Exercises for Sketching Cubics

(1) Sketch the graph of $y = f(x) = x^3 - x$. Solution

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(2) Sketch the graph of $y = f(x) = x^3 - x^2 - 2x$ Solution

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(3) Sketch the graph of $y = f(x) = x^3 - 3x^2 + 2x$. Solution

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(4) Sketch the graph of $y = f(x) = -x^3 + 3x^2 + 2x + 1$. Solution

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Solutions

(1)

$f(x) = x^3 - x = x(x+1)(x-1)$,

$f'(x) = 3x^2 - 1 = 3\left(x^2 - \frac{1}{3}\right) = 0$ if $x = \pm\frac{\sqrt{3}}{3}$,

$f''(x) = 6x = 0$ if $x = 0$

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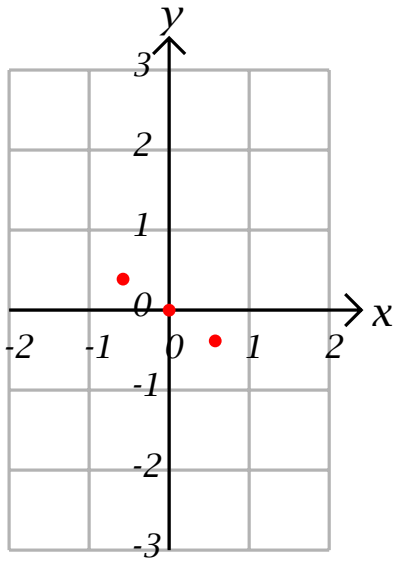
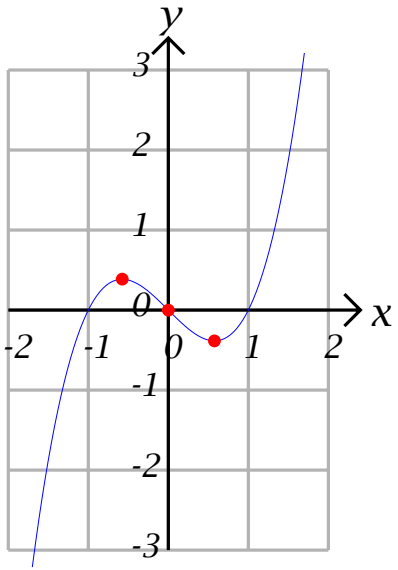
Since f is odd, we only construct a table for non-negative interesting values:

	0	$\left(0, \frac{\sqrt{3}}{3}\right)$	$\frac{\sqrt{3}}{3}$	$\left(\frac{\sqrt{3}}{3}, \infty\right)$	∞
$f''(x)$	0	$+$	$2\sqrt{3}$	$+$	∞
$f'(x)$	-1	$-$	0	$+$	∞
$f(x)$	0	$+$	$-\frac{2\sqrt{3}}{9}$	$-$	∞





(2)

$f(x) = x^3 - x^2 - 2x = x(x+1)(x-2) = x(x^2 - x - 2),$

$f'(x) = 3x^2 - 2x - 2 = 0$ if $x = \frac{2 \pm \sqrt{28}}{6} = \frac{1 \pm \sqrt{7}}{3},$

$f''(x) = 6x - 2 = 0$ if $x = \frac{1}{3}$

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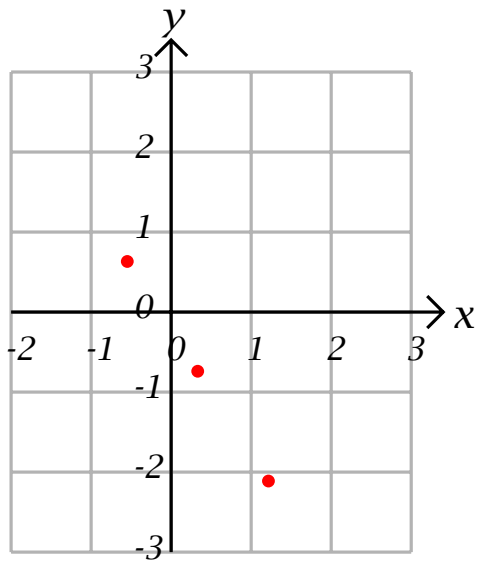
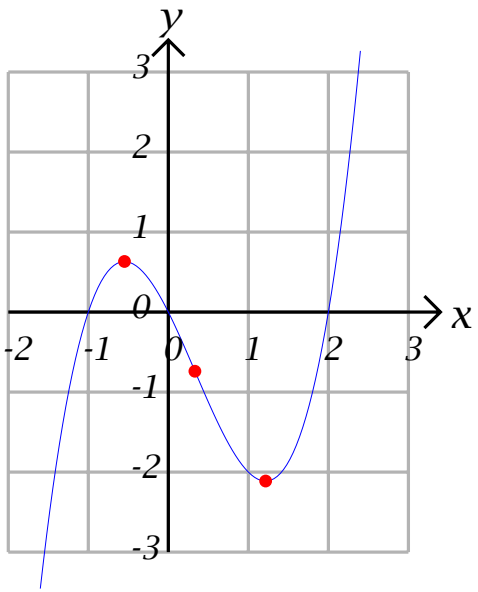
	$-\infty$	$\left(-\infty, \frac{1-\sqrt{7}}{3}\right)$	$\frac{1-\sqrt{7}}{3}$	$\left(\frac{1-\sqrt{7}}{3}, \frac{1}{3}\right)$	$\frac{1}{3}$	$\left(\frac{1}{3}, \frac{1+\sqrt{7}}{3}\right)$	$\frac{1+\sqrt{7}}{3}$	$\left(\frac{1+\sqrt{7}}{3}, \infty\right)$
$f''(x)$	$-\infty$	$-$	$-$	$-$	0	$+$	$+$	$+$
$f'(x)$	∞	$+$	0	$-$	$-$	$-$	0	$+$
$f(x)$	$-\infty$	?	$-\frac{20+14\sqrt{7}}{27}$	$-$	$-\frac{20}{27}$	$-$	$-\frac{20-14\sqrt{7}}{27}$	?





(3)

$f(x) = x^3 - 3x^2 + 2x = x(x-1)(x-2),$

$f'(x) = 3x^2 - 6x + 2 = 0$ if $x = \frac{6 \pm \sqrt{36-24}}{6} = \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3} = 1 \pm \frac{\sqrt{3}}{3},$

$f''(x) = 6x - 6 = 0$ if $x = 1$

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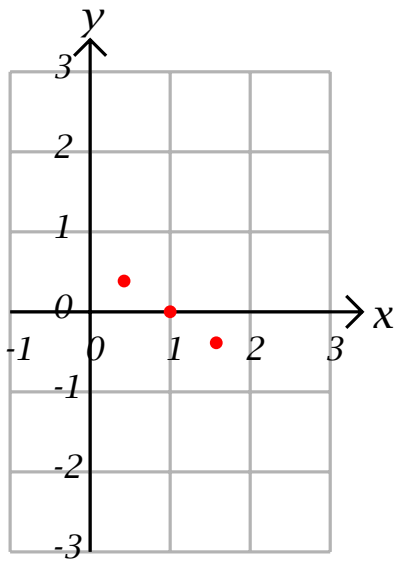
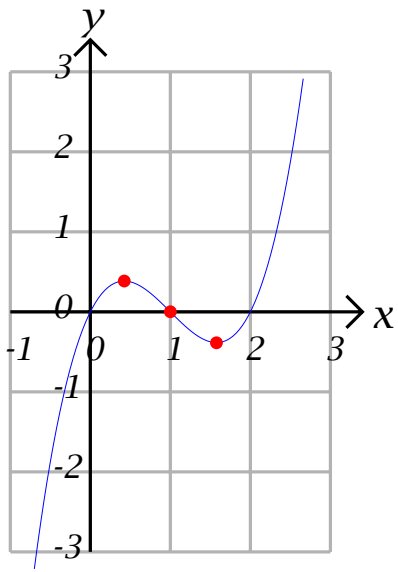
	$\left(-\infty, 1-\frac{\sqrt{3}}{3}\right)$	$1-\frac{\sqrt{3}}{3}$	$\left(1-\frac{\sqrt{3}}{3}, 1\right)$	1	$\left(1, 1+\frac{\sqrt{3}}{3}\right)$	$1+\frac{\sqrt{3}}{3}$	$\left(1+\frac{\sqrt{3}}{3}, \infty\right)$	∞
$f''(x)$	$-$	$-$	$-$	0	$+$	$+$	$+$	∞
$f'(x)$	$-$	0	$+$	-1	$-$	0	$+$	∞
$f(x)$	?	$\frac{2\sqrt{3}}{9}$	$+$	0	$+$	$-\frac{2\sqrt{3}}{9}$	?	∞





(4)

$f(x) = -x^3 + 3x^2 + 2x + 1,$

$f'(x) = -3x^2 + 6x + 2 = 0$ if $x = \frac{-6 \pm \sqrt{36+24}}{-6} = \frac{-6 \pm \sqrt{60}}{-6} == 1 \pm \frac{\sqrt{15}}{3},$

$f''(x) = -6x + 6 = 0$ if $x = 1$

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	$\left(-\infty, 1-\frac{\sqrt{15}}{3}\right)$	$1-\frac{\sqrt{15}}{3}$	$\left(1-\frac{\sqrt{15}}{3}, 1\right)$	1	$\left(1, 1+\frac{\sqrt{15}}{3}\right)$	$1+\frac{\sqrt{15}}{3}$	$\left(1+\frac{\sqrt{15}}{3}, \infty\right)$	∞
$f''(x)$	+	+	+	0	−	−	−	∞
$f'(x)$	−	0	+	5	+	0	−	∞
$f(x)$	$+(Why?)$	$5-\frac{10\sqrt{15}}{9} > 0$	+	5	+	$5+\frac{10\sqrt{15}}{9}$	?	∞