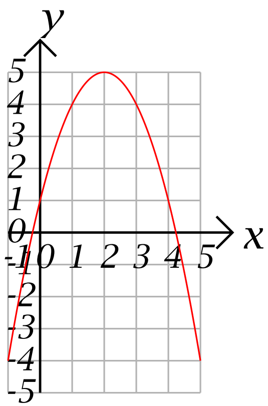
Polynomial Graphing Exercises

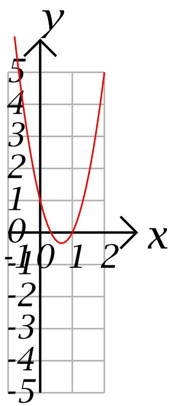
Parts of the graphs of the functions in question are shown below. Match them.

(1) $f(x) = 4x - x^2 + 1$ **(2)** $f(x) = 3x^2 - 4x + 1$ **(3)** $f(x) = \frac{1}{10}\left(x^4 - 4x^2\right)$ **(4)** $f(x) = \frac{1}{16}x^4$ **(5)** $f(x) = \frac{1}{10}\left(x^3 + x^2\right)$

(6) $f(x) = 2x^3 - x^4$ **(7)** $f(x) = \frac{1}{10}\left(2x^3 - 6x^2 + 3\right)$ **(8)** $f(x) = \frac{1}{2}\left(3x - x^3\right)$ **(9)** $f(x) = \frac{1}{8}x^5$ **(10)** $f(x) = \frac{1}{6}\left(x^5 - x\right)$



A



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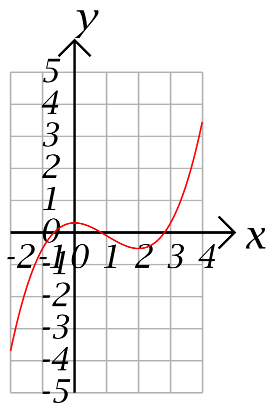

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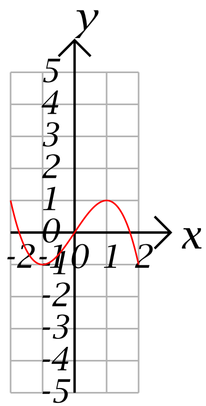

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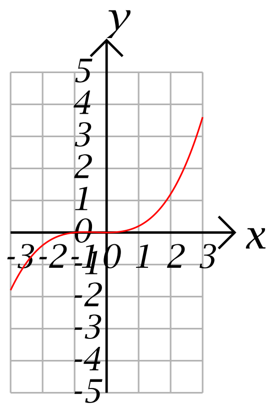

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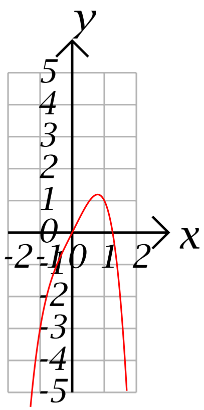

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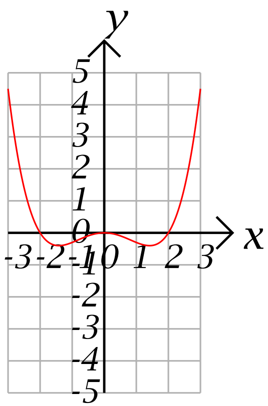

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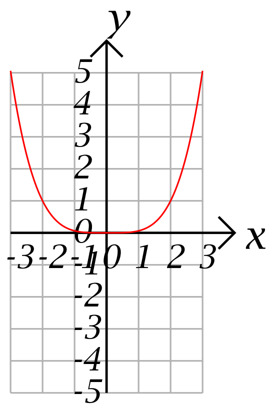

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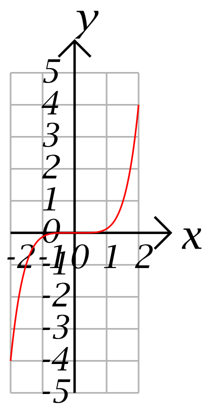

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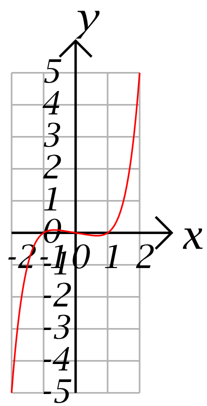
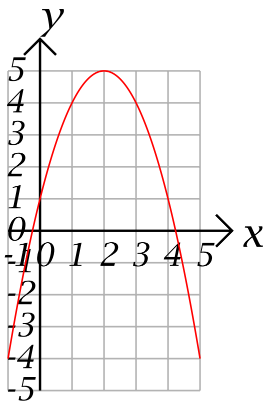


J

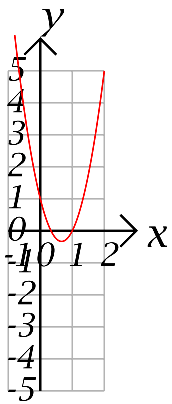
…Answers→

Answers: (1) **(A)** (2) **(B)** (3) **(G)** (4) **(H)** (5) **(E)**

(6) **(F)** (7) **(C)** (8) **(D)** (9) **(I)** (10) **(J)**



A



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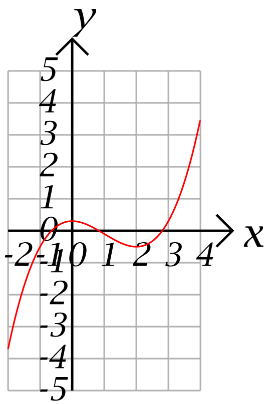

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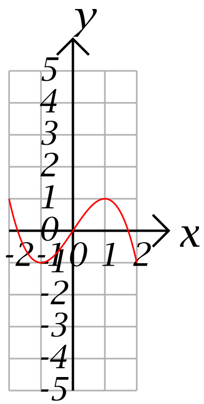

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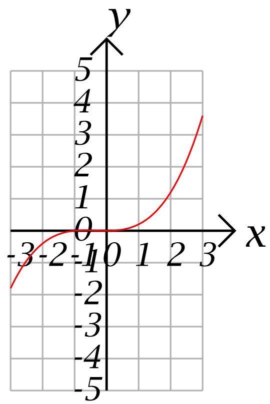

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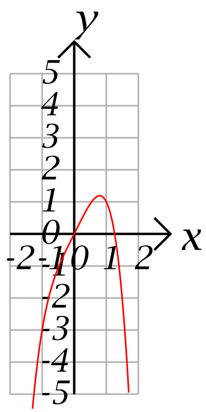

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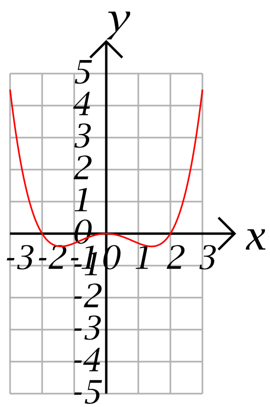

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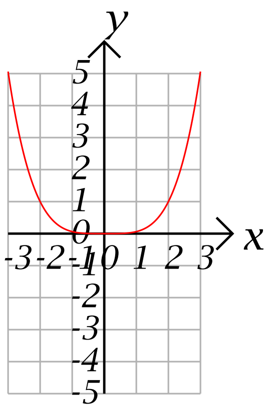

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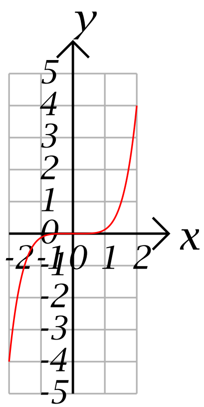
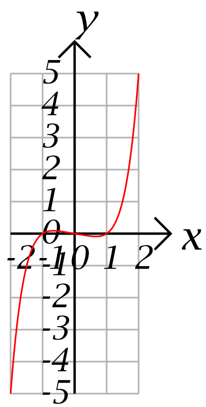


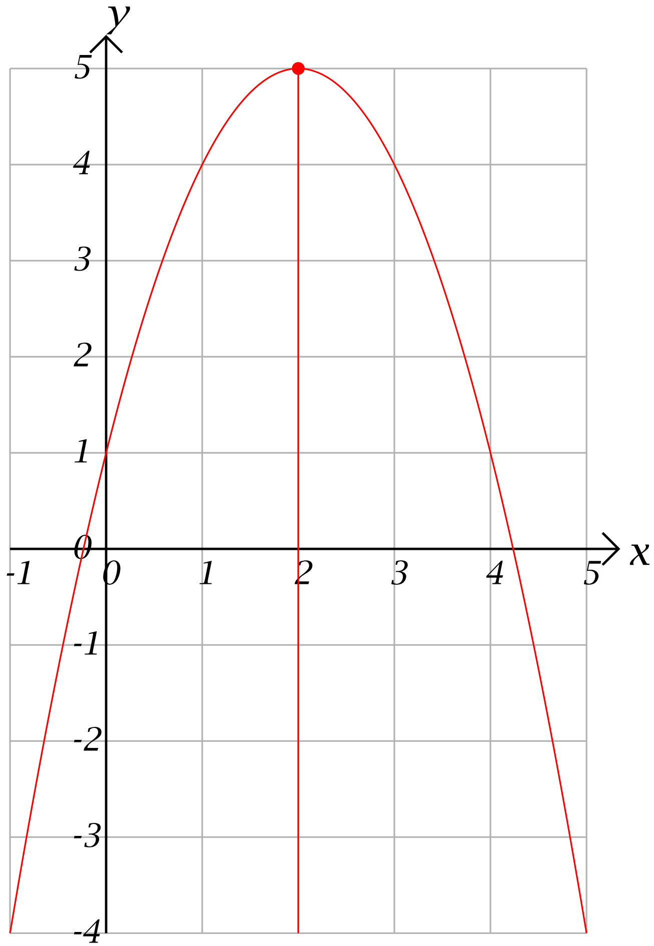
I



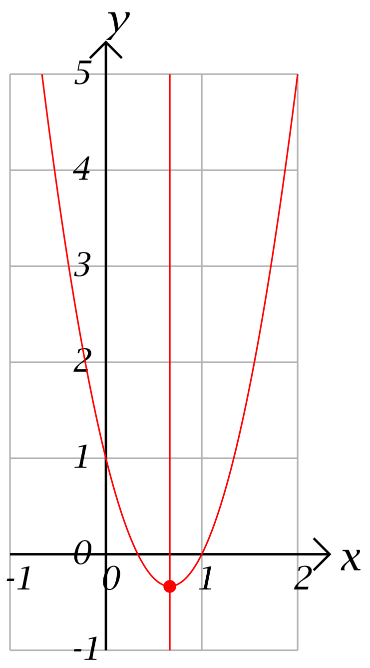
J

Solutions: **(1) (A)** $f(x) = 4x - x^2 + 1$

Solution: $f'(x) = 4 - 2x = 2(2 - x) < 0$ if $x > 2$ and $f'(x) > 0$ if $x < 2$.
$f''(x) = -2 < 0$, so the graph is concave down. It is actually a parabola with vertical axis $x = 2$ and vertex at $(2, 5)$.



(2) (B) $f(x) = 3x^2 - 4x + 1$ **Solution:** $f'(x) = 6x - 4 = 6\left(x - \frac{2}{3}\right) < 0$ if $x < \frac{2}{3}$ and $f'(x) > 0$ if $x > \frac{2}{3}$. $f''(x) = 6 > 00$, so the graph is concave up. It is actually a parabola with vertical axis $x = \frac{2}{3}$ and vertex at $\left(\frac{2}{3}, -\frac{1}{3}\right)$.



(3) (G) $f(x) = \frac{1}{10}\left(x^4 - 4x^2\right)$

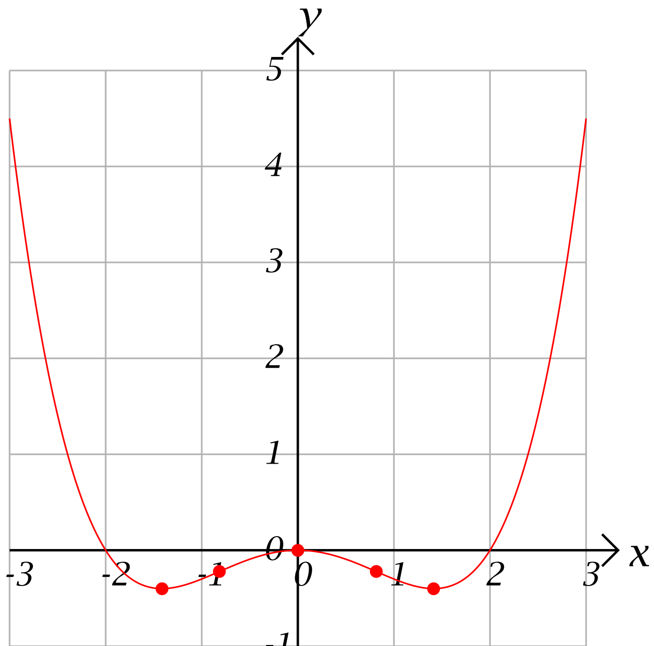
Solution: $f'(x) = \frac{1}{10}\left(4x^3 - 8x\right) = 4x\left(x^2 - 2\right) = 0$ if $x = 0$ or $x = \pm\sqrt{2}$. Thus
$f'(x) = \frac{1}{10}\left(x - \left(-\sqrt{2}\right)\right)(x - 0)\left(x - \sqrt{2}\right)$.

Thus $f'(x) < 0$ on $\left(-\infty, -\sqrt{2}\right) \cup \left(0, \sqrt{2}\right)$, and $f'(x) > 0$ on $\left(-\sqrt{2}, 0\right) \cup \left(\sqrt{2}, \infty\right)$.

Next, $f''(x) = \frac{1}{10}\left(12x^2 - 8\right) = \frac{12}{10}\left(x^2 - \frac{2}{3}\right)$, so $f''(x) < 0$ on $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$ and $f''(x) > 0$ on
$\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup, \left(\sqrt{\frac{2}{3}}, \infty\right)$.

f has inflection points at $\left(-\sqrt{\frac{2}{3}}, f\left(-\sqrt{\frac{2}{3}}\right)\right) = \left(-\sqrt{\frac{2}{3}}, -\frac{2}{9}\right)$ and $\left(\sqrt{\frac{2}{3}}, f\left(\sqrt{\frac{2}{3}}\right)\right) = \left(\sqrt{\frac{2}{3}}, -\frac{2}{9}\right)$.

It has a relative maximum at $(0, 0)$ and relative minima at $\left(-\sqrt{2}, f\left(-\sqrt{2}\right)\right) = \left(-\sqrt{2}, -\frac{2}{5}\right)$ and $\left(\sqrt{2}, f\left(\sqrt{2}\right)\right) = \left(\sqrt{2}, -\frac{2}{5}\right)$. It is an even function.

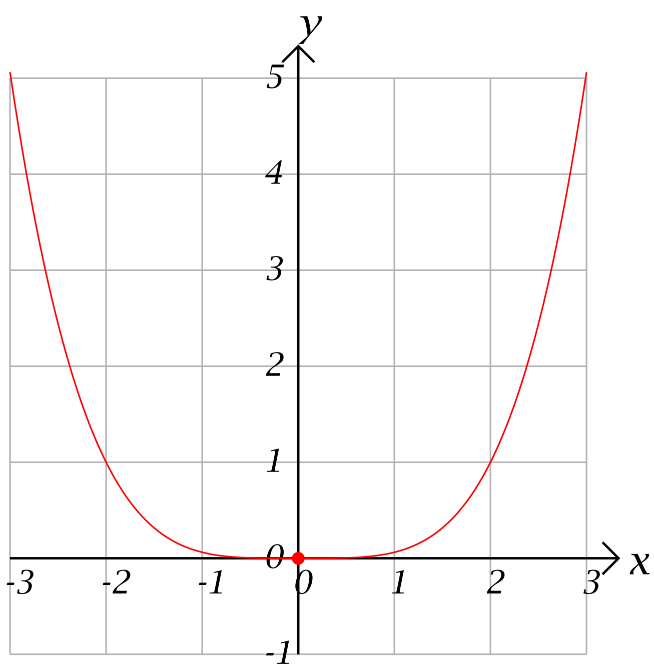


(4) **(H)** $f(x) = \frac{1}{16}x^4$

Solution: $f'(x) = \frac{1}{16}4x^3 = \frac{1}{4}x^3$, so $f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$.

$f''(x) = \frac{3}{4}x^2 > 0$ if $x \neq 0$, so the graph is concave up with an absolute minimum at $(0, 0)$.

f is an even function.



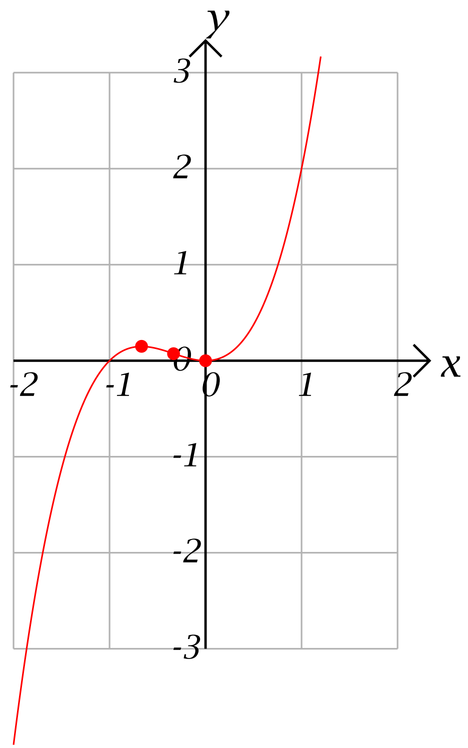
(5) (E) $f(x) = x^3 + x^2$

Solution: $f'(x) = \left(3x^2 + 2x\right) = 3x\left(x + \frac{2}{3}\right) > 0$ on $\left(-\infty, -\frac{2}{3}\right) \cup (0, \infty)$. $f'(x) < 0$ on $\left(-\frac{2}{3}, 0\right)$.

$f''(x) = 6x + 2 = 6\left(x + \frac{1}{3}\right) > 0$ on $\left(-\frac{1}{3}, \infty\right)$. $f''(x) < 0$ on $\left(-\infty, -\frac{1}{3}\right)$.

There is an inflection point at $\left(-\frac{1}{3}, f\left(-\frac{1}{3}\right)\right) = \left(-\frac{1}{3}, \frac{2}{27}\right)$.

There is a relative maximum at $\left(-\frac{2}{3}, f\left(-\frac{2}{3}\right)\right) = \left(-\frac{2}{3}, \frac{20}{27}\right)$, and a relative minimum at $(0, 0$.

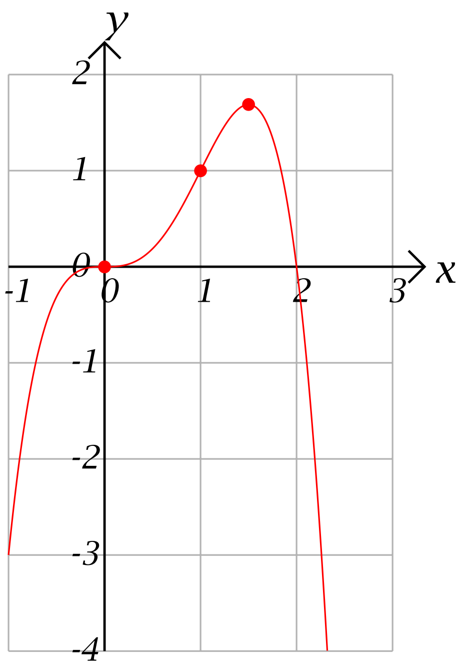


(6) **(F)** $f(x) = 2x^3 - x^4$

Solution: $f'(x) = 6x^2 - 4x^3 = -4x^2\left(x - \frac{3}{2}\right)$, so $f'(x) \geq 0$ on $\left(-\infty, \frac{3}{2}\right]$ and $f'(x) < 0$ on $\left(\frac{3}{2}, \infty\right)$.

$f''(x) = 12x - 12x^2 = -12x(x-1)$ so $f''(x) > 0$ on $(0,1)$ and $f''(x) < 0$ on $(-\infty, 0) \cup (1, \infty)$.

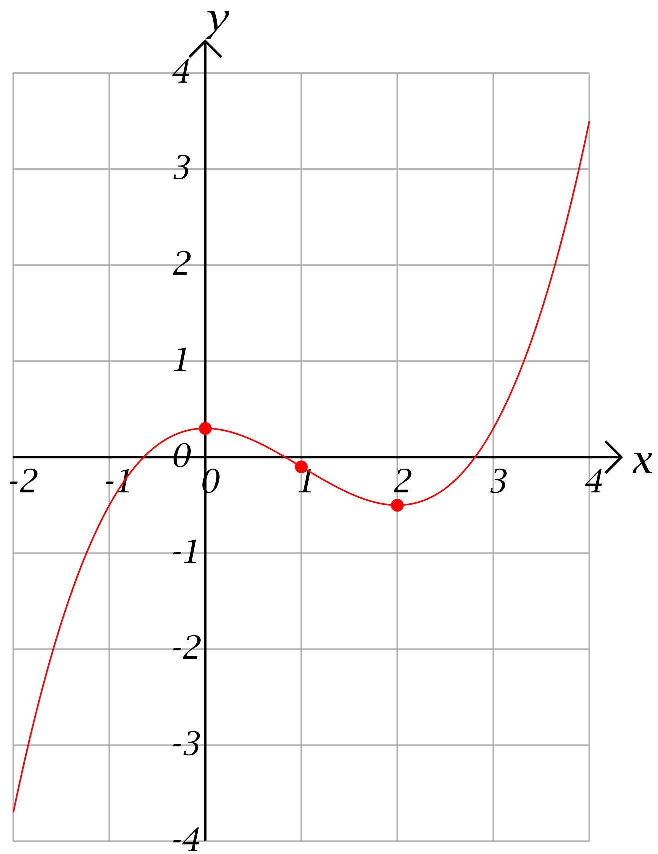
There is an inflection point at $(1,1)$ and an absolute maximum at $\left(\frac{3}{2}, \frac{27}{16}\right)$.



(7) (C) $f(x) = \frac{1}{10}\left(2x^3 - 6x^2 + 3\right)$

Solution: $f'(x) = \frac{1}{10}\left(6x^2 - 12x\right) = \frac{3}{5}x(x-2) > 0$ on $(-\infty, 0) \cup (2, \infty)$. $f'(x) < 0$ on $(0, 2)$.

$f''(x) = \frac{1}{10}(12x - 12) = \frac{6}{5}(x-1) > 4$ on $(1, \infty)$. $f''(x) < 0$ on $(-\infty, 1)$, so there is an inflection point at $\left(1, -\frac{1}{10}\right)$. There is a relative maximum at $\left(0, \frac{3}{10}\right)$ and a relative minimum at $\left(2, -\frac{1}{2}\right)$.

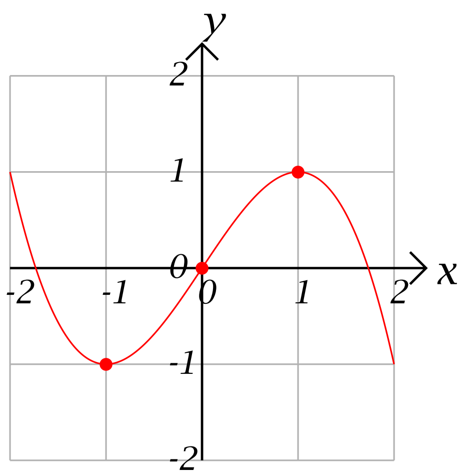


(8) **(D)** $f(x) = \frac{1}{2}\left(3x - x^3\right)$

Solution: $f'(x) = \frac{1}{2}\left(3 - 3x^2\right) = \frac{3}{2}\left(1 - x^2\right) > 0$ on $(-1, 1)$. $f'(x) < 0$ on $(-\infty, -1) \cup (1, \infty)$.

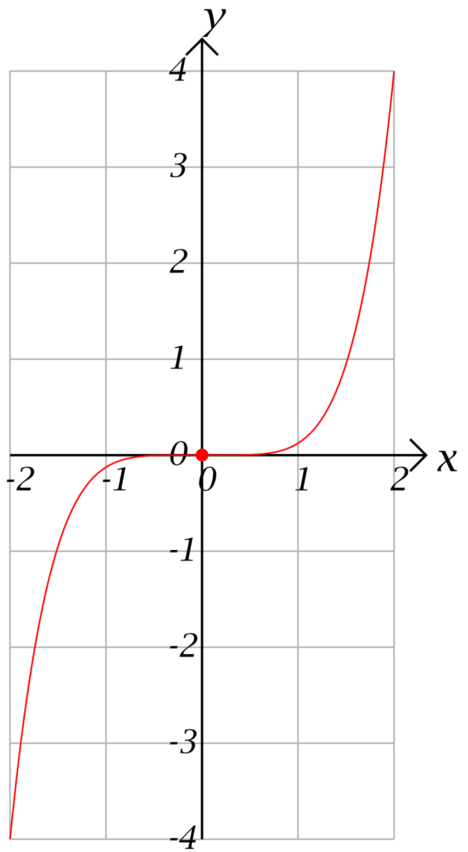
$f''(x) = -3x > 0$ on $(-\infty, 0)$ and $f''(x) < 0$ on $(0, \infty)$.

There is an inflection point at $(0, 0)$, a relative maximum at $(1, 1)$, and a relative minimum at $(-1, -1)$.



(9) (I) $f(x) = \frac{1}{8}x^5$

Solution: $f'(x) = \frac{5}{8}x^4 \geq 0$ and $f''(x) = \frac{5}{2}x^3 > 0$ on $(0,\infty)$ and < 0 on $(-\infty, 0)$. There is an inflection point at (0,0).



(10) **(J)** $f(x) = \frac{1}{6}\left(x^5 - x\right)$

Solution: $f'(x) = \frac{1}{6}\left(5x^4 - 1\right) = \frac{5}{6}\left(x^4 - \frac{1}{5}\right) > 0$ on $\left(-\infty, -\sqrt{\frac{1}{5}}\right) \cup \left(\sqrt{\frac{1}{5}}, \infty\right)$. $f'(x) < 0$ on $\left(-\sqrt{\frac{1}{5}}, \sqrt{\frac{1}{5}}\right)$. $f''(x) = \frac{10}{3}x^3 > 0$ and $(0, \infty)$ and < 0 on $(-\infty, 0)$. There is an inflection point at (0,0), a relative maximimu at $\left(-\sqrt{\frac{1}{5}}, f\left(-\sqrt{\frac{1}{5}}\right)\right)$ and a relative minimimu at $\left(\sqrt{\frac{1}{5}}, f\left(\sqrt{\frac{1}{5}}\right)\right)$