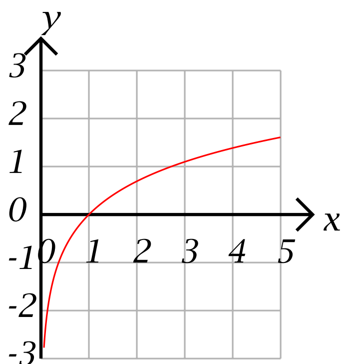
Logarithm Graphing Exercises

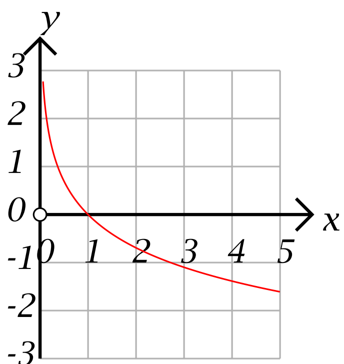
Parts of the graphs of the functions in questions 1 to 10 are shown below. Match them.

(1) $f(x) = \ln x^2$ **(2)** $f(x) = \ln x^x$ **(3)** $f(x) = \ln \frac{1}{x}$ **(4)** $f(x) = x \ln x$ **(5)** $f(x) = \frac{\ln x}{x}$

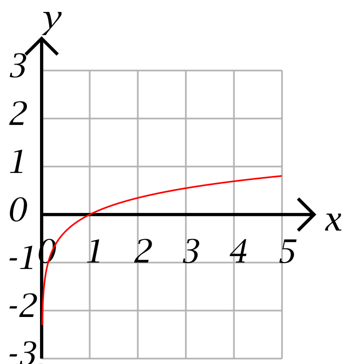
(6) $f(x) = x^2 \ln x$ **(7)** $f(x) = \ln ex$ **(8)** $f(x) = \ln x$ **(9)** $f(x) = \ln \sqrt{x}$ **(10)** $f(x) = \ln \frac{x}{e}$



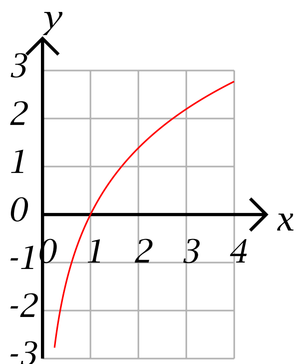
A



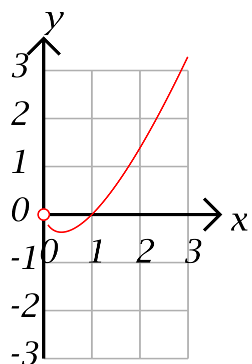
B



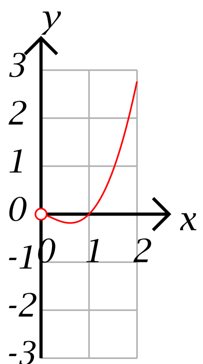
C



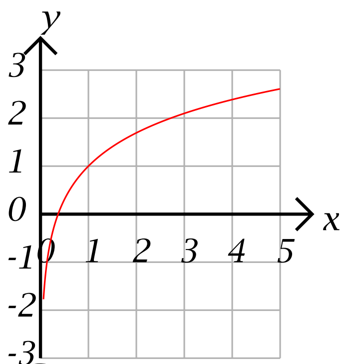
D



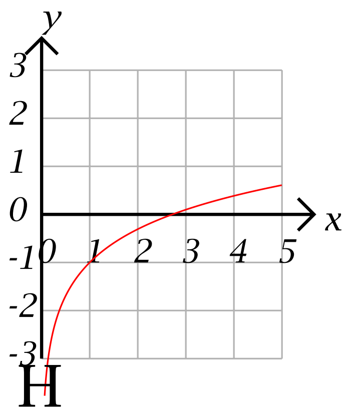
E



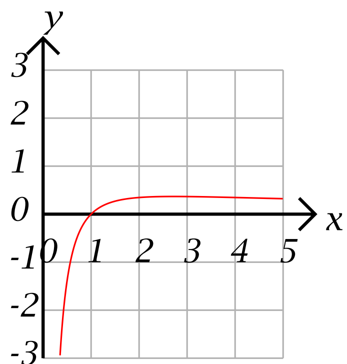
F



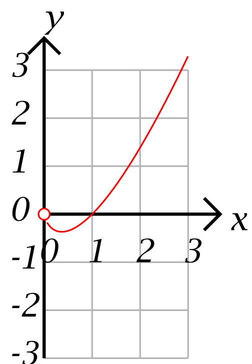
G



H



I



J

…Answers→

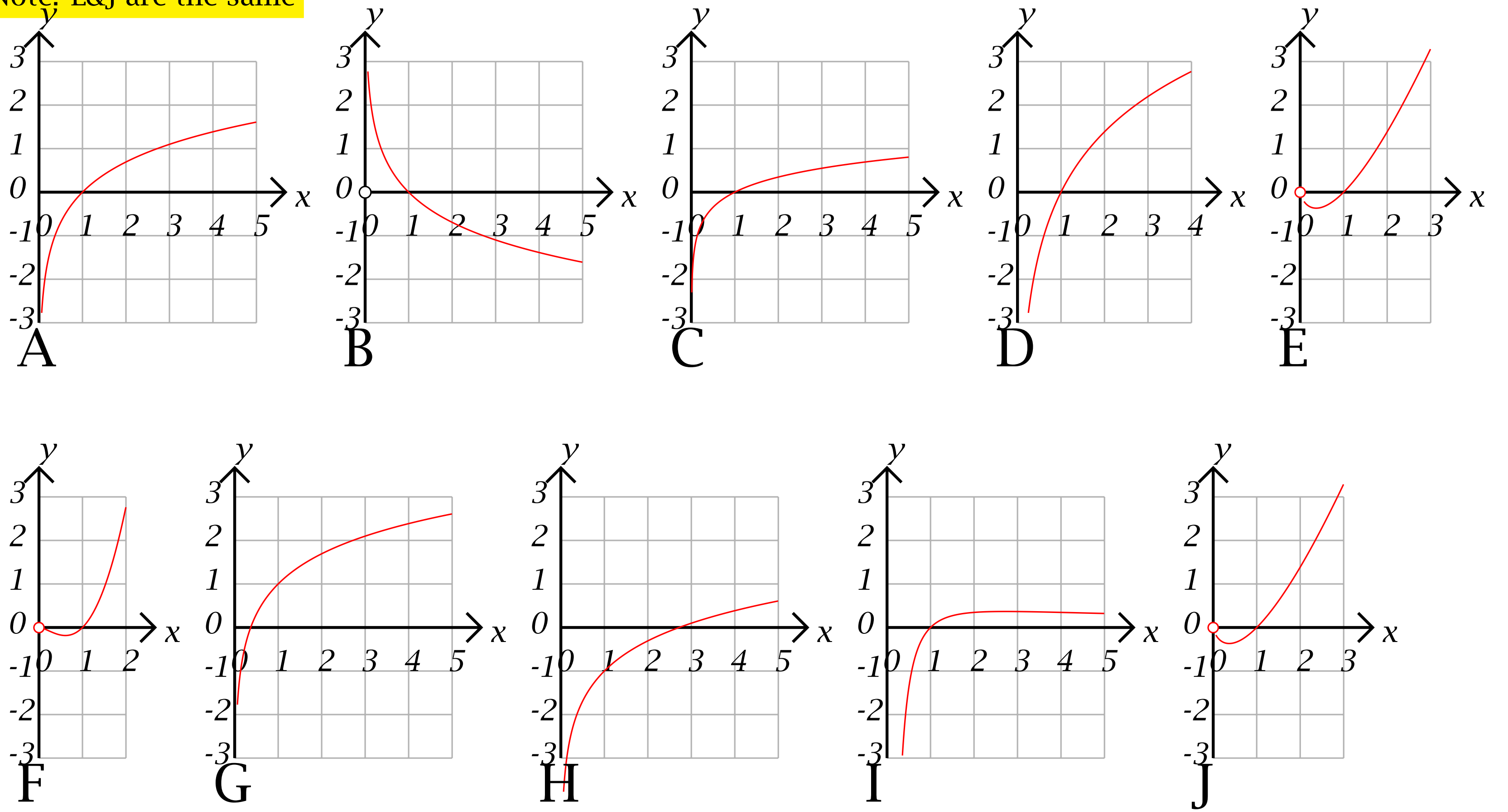
Answers

Note: E&J are the same

(1) (D) $f(x) = \ln x^2$ **(2) (J)** $f(x) = \ln x^x$ **(3) (B)** $f(x) = \ln \frac{1}{x}$ **(4) (E)** $f(x) = x \ln x$ **(5) (I)** $f(x) = \frac{\ln x}{x}$

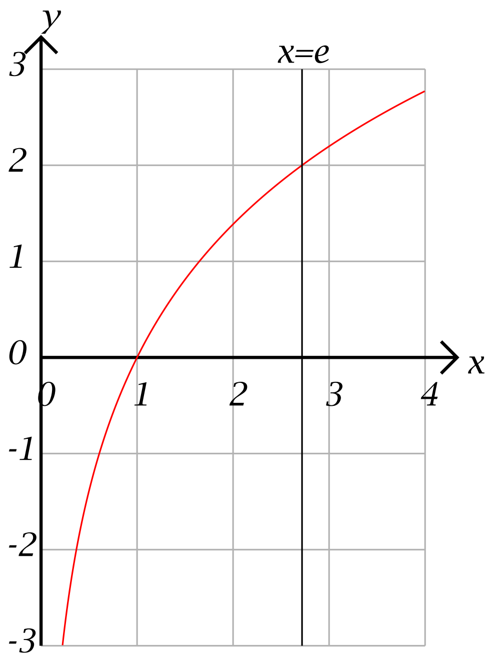
(6) (F) $f(x) = x^2 \ln x$ **(7) (G)** $f(x) = \ln ex$ **(8) (A)** $f(x) = \ln x$ **(9) (C)** $f(x) = \ln \sqrt{x}$ **(10) (H)** $f(x) = \ln \frac{x}{e}$

Note: E&J are the same



(1) **(D)** $f(x) = \ln x^2 = 2\ln x$

Solution: $f'(x) = \dfrac{2}{x} > 0,\ f''(x) = -\dfrac{2}{x^2} < 0$

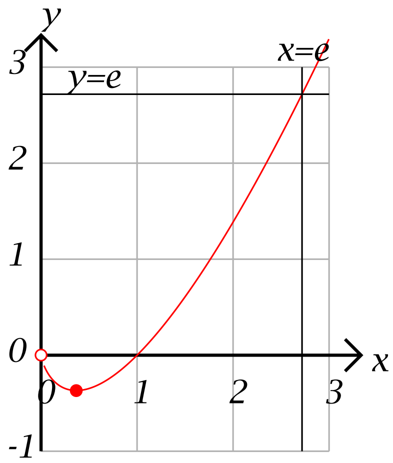


(2) (J) $f(x) = \ln x^x = x \ln x$

Solution: $f'(x) = (x)' + x(\ln x)' = (1)\ln x + x\dfrac{1}{x} = \ln x + 1 > 0$ if $\ln x > 1$ or $e^{\ln x} = x > e^{-1} = \dfrac{1}{e}$, so
$f'(x) > 0$ on $\left(\dfrac{1}{e}, \infty\right)$ and $f'(x) < 0$ on $\left(0, \dfrac{1}{e}\right)$.

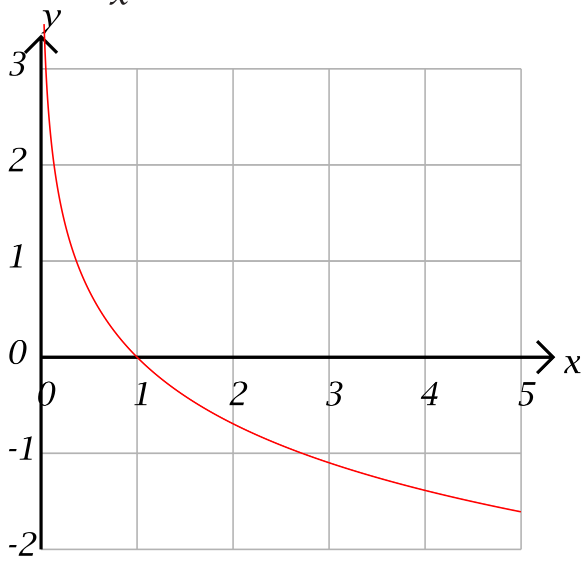
$f''(x) = \dfrac{1}{x} > 0$. There is an absolute minimum at $\left(\dfrac{1}{e}, -\dfrac{1}{e}\right)$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \text{(by L'Hôpital's Rule)} \lim_{x\to 0^+} \frac{(\ln x)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} (-x) = 0.$$



(3) **(B)** $f(x) = \ln\frac{1}{x} = -\ln x$

Solution: $f'(x) = -\frac{1}{x} < 0,\ f''(x) = \frac{1}{x^2} > 0.$



(4) **(E)** $f(x) = x \ln x$

Solution: See (1).

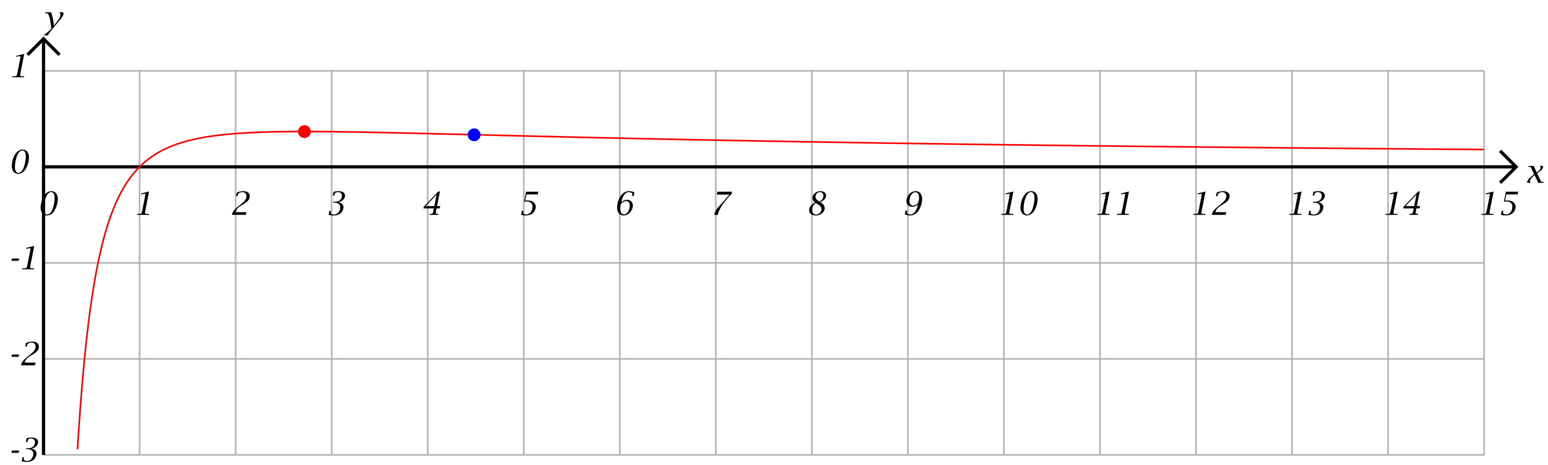
(5) **(I)** $f(x) = \dfrac{\ln x}{x}$

Solution: $f'(x) = \dfrac{x(\ln x)' - \ln x(x)'}{x^2} = \dfrac{x\dfrac{1}{x} - \ln x}{x^2} = \dfrac{1 - \ln x}{x^2} > 0$ if $1 - \ln x > 0$ or $\ln x < 1$ or $x = e^{\ln x} < e^1 = e$. Thus $f'(x) > 0$ on $(0, e)$.

$f''(x) = \dfrac{x^2(1-\ln x)' - (1-\ln x)\left(x^2\right)'}{\left(x^2\right)^2} = \dfrac{x^2\left(-\frac{1}{x}\right) - (1-\ln x)(2x)}{x^4} = \dfrac{-x-(1-\ln x)(2x)}{x^4} = \dfrac{-1-(1-\ln x)(2)}{x^3} = \dfrac{-3+2\ln x}{x^3} > 0$ if $-3 + 2\ln x > 0$ or $\ln x > \dfrac{3}{2}$ or $x = e^{\ln x} > e^{\frac{3}{2}}$.

$f''(x) < 0$ on $\left(0, e^{\frac{3}{2}}\right)$.

There is an absolute maximum at $\left(e, \dfrac{1}{e}\right)$ and an inflection point at $\left(e^{\frac{3}{2}}, \dfrac{3}{2e^{\frac{3}{2}}}\right)$.



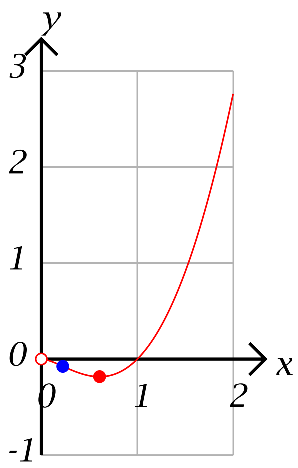
(6) (F) $f(x) = x^2 \ln x$

Solution: $f'(x) = \left(x^2\right)' \ln x + x^2 (\ln x)' = 2x \ln x + x^2 \frac{1}{x} = 2x \ln x + x = x(1 + 2\ln x) > 0$ if $1 + 2\ln x > 0$ or $\ln x > -\frac{1}{2}$ or $x = e^{\ln x} > e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

$f''(x) = (2x \ln x + x)' = 2\ln x + 2 + 1 = 2\ln x + 3 > 0$ if $\ln x > -\frac{3}{2}$ or $x = e^{\ln x} > e^{-\frac{3}{2}}$.

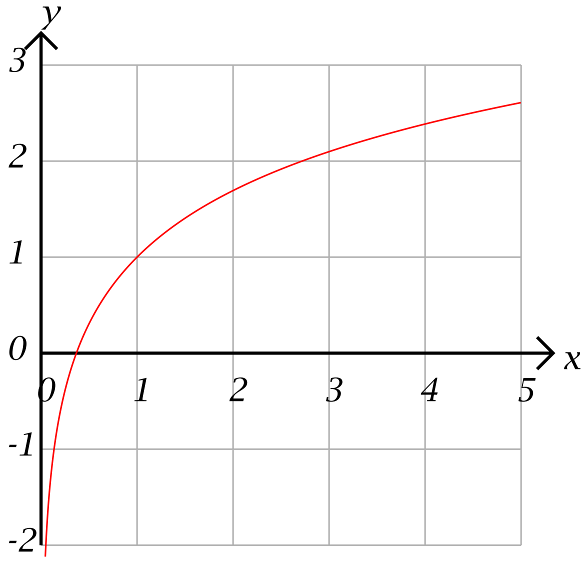
There is an absolute minimum at $\left(\frac{1}{\sqrt{e}}, \frac{-1}{2e}\right)$ and an inflection point at $\left(e^{-\frac{3}{2}}, -\frac{3}{2e^3}\right)$.

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} =$ (by L'Hôpital's Rule) $\lim_{x \to 0^+} \frac{(\ln x)'}{\left(\frac{1}{x^2}\right)'} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^3}} = \lim_{x \to 0^+} (-x^2) = 0$



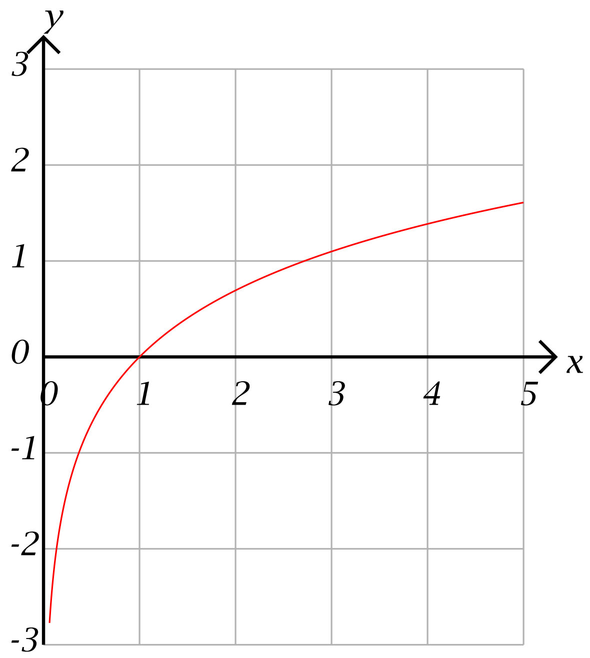
(7) **(G)** $f(x) = \ln ex = 1 + \ln x$

Solution: $f'(x) = \dfrac{1}{x} > 0$, $f''(x) = -\dfrac{1}{x^2} < 0$. This is the graph of $y = \ln x$ shifted one unit vertically.



(8) (A) $f(x) = \ln x$

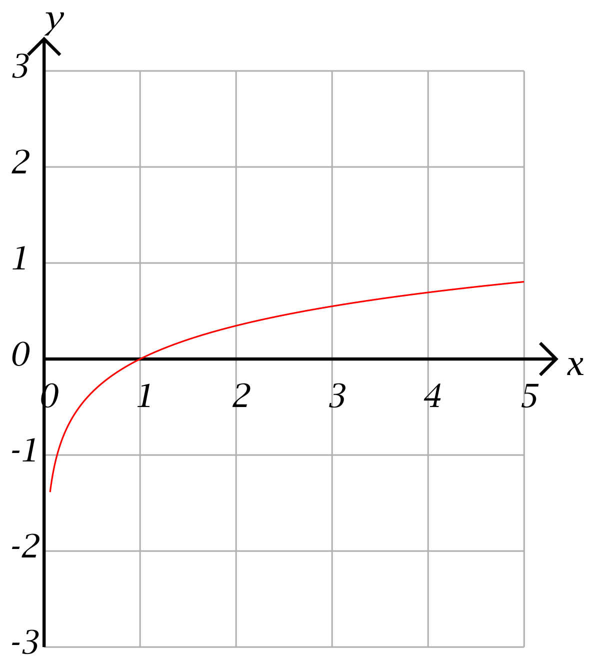
Solution: $f'(x) = \dfrac{1}{x} > 0,\ f''(x) = -\dfrac{1}{x^2} < 0.$



A

(9) **(C)** $f(x) = \ln\sqrt{x} = \frac{1}{2}\ln x$

Solution: $f'(x) = \frac{1}{2x} > 0$, $f''(x) = -\frac{1}{2x^2} < 0$.



C

(10) **(H)** $f(x) = \ln \frac{x}{e} = \ln x - 1$

Solution: $f'(x) == \frac{1}{x} > 0$, $f''(x) = -\frac{1}{x^2} < 0$.