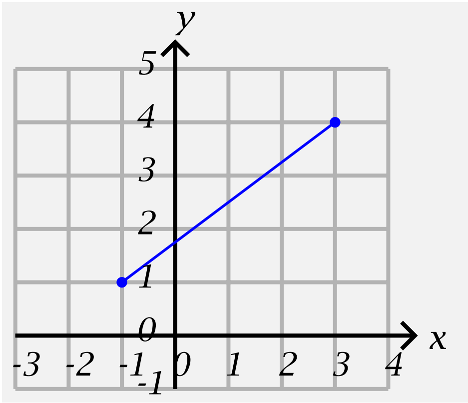
Coordinate Geometry & Lines

It is essential that the student be able to automatically apply the very basic formulas of elementary Analytic Geometry:

Distance Formula



The distance between the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In our diagram, we take $P_1 = (x_1, y_1)$ to be $(-1, 1)$ and $P_2 = (x_2, y_2)$ to be $(3, 4)$.

Then $$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$$

$$\sqrt{(3 - (-1))^2 + (4 - 1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

Slope

The slope of the line passing through the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

so in our example $m = \dfrac{4 - 1}{3 - (-1)} = \dfrac{3}{4}$

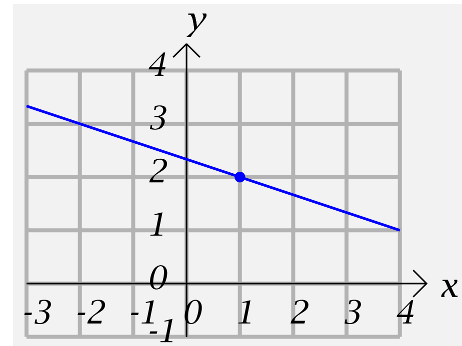
Equations of Lines

These come in many useful forms:

Point-Slope Form

The equation of the line passing through the point $P_1 = (x_1, y_1)$ with slope m is $y - y_1 = m(x - x_1)$

Thus given the point $P_1 = (1, 2)$ and the slope $m = -\frac{1}{3}$ the equation of the line is $y - 2 = -\frac{1}{3}(x - 1)$

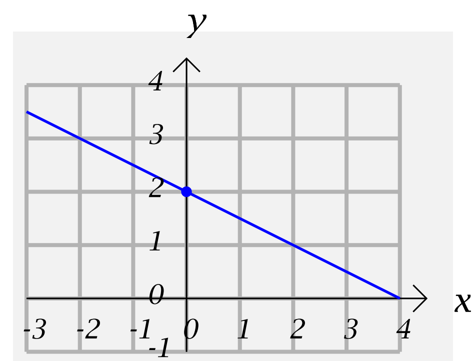


Point-Point Form

The equation of the line passing through the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

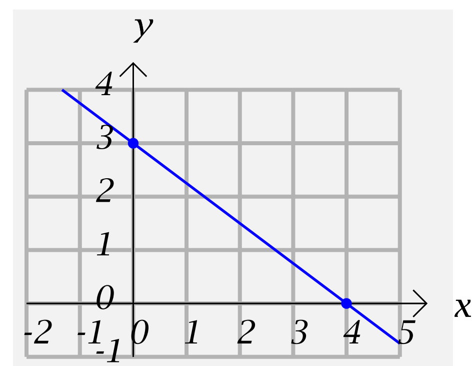
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

This just comes from putting the two previous formulas together.



Slope-Intercept Form

The equation of the line passing through the y-axis at the point $(0, b)$ with slope m is $y = mx + b$. For example, if $m = -\frac{1}{2}$ and $b = 2$, the equation is $y = -\frac{1}{2}x + 2$



Intercept-Intercept Form

The equation of the line passing through the intercepts $(a, 0)$ and $(0, b)$ is $\frac{x}{a} + \frac{y}{b} = 1$. For example, the equation of the line through $(4, 0)$ and $(0, 3)$ is $\frac{x}{4} + \frac{y}{3} = 1$.

General Form

Every line has infinitely many equations of the form

$$Ax + By + C = 0.$$

For any fixed line, they are non-zero multiples of each other.

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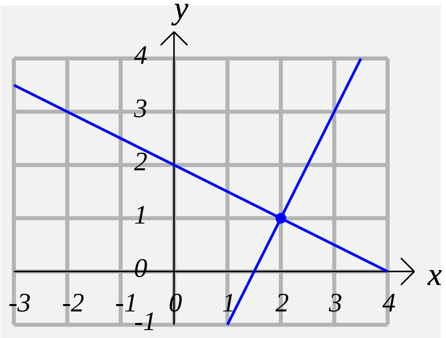
Parallel & Perpendicular Lines

Two lines with slopes m_1 and m_2 are **parallel** if $m_1 = m_2$.

perpendicular if $m_1 m_2 = -1$.

Example: Find the equation of the line through the point $(2, 1)$ which is perpendicular to the line $y = -\frac{1}{2}x + 2$.

Solution: The slope of the perpendicular line is $-\frac{1}{-\frac{1}{2}} = 2$, so the equation of the perpendicular line is, using the Point-Slope Form: $y - 1 = 2(x - 2)$



Distance from a Point to a Line

The distance from the point $P_0 = (x_0, y_0)$ to a line ℓ with equation $Ax + By + C = 0$ is

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Example: Find the distance from the point $(3, 4)$ to the line with equation $y = -\frac{1}{2}x + 2$.

Solution: We must rewrite the equation of the line in General form:

$y = -\frac{1}{2}x + 2$. becomes $2y = -x + 2$ or $x + 2y - 2 = 0$, so we apply the Distance Formula with $x_0 = 3$, $y_0 = 4$, $A = 1$, $B = 2$, and $C = -2$:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{|(1)(3) + (2)(4) + (-2)|}{\sqrt{(1)^2 + (2)^2}} = \frac{|3 + 8 - 2|}{\sqrt{1 + 4}} = \frac{9}{\sqrt{5}}$$