

Coordinate Geometry & Lines

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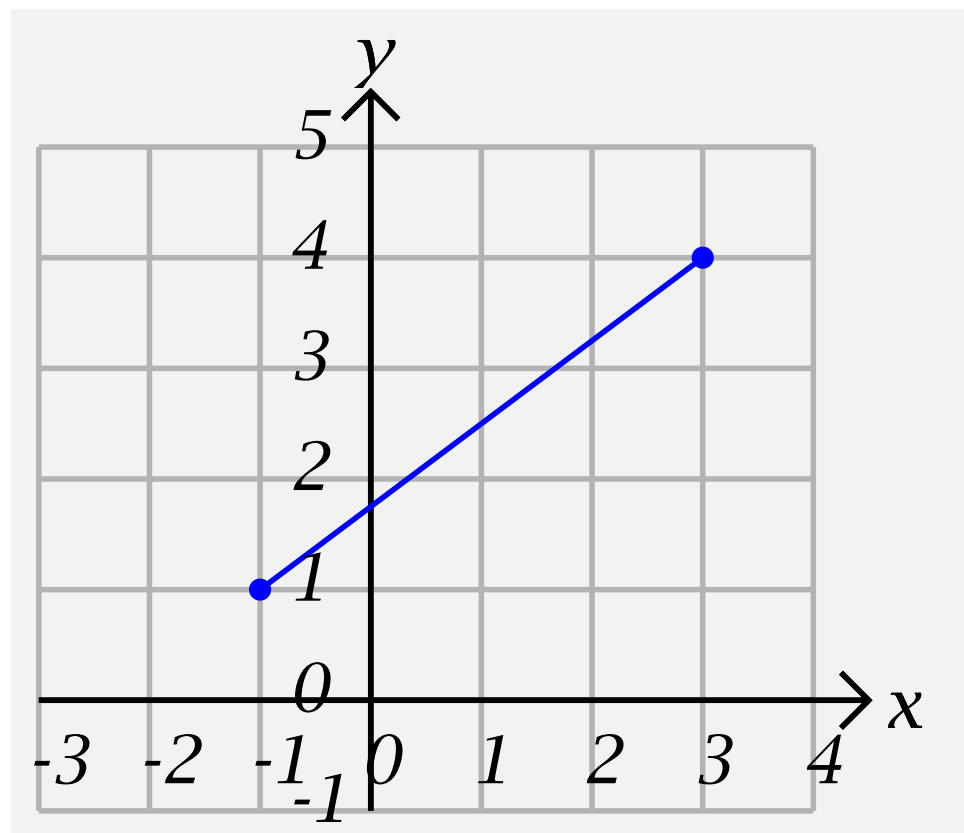
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In our diagram, we take $P_1 = (x_1, y_1)$ to be $(-1, 1)$ and $P_2 = (x_2, y_2)$ to be $(3, 4)$.



Coordinate Geometry & Lines-2

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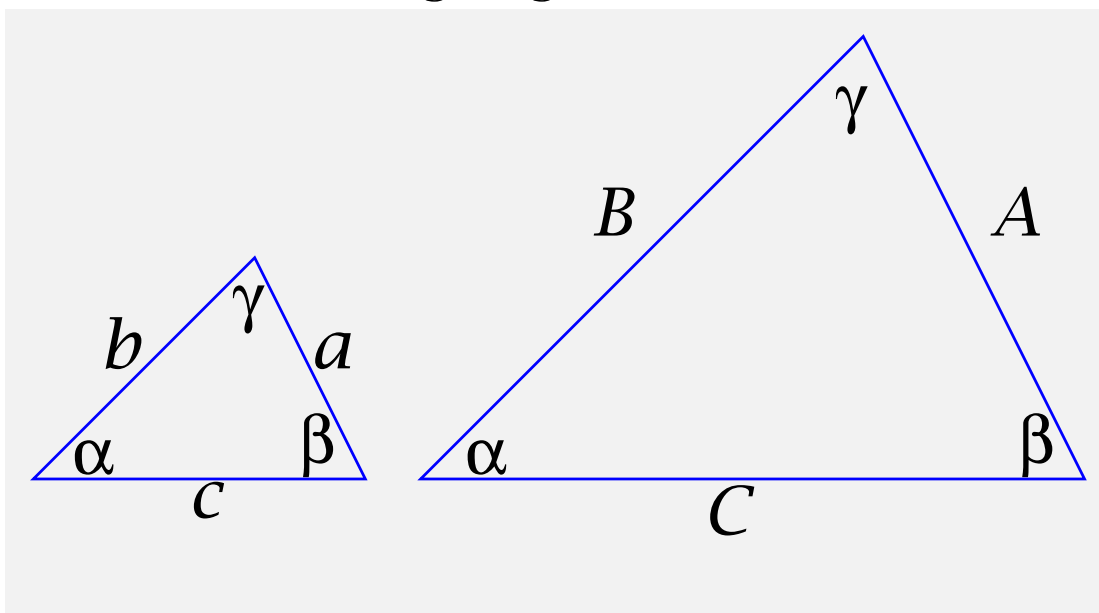
$$\sqrt{(3 - (-1))^2 + (4 - 1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} =$$

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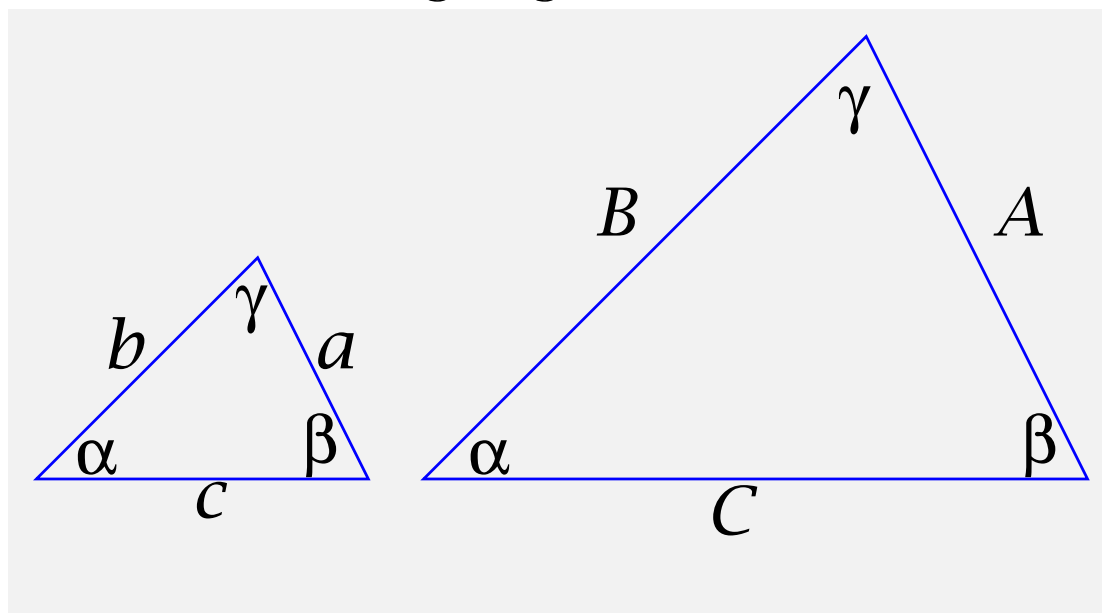
Similar Triangles

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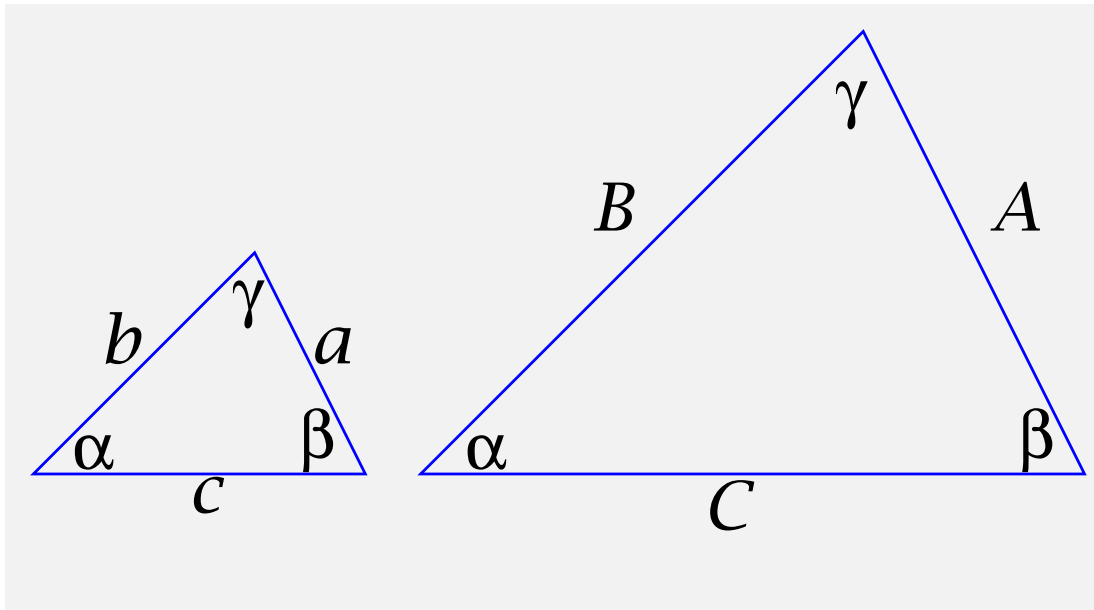


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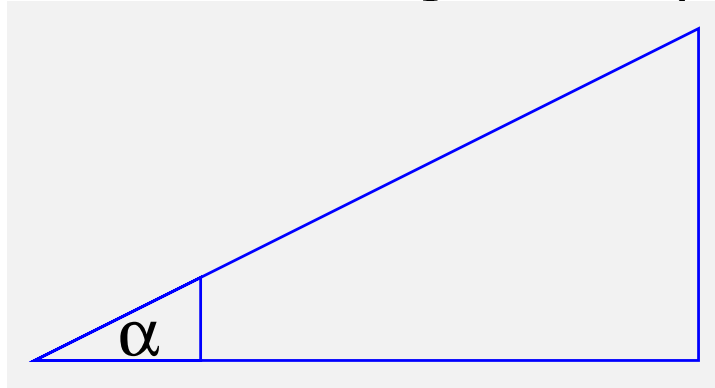
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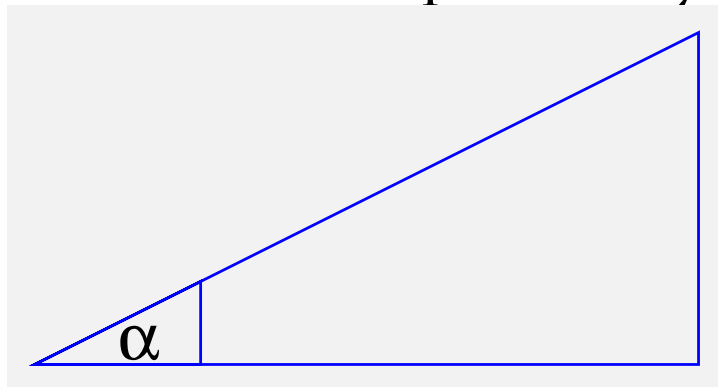
In Calculus we will mostly deal with right-angled triangles

having a horizontal and a vertical side. The ratio of the vertical side to the horizontal side depends only on the angle α .

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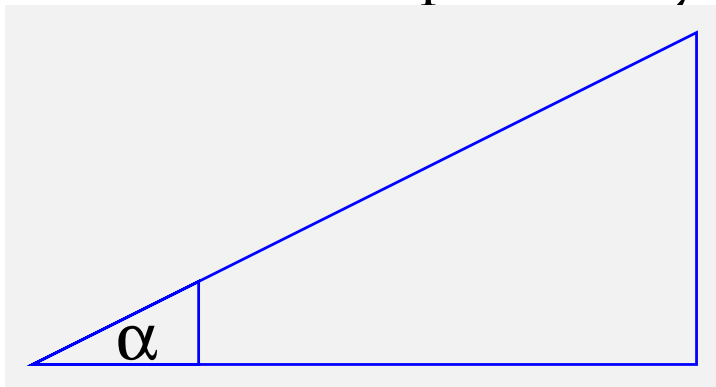


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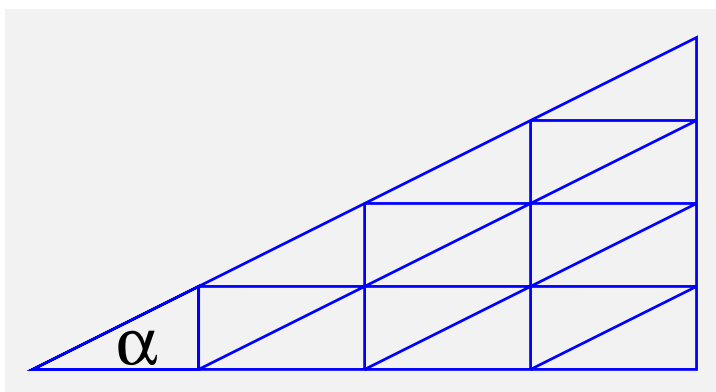


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Equations of Lines

These come in many useful forms:

Coordinate Geometry & Lines-6

Point-Slope Form

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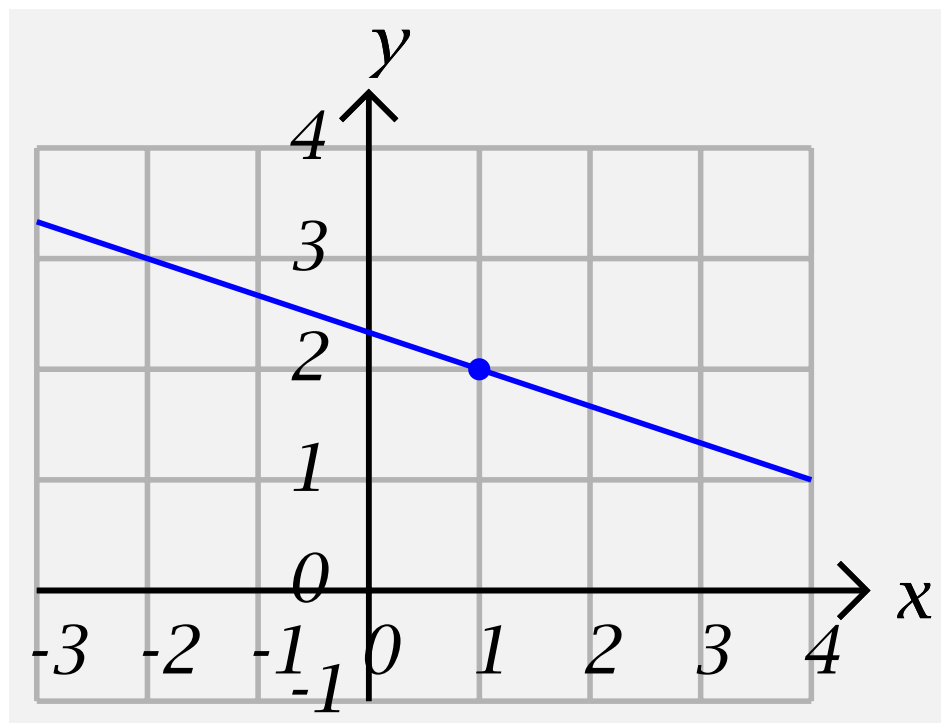
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$$y - 2 = -\frac{1}{3}(x - 1)$$



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This just comes from putting the two previous formulas together.

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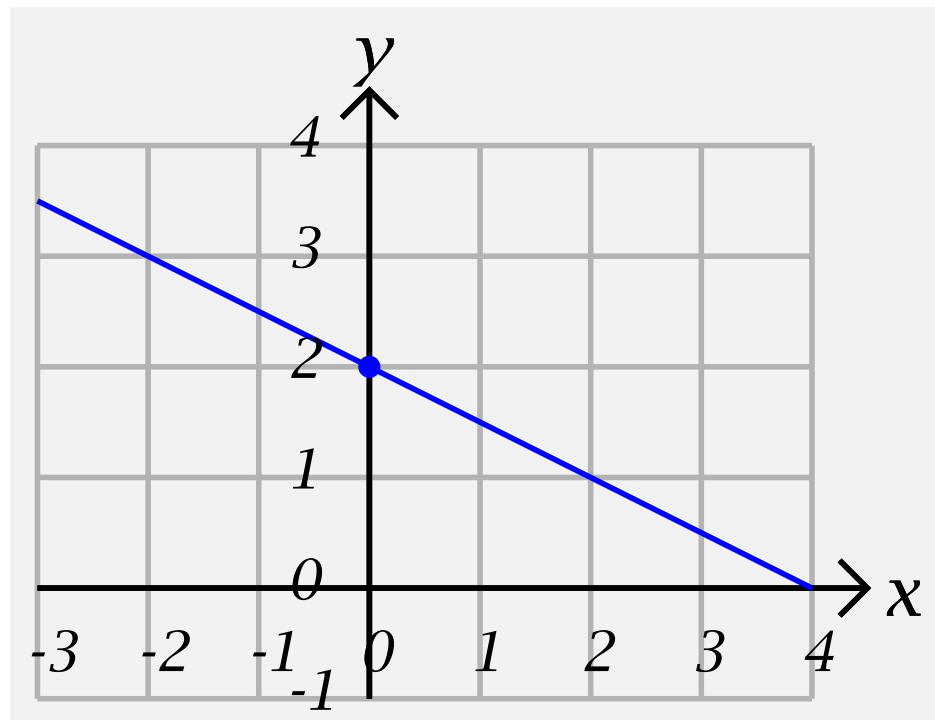
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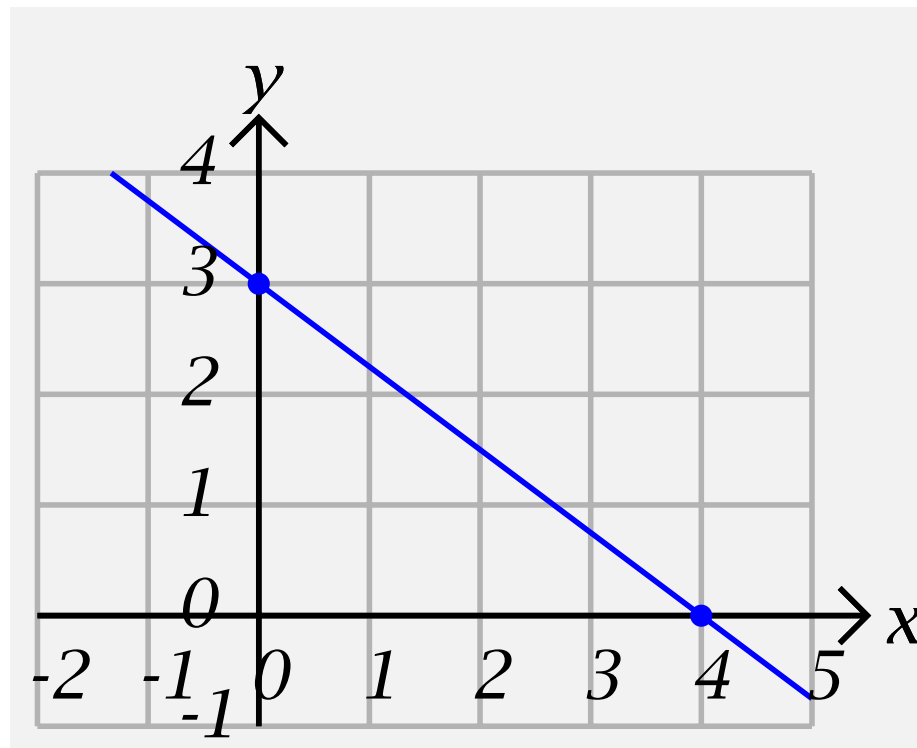
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General Form

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$$Ax + By + C = 0.$$

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For any fixed line, they are non-zero multiples of each other.

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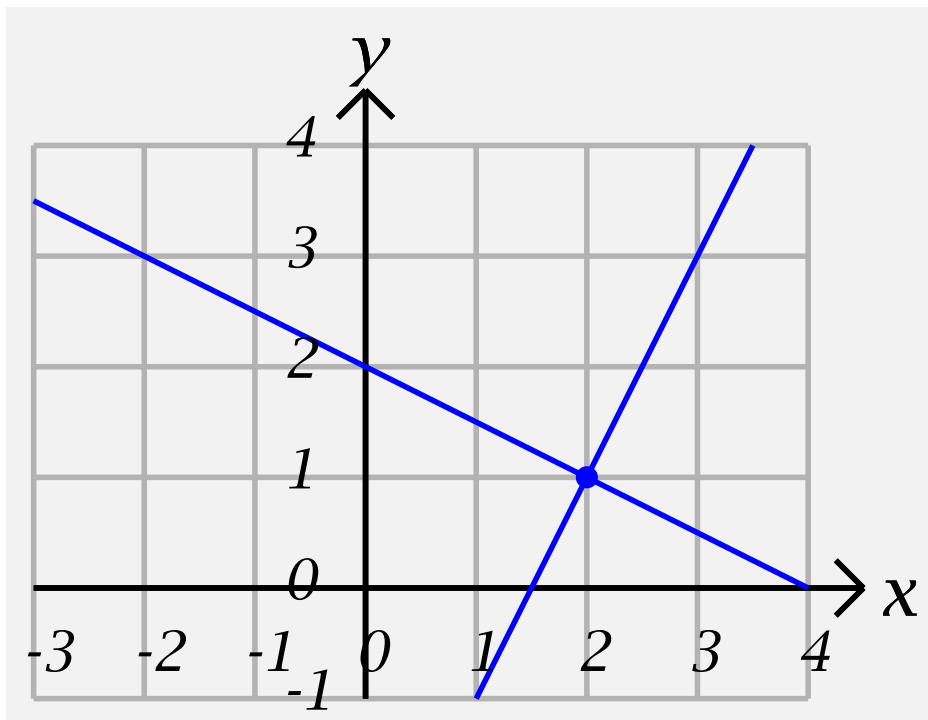
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$$y - 1 = 2(x - 2)$$



Distance from a Point to a Line

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Coordinate Geometry & Lines-14

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