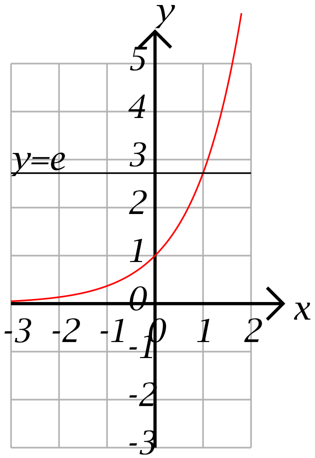
Exponential Graphing Exercises

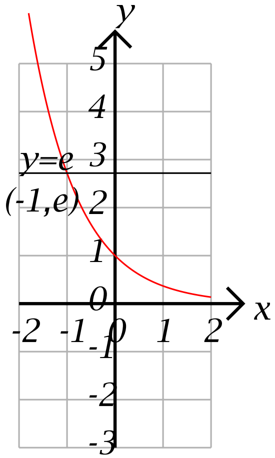
Parts of the graphs of the functions in questions 1 to 10 are shown below. Match them.

(1) $f(x) = xe^{x^2}$ **(2)** $f(x) = xe^{x}$ **(3)** $f(x) = x^2e^{-x^2}$ **(4)** $f(x) = xe^{-x}$ **(5)** $f(x) = x^2e^{x^2}$

(6) $f(x) = x^2e^{x}$ **(7)** $f(x) = e^{-x}$ **(8)** $f(x) = x^2e^{-x}$ **(9)** $f(x) = e^{x}$ **(10)** $f(x) = xe^{-x^2}$



A



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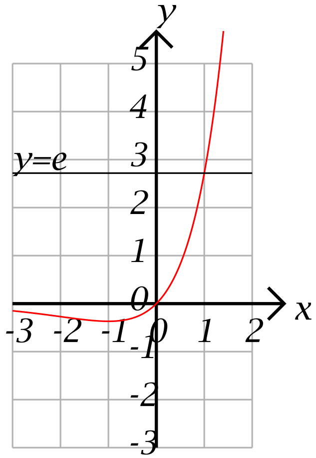
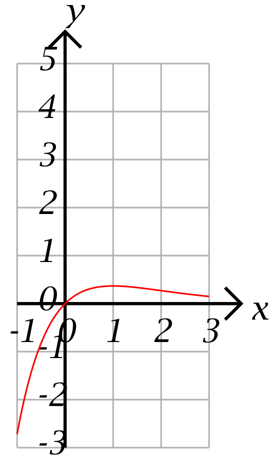
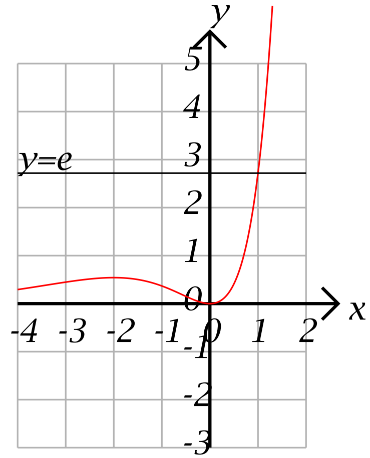


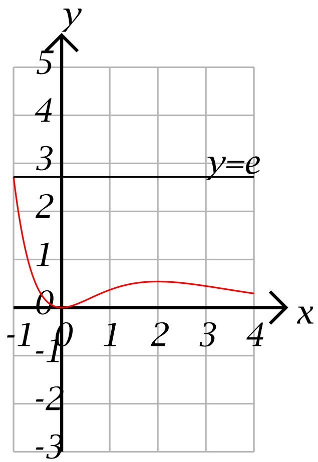
C



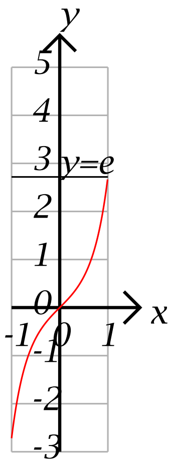
D



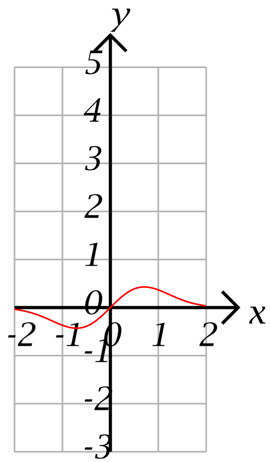
E



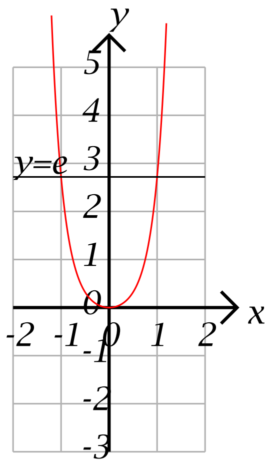
F



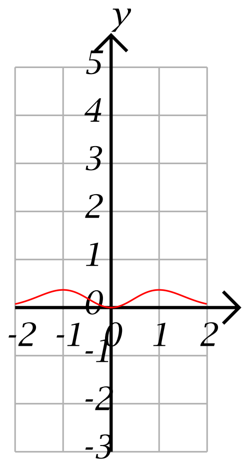
G



H



I

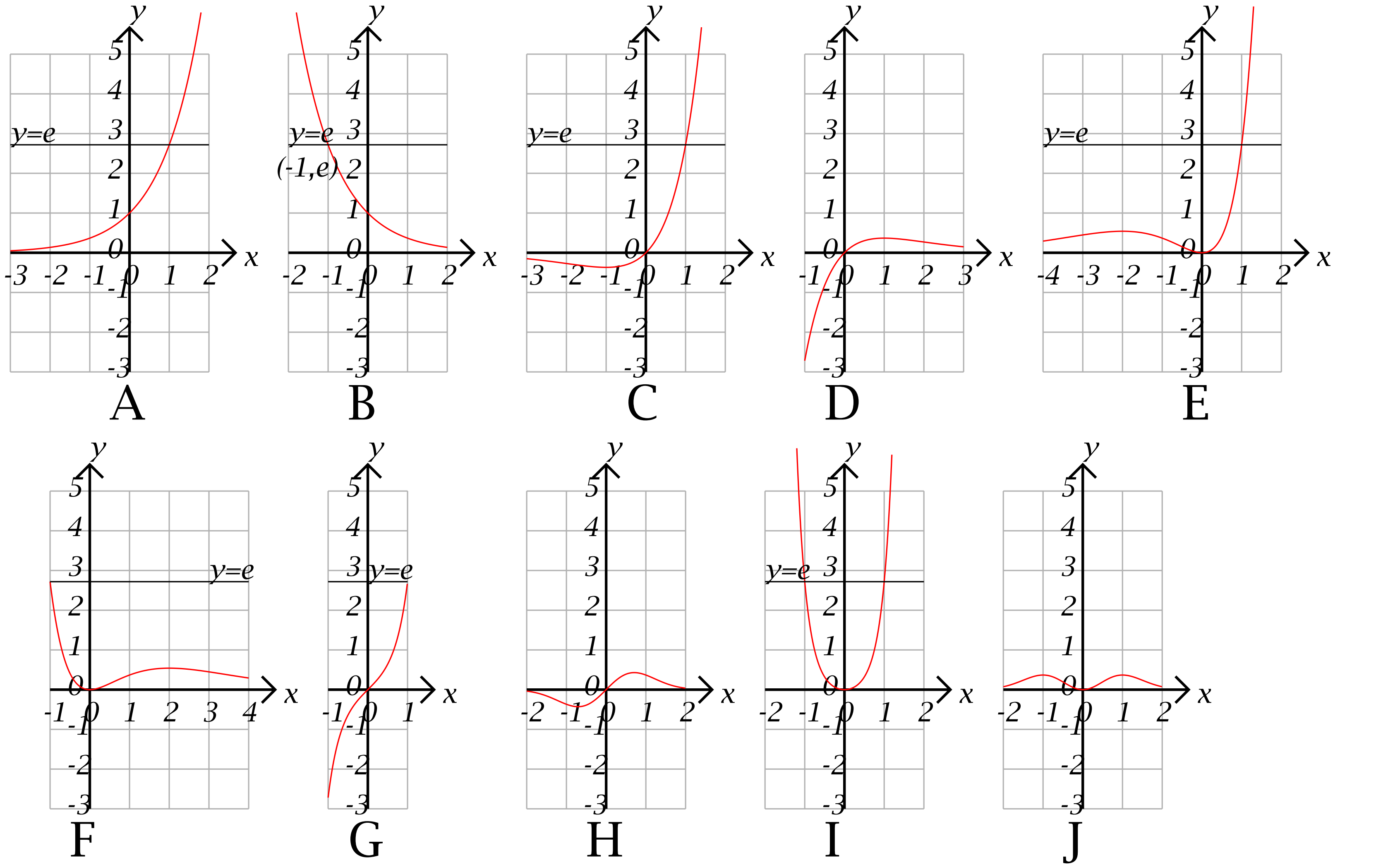


J

…Answers→

(1) (G) $f(x) = xe^{x^2}$ **(2) (C)** $f(x) = xe^{x}$ **(3) (J)** $f(x) = x^2e^{-x^2}$ **(4) (D)** $f(x) = xe^{-x}$ **(5) (I)** $f(x) = x^2e^{x^2}$

(6) (E) $f(x) = x^2e^{x}$ **(7) (B)** $f(x) = e^{-x}$ **(8) (F)** $f(x) = x^2e^{-x}$ **(9) (A)** $f(x) = e^{x}$ **(10) (H)** $f(x) = xe^{-x^2}$



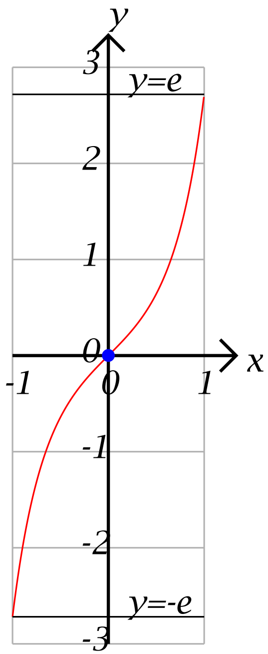
Solutions:

(1) (G) $f(x) = xe^{x^2}$

Solution: $f'(x) = (x)'e^{x^2} + x\left(e^{x^2}\right)' = e^{x^2} + xe^{x^2}2x = (1+2x^2)e^{x^2} > 0$

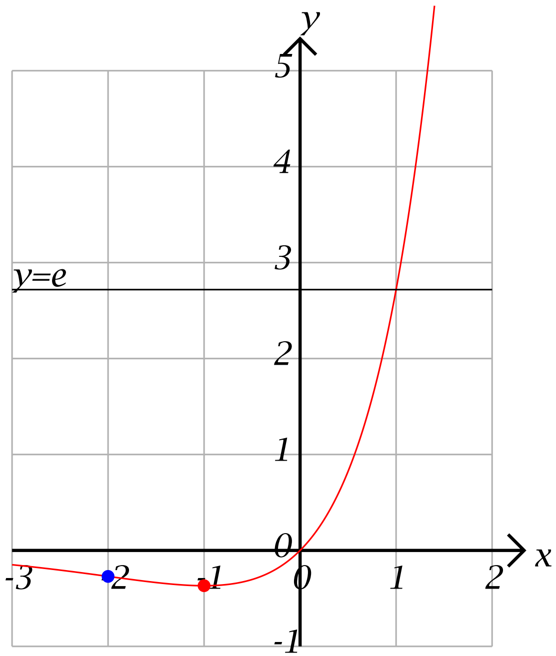
$f''(x) = (1+2x^2)'e^{x^2} + (1+2x^2)\left(e^{x^2}\right)' = 4xe^{x^2} + (1+2x^2)e^{x^2}2x = \left(6x+4x^3\right)e^{x^2} = 2x\left(2x^2+3\right)e^{x^2} > 0$ on $(0,\infty)$ and < 0 on $(-\infty)$.

There is an inflection point at (0,0).



(2) (C) $f(x) = xe^x$

Solution: $f'(x) = (x)'e^x + x(e^x)' = e^x + xe^x = (1+x)e^x > 0$ on $(-1, \infty)$ and < 0 on $(-\infty, -1)$.
$f''(x) = (2+x)e^x > 0$ on $(-2, \infty)$ and < 0 on $(-\infty, -2)$. There is an absolute minimum at $\left(-1, -\frac{1}{e}\right)$ and an inflection point at $\left(-2, -\frac{2}{e^2}\right)$



(3) (J) $f(x) = x^2 e^{-x^2}$

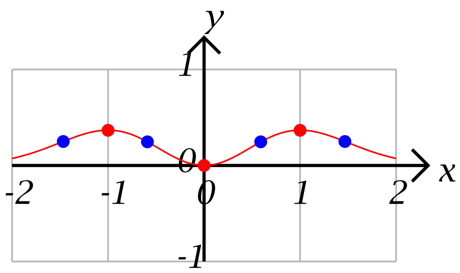
Solution: $f'(x) = (x^2)' e^{-x^2} + x^2 \left(e^{-x^2}\right)' = 2xe^{-x^2} + x^2\left(e^{-x^2}(-2x)\right) = 2xe^{-x^2} - 2x^3 e^{-x^2} =$
$2\left(x - x^3\right)e^{-x^2} = 2x\left(1 - x^2\right)e^{-x^2} = 2x(1-x)(1+x)e^{-x^2} > 0$ on $(-\infty, -1) \cup (0,1)$ and < 0 on $(-1,0) \cup (1,\infty)$.

There are absolute maxima at $\left(-1, \frac{1}{e}\right)$ and $\left(1, \frac{1}{e}\right)$ and there is an absolute minimum at (0,0).

$f''(x) = 2\left(x - x^3\right)' e^{-x^2} + 2\left(x - x^3\right)\left(e^{-x^2}\right)' = 2\left[2x^4 - 5x^2 + 1\right]e^{-x^2} = 2\left[2\left(x^2\right)^2 - 5x^2 + 1\right]e^{-x^2} = 0$ if

$x^2 = \frac{-(-5) \pm \sqrt{5^2 - 4(2)1}}{2(2)} = \frac{5 \pm \sqrt{25-8}}{4} = \frac{5 \pm \sqrt{13}}{4}$, so there are four inflection points at

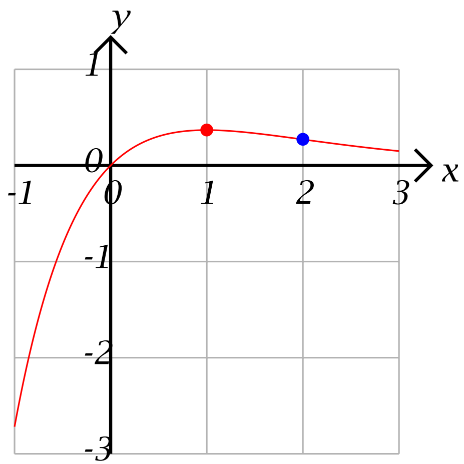
$\left(-\sqrt{\frac{5+\sqrt{13}}{4}}, f\left(-\sqrt{\frac{5+\sqrt{13}}{4}}\right)\right)$, $\left(-\sqrt{\frac{5-\sqrt{13}}{4}}, f\left(-\sqrt{\frac{5-\sqrt{13}}{4}}\right)\right)$, $\left(\sqrt{\frac{5-\sqrt{13}}{4}}, f\left(\sqrt{\frac{5-\sqrt{13}}{4}}\right)\right)$, and
$\left(\sqrt{\frac{5+\sqrt{13}}{4}}, f\left(\sqrt{\frac{5+\sqrt{13}}{4}}\right)\right)$.



(4) **(D)** $f(x) = xe^{-x}$

Solution: $f'(x) = (x)'e^{-x} + x\left(e^{-x}\right)' = (1)e^{-x} + xe^{-x}(-1) = (1-x)e^{-x} >= $ on $(-\infty, 1)$ and < 0 on $(1, \infty)$.

$f''(x) = (1-x)'e^{-x} + (1-x)\left(e^{-x}\right)' = (-1)e^{-x} + (1-x)e^{-x}(-1) = (x-2)e^{-x} > 0$ on $(2, \infty)$ and < 0 on $(-\infty, 2)$.



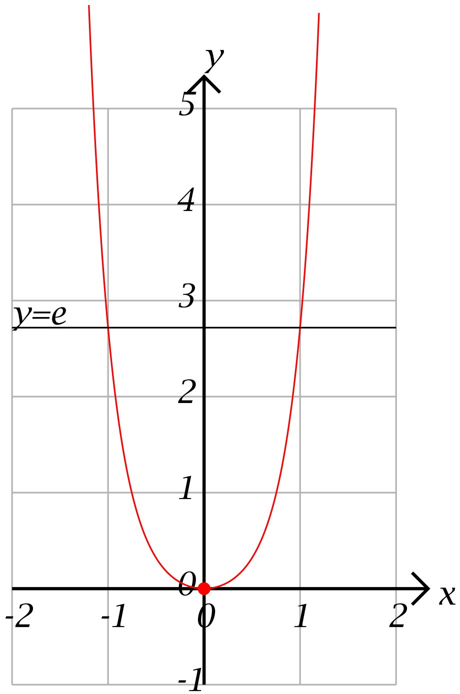
(5) **(I)** $f(x) = x^2 e^{x^2}$

Solution:

$f'(x) = \left(x^2\right)' e^{x^2} + x^2\left(e^{x^2}\right)' = 2xe^{x^2} + x^2 e^{x^2}\left(x^2\right)' = 2xe^{x^2} + x^2 e^{x^2}(2x) = 2\left(x + x^3\right)e^{x^2} = 2x(1+x^2)e^{x^2} > 0$ on $(0, \infty)$ and < 0 on $(-\infty, 0)$.

$f''(x) = 2\left(x + x^3\right)' e^{x^2} + 2\left(x + x^3\right)\left(e^{x^2}\right)' = 2\left(1 + 3x^2\right)e^{x^2} + 2\left(x + x^3\right)e^{x^2}(2x) =$

$2\left[1 + 3x^2 + \left(x + x^3\right)(2x)\right]e^{x^2} = 2\left[1 + 3x^2 + 2x^2 + 2x^4\right]e^{x^2} = 2\left[2x^4 + 5x^2 + 1\right]e^{x^2} > 0$. There is an absolute minimum at (0,0).



(6) **(E)** $f(x) = x^2 e^x$

Solution: $f'(x) = \left(x^2\right)' e^x + x^2 (e^x)' = 2xe^x + x^2 e^x = (x^2 + 2x)e^x = x(x+2)e^x > 0$ on $(-\infty, -2) \cup (0, \infty)$ and < 0 on $(0, 2)$.

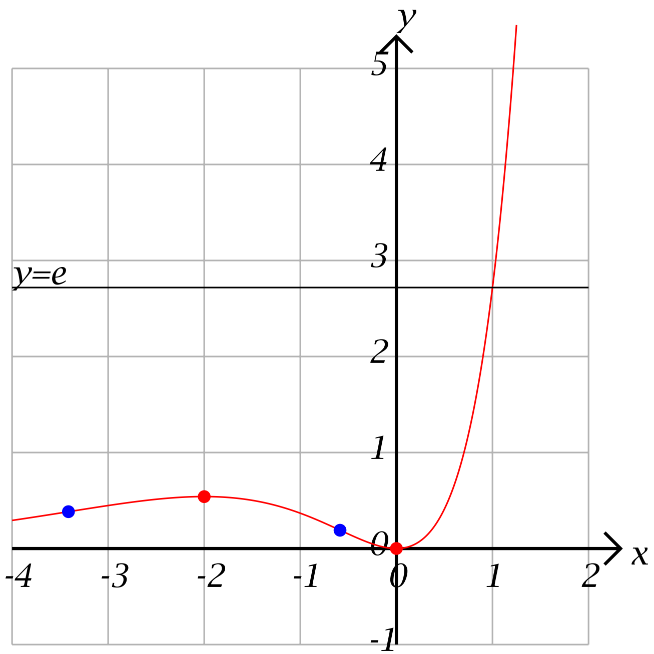
$f''(x) = \left(x^2 + 2x\right)' e^x + (x^2 + 2x)(e^x)' = (2x + 2)e^x + (x^2 + 2x)e^x = (x^2 + 4x + 2)e^x = 0$ if

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2} = \frac{-4 \pm \sqrt{16 - 8}}{2} = \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}.$$

$f''(x) > 0$ on $\left(-\infty, -2 - \sqrt{2}\right) \cup \left(-2 + \sqrt{2}, \infty\right)$ and < 0 on $\left(-2 - \sqrt{2}, -2 + \sqrt{2}\right)$.

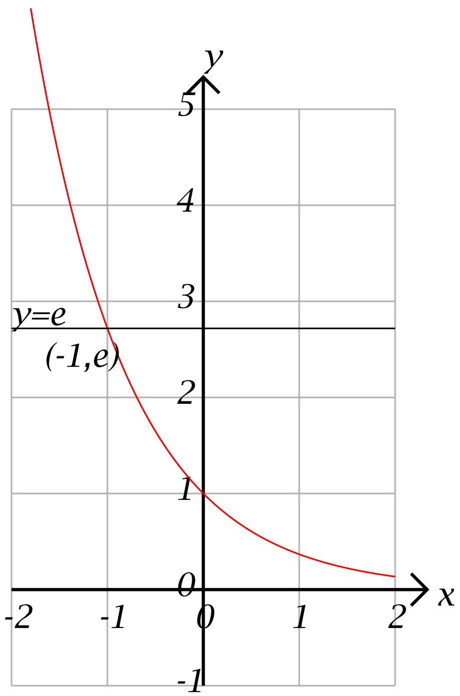
There is an absolute minimum at (0,0) and a relative maximum at $\left(-2, \frac{4}{e^2}\right)$.

There are inflection points at $\left(-2 - \sqrt{2}, f\left(-2 - \sqrt{2}\right)\right)$ and $\left(-2 + \sqrt{2}, f\left(-2 + \sqrt{2}\right)\right)$.



(7) **(B)** $f(x) = e^{-x}$

Solution: $f'(x) = -e^{-x} < 0$, $f''(x) = e^{-x} > 0$.

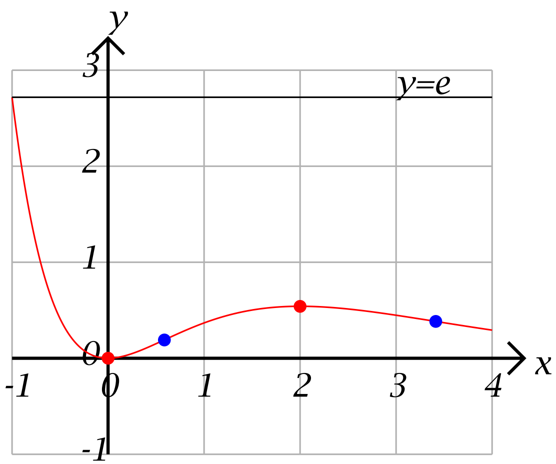


(8) **(F)** $f(x) = x^2e^{-x}$

Solution: $f'(x) = \left(x^2\right)' e^{-x} + x^2\left(e^{-x}\right)' = 2xe^{-x} + x^2e^{-x}(-1) = (2x - x^2)e^{-x} = x(2-x)e^{-x} > 0$ on $(0,2)$ and < 0 on $(-\infty, 0) \cup (2, \infty)$.

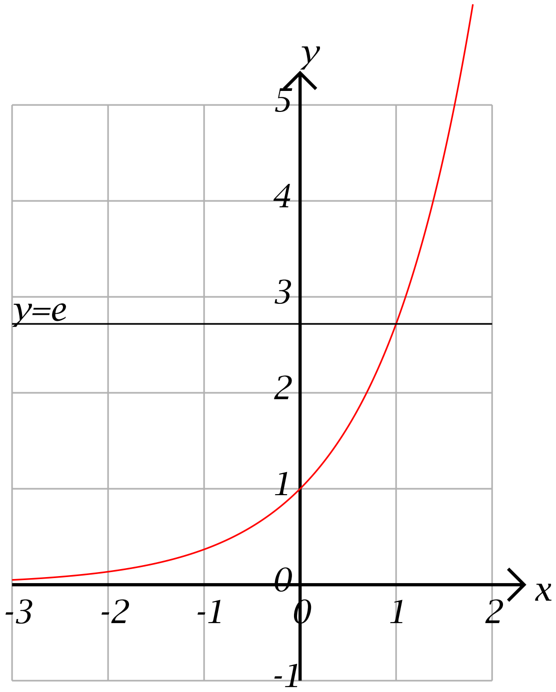
$f''(x) = \left(2x - x^2\right)' e^{-x} + (2x - x^2)\left(e^{-x}\right)' = (2-2x)e^{-x} + (2x - x^2)e^{-x}(-1) = (x^2 - 4x + 2)e^{-x} = 0$ if $x = \dfrac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$. There is an absolute minimum at (0,0) and a relative maximum at $\left(2, \dfrac{4}{e^2}\right)$.

There are inflection points at $\left(2 - \sqrt{2}, f\left(2 - \sqrt{2}\right)\right)$ and $\left(2 + \sqrt{2}, f\left(2 + \sqrt{2}\right)\right)$.



(9) **(A)** $f(x) = e^x$

Solution: $f'(x) = e^x > 0$, $f''(x) = e^x > 0$.



(10) (H) $f(x) = xe^{-x^2}$

Solution:

$f'(x) = (x)'e^{-x^2} + x\left(e^{-x^2}\right)' = (1)e^{-x^2} + xe^{-x^2}\left(-x^2\right)' = (1)e^{-x^2} + xe^{-x^2}(-2x) = (1-2x^2)e^{-x^2} > 0$ on $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and < 0 on $\left(-\infty, -\frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}, \infty\right)$

$f''(x) = \left(1-2x^2\right)' e^{-x^2} + (1-2x^2)\left(e^{-x^2}\right)' = (-4x)e^{-x^2} + (1-2x^2)e^{-x^2}(-2x) = (4x^3-6x)e^{-x^2} = 2x(2x^2-3)e^{-x^2} > 0$ on $\left(-\sqrt{\frac{3}{2}}, 0\right) \cup \left(\sqrt{\frac{3}{2}}, \infty\right)$ and < 0 on $\left(-\infty, -\sqrt{\frac{3}{2}}\right) \cup \left(0, \sqrt{\frac{3}{2}}\right)$.

There is an absolute mimimum at $\left(-\frac{\sqrt{2}}{2}, f\left(-\frac{\sqrt{2}}{2}\right)\right)$ and and an absolute maximum at $\left(\frac{\sqrt{2}}{2}, f\left(\frac{\sqrt{2}}{2}\right)\right)$