

## Coordinate Geometry & Lines

It is essential that the student be able to automatically apply the very basic formulas of elementary Analytic Geometry:

### Distance Formula

The distance between the points  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  is

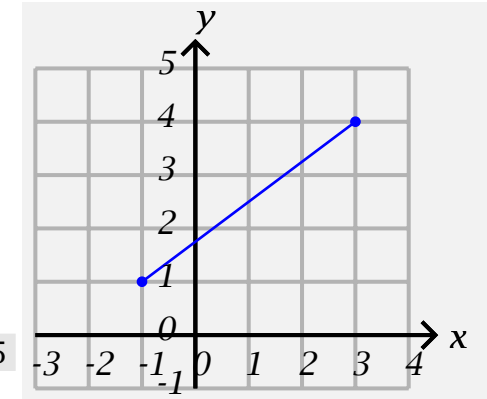
$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In our diagram, we take  $P_1 = (x_1, y_1)$  to be  $(-1, 1)$  and  $P_2 = (x_2, y_2)$  to be  $(3, 4)$ .

Then

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$$

$$\sqrt{(3 - (-1))^2 + (4 - 1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$



### Slope

The slope of the line passing through the points  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

so in our example  $m = \frac{4 - 1}{3 - (-1)} = \frac{3}{4}$

## Equations of Lines

These come in many useful forms:

### Point-Slope Form

The equation of the line passing through the point  $P_1 = (x_1, y_1)$  with slope  $m$  is  $y - y_1 = m(x - x_1)$

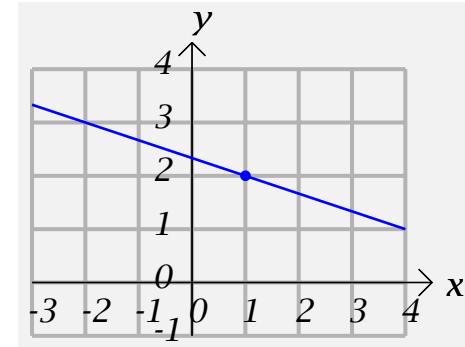
Thus given the point  $P_1 = (1, 2)$  and the slope  $m = -\frac{1}{3}$  the equation of the line is  $y - 2 = -\frac{1}{3}(x - 1)$

### Point-Point Form

The equation of the line passing through the points  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  is

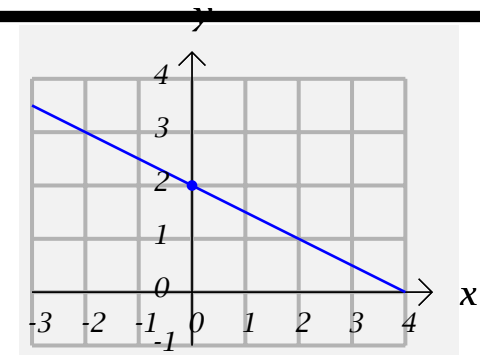
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

This just comes from putting the two previous formulas together.



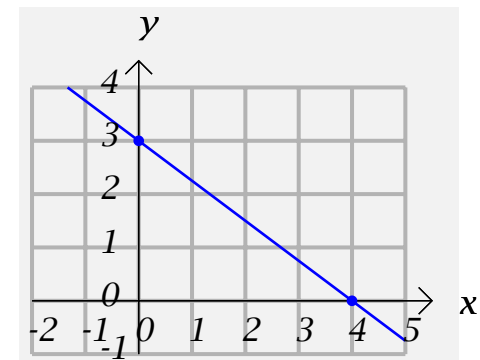
### Slope-Intercept Form

The equation of the line passing through the  $y$ -axis at the point  $(0, b)$  with slope  $m$  is  $y = mx + b$ . For example, if  $m = -\frac{1}{2}$  and  $b = 2$ , the equation is  $y = -\frac{1}{2}x + 2$



### Intercept-Intercept Form

The equation of the line passing through the intercepts  $(a, 0)$  and  $(0, b)$  is  $\frac{x}{a} + \frac{y}{b} = 1$ . For example, the equation of the line through  $(4, 0)$  and  $(0, 3)$  is  $\frac{x}{4} + \frac{y}{3} = 1$ .



## General Form

Every line has infinitely many equations of the form

$$Ax + By + C = 0.$$

For any fixed line, they are non-zero multiples of each other.

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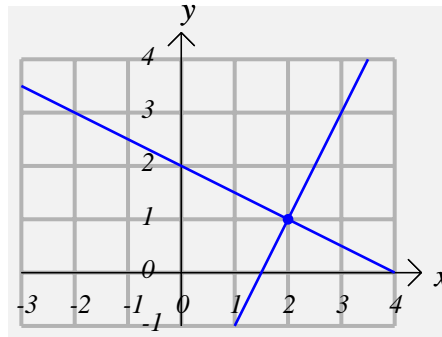
## Parallel & Perpendicular Lines

Two lines with slopes  $m_1$  and  $m_2$  are **parallel** if  $m_1 = m_2$ .

**perpendicular** if  $m_1 m_2 = -1$ .

**Example:** Find the equation of the line through the point  $(2, 1)$  which is perpendicular to the line  $y = -\frac{1}{2}x + 2$ .

**Solution:** The slope of the perpendicular line is  $-\frac{1}{-\frac{1}{2}} = 2$ , so the equation of the perpendicular line is, using the Point-Slope Form:  $y - 1 = 2(x - 2)$



## Distance from a Point to a Line

The distance from the point  $P_0 = (x_0, y_0)$  to a line  $\ell$  with equation  $Ax + By + C = 0$  is

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

**Example:** Find the distance from the point  $(3, 4)$  to the line with equation  $y = -\frac{1}{2}x + 2$ .

**Solution:** We must rewrite the equation of the line in General form:

$y = -\frac{1}{2}x + 2$  becomes  $2y = -x + 2$  or  $x + 2y - 2 = 0$ , so we apply the Distance Formula with  $x_0 = 3$ ,  $y_0 = 4$ ,  $A = 1$ ,  $B = 2$ , and  $C = -2$ :

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{|(1)(3) + (2)(4) + (-2)|}{\sqrt{(1)^2 + (2)^2}} = \frac{|3 + 8 - 2|}{\sqrt{1 + 4}} = \frac{9}{\sqrt{5}}$$

