2x-3}{x+1}$

$f'(x) = \dfrac{(x+1)(2x-3)' - (2x-3)(x+1)'}{(x+1)^2} = \dfrac{(x+1)(2) - (2x-3)(1)}{(x+1)^2} = \dfrac{2x+2-2x+3}{(x+1)^2} = \dfrac{5}{(x+1)^2} =$ Back to Questions
$5(x+1)^{-2} > 0$,

$f''(x) = 5(-2)(x+1)^{-3}(x+1)' = -\dfrac{10}{(x+1)^3}$.

Since neither derivative is ever 0, the only interesting number is -1. The line $x = -1$ is a Vertical Asymptote, and the line $y = 2$ is a Horizontal Asymptote.



(2) $f(x) = \dfrac{x+1}{3x-1}$

$f'(x) = \dfrac{(3x-1)(x+1)' - (x+1)(3x-1)'}{(3x-1)^2} = \dfrac{(3x-1)(1) - (x+1)(3)}{(3x-1)^2} = \dfrac{3x-1-3x-3}{(3x-1)^2} = \dfrac{-4}{(3x-1)^2} =$
$-4(3x-1)^{-2} < 0$,

$f''(x) = -4(-2)(3x-1)^{-2-1}(3x-1)' = 8(3x-1)^{-3}(3) = \frac{24}{(3x-1)^3}$.

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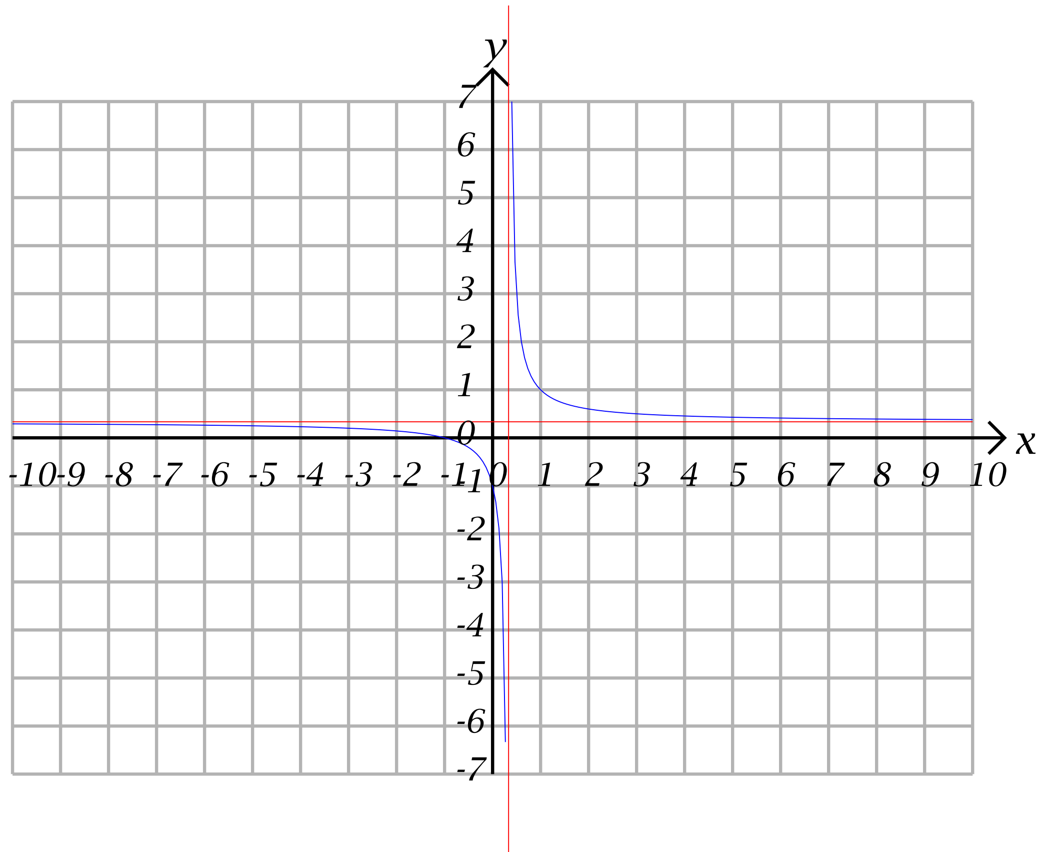
Since neither derivative is ever 0, the only interesting number is $\dfrac{1}{3}$. The line $x = \dfrac{1}{3}$ is a Vertical Asymptote,

and the line $y = \dfrac{1}{3}$ is a Horizontal Asymptote.



(3) $f(x) = \dfrac{ax+b}{cx+d}$

$f'(x) = \dfrac{(cx+d)(ax+b)' - (ax+b)(cx+d)'}{(cx+d)^2} = \dfrac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} = \dfrac{acx + ad - acx - bc}{(cx+d)^2} =$

$\dfrac{ad-bc}{(cx+d)^2} = (ad-bc)(cx+d)^{-2}$, which is never 0 unless $ad - bc = 0$, in which case the graph is the horizontal line $y = \dfrac{b}{d}$ minus the point $\left(-\dfrac{d}{c}, \dfrac{b}{d}\right)$.

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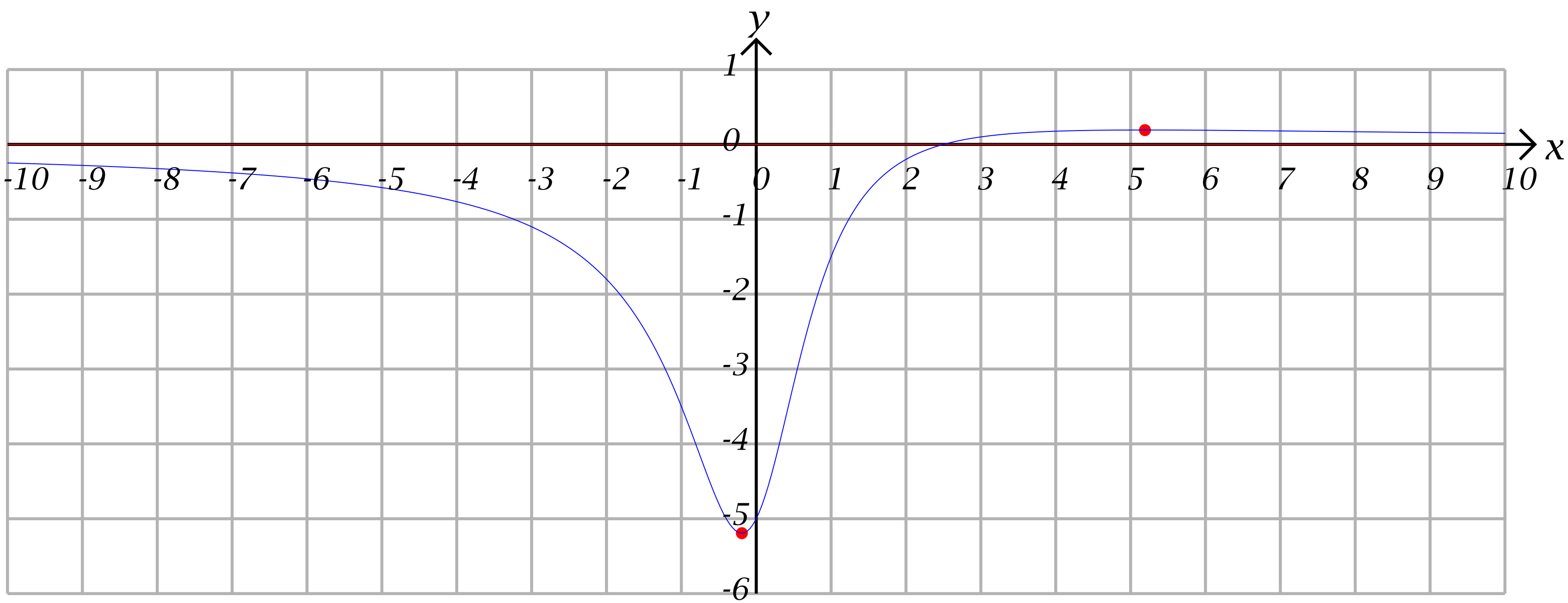
If $ad - bc \neq 0$, then the graph is a hyperbola with Vertical Asymptote $x = -\dfrac{d}{c}$, and Horizontal Asymptote $y = \dfrac{a}{c}$.

(4) $f(x) = \dfrac{2x-5}{x^2+1}$

$f'(x) = \dfrac{(x^2+1)(2x-5)' - (2x-5)(x^2+1)'}{(x^2+1)^2} = \dfrac{(x^2+1)(2) - (2x-5)(2x)}{(x^2+1)^2} = \dfrac{2x^2+2-4x^2+10x}{(x^2+1)^2} =$
$\dfrac{-2x^2+10x+2}{(x^2+1)^2} = -2\dfrac{x^2-5x-1}{(x^2+1)^2} = 0 \text{ if } x = \dfrac{-(-5) \pm \sqrt{(-5)^2-4(1)(-1)}}{2(-1)} = \dfrac{5 \pm \sqrt{25+4}}{2} = \dfrac{5 \pm \sqrt{29}}{2}$ Back to Questions

$f''(x) = -2\dfrac{(x^2+1)^2)(x^2-5x-1)' - (x^2-5x-1)\left((x^2+1)^2\right)'}{((x^2+1)^2)^2} =$
$-2\dfrac{(x^2+1)^2(2x-5) - (x^2-5x-1)2(x^2+1)(x^2+1)'}{(x^2+1)^4} = -2\dfrac{(x^2+1)(2x-5) - (x^2-5x-1)2(x^2+1)'}{(x^2+1)^3} =$
$-2\dfrac{(x^2+1)(2x-5) - (x^2-5x-1)2(2x)}{(x^2+1)^3} = -2\dfrac{(x^2+1)(2x-5) - (x^2-5x-1)(4x)}{(x^2+1)^3} =$
$-2\dfrac{2x^3-5x^2+2x-5-(4x^3-20x^2-4x)}{(x^2+1)^3} = -2\dfrac{-2x^3+15x^2+6x-5}{(x^2+1)^3}$. The three roots of the cubic $-2x^3+15x^2+6x-5$ are not easy to find numerically, but can easily be picked out on the graph,.



(5) $f(x) = \dfrac{2x-5}{x^2-5x+4}$

$$f'(x) = \frac{(x^2-5x+4)(2x-5)' - (2x-5)(x^2-5x+4)'}{(x^2-5x+4)^2} = \frac{(x^2-5x+4)(2) - (2x-5)(2x-5)}{(x^2-5x+4)^2} =$$

$$\frac{2x^2-10x+8-(4x^2-20x+25)}{(x^2-5x+4)^2} = \frac{-2x^2+10x-13}{(x^2-5x+4)^2} < 0, \text{ since } -2x^2+10x-13 \text{ has no real roots.}$$

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$$f''(x) = \frac{(x^2-5x+4)^2(-2x^2+10x-13)' - (-2x^2+10x-13)\left((x^2-5x+4)^2\right)'}{\left((x^2-5x+4)^2\right)^2} =$$

$$\frac{(x^2-5x+4)^2(-4x+10) - (-2x^2+10x-13)2(x^2-5x+4)\left(x^2-5x+4\right)'}{(x^2-5x+4)^4} =$$

$$\frac{(x^2-5x+4)(-4x+10) - 2(-2x^2+10x-13)\left(x^2-5x+4\right)'}{(x^2-5x+4)^3} = -2(2x-5)\frac{-x^2+5x-9}{(x^2-5x+4)^3} = 0 \text{ if } x = \frac{5}{2}.$$

There are Vertical Asymptotes at $x = 1$ and $x = 4$ and the x=axis is a Horixontal Asymotote. There are no relative extrema, and the f inflection numbers are 1, $\frac{5}{2}$, and 4.