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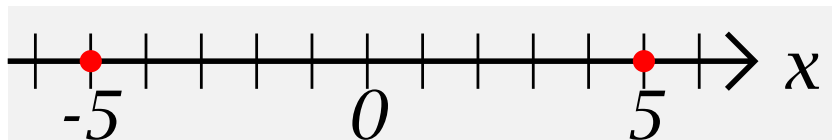
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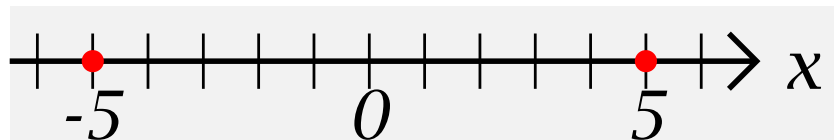
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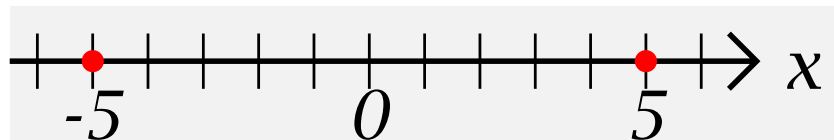


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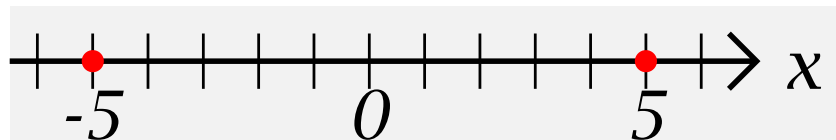
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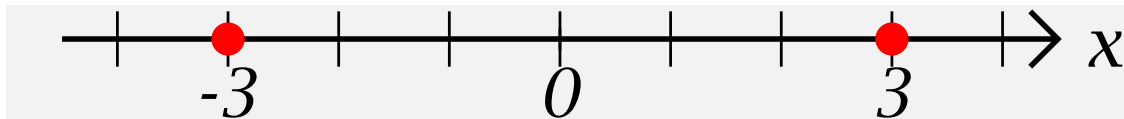
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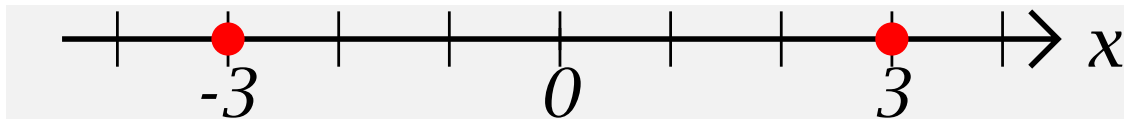
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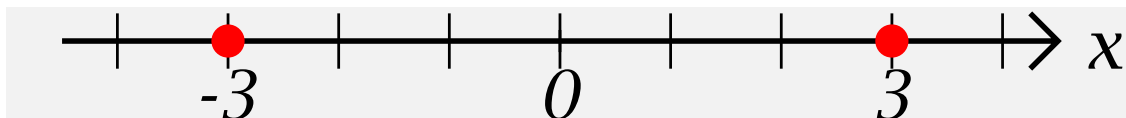
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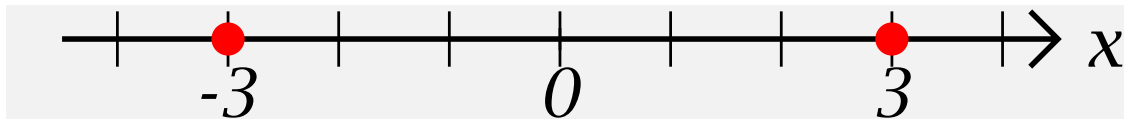


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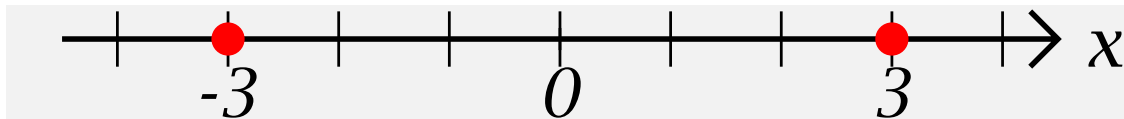


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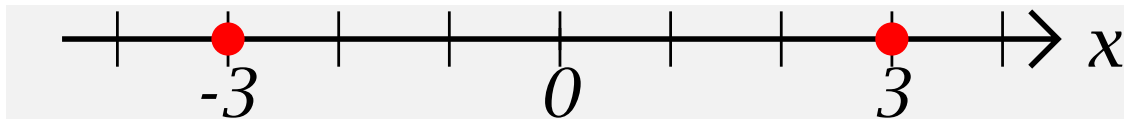


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**Note:**  $|a - b| = |b - a|$  is the distance of the number  $b$  from the number  $a$ .

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$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|} \text{ (if } b \neq 0.)$$

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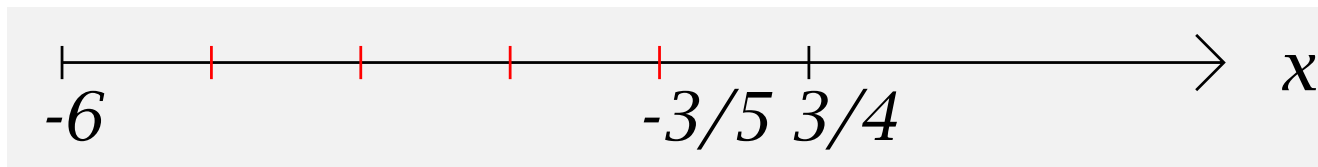
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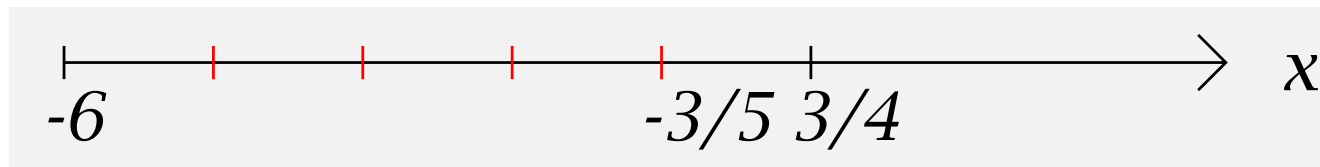
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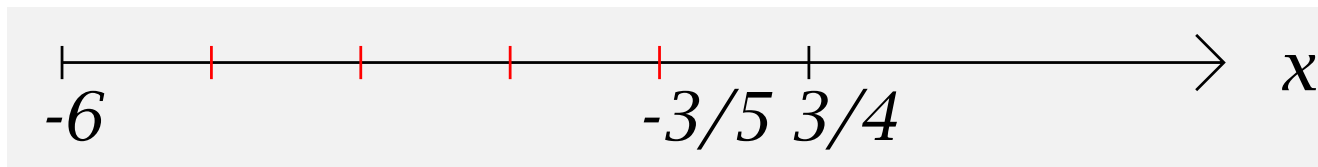
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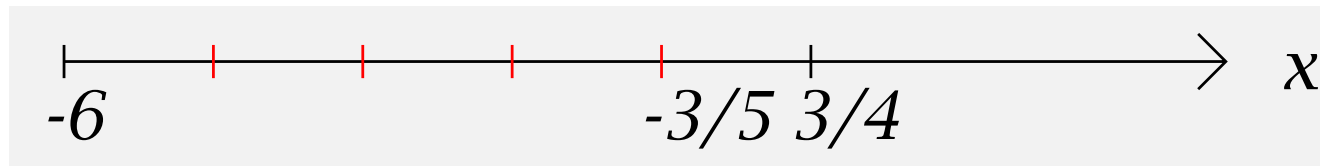
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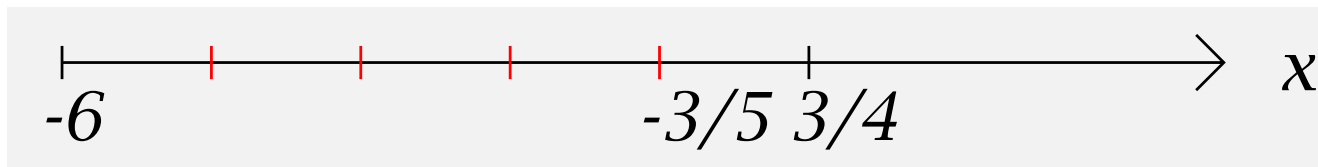
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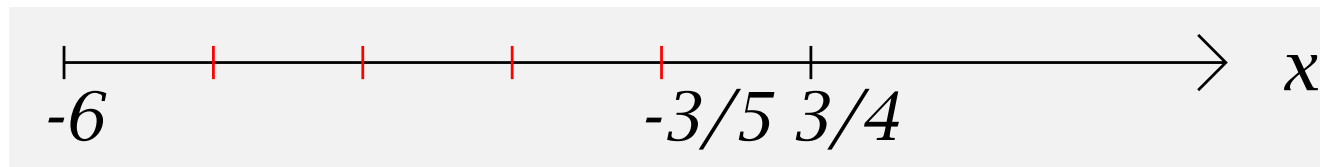
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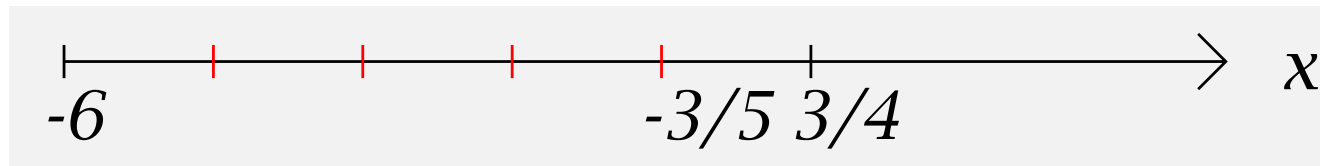
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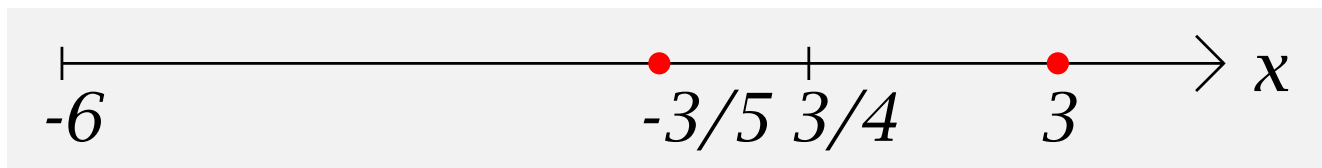
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Absolute Value-13

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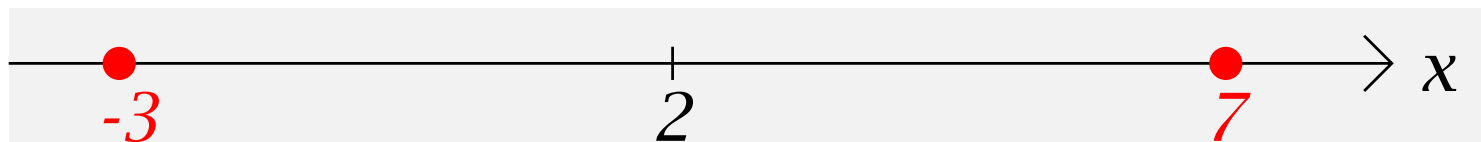
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Thus the solution set consists of all numbers which are further than  $\frac{5}{7}$  from  $\frac{2}{7}$ .

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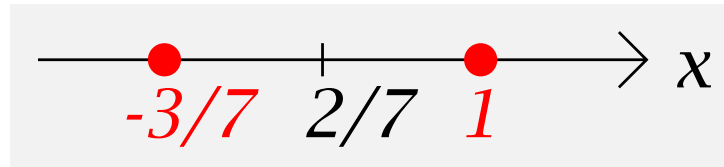
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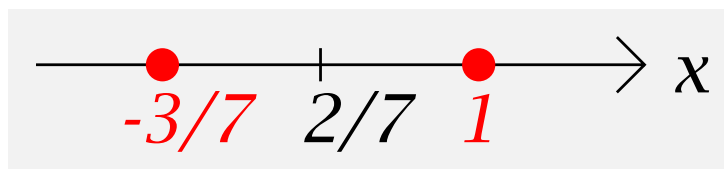
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**Example 9:** Express the condition  $x < 0$  or  $100 < x$  using absolute values and an inequality.

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$$|x - 122| < 90$$

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**Example 9:** Express the condition  $x < 0$  or  $100 < x$  using absolute values and an inequality.

The midpoint of the interval (0,100) is 50 and its length is 100, so our solution is

**Example 8:** Express the condition  $32 < x < 212$  using absolute values and an inequality.

The midpoint of the interval (32,212) is 122, and its length is 180, so our solution is

$$|x - 122| < 90$$

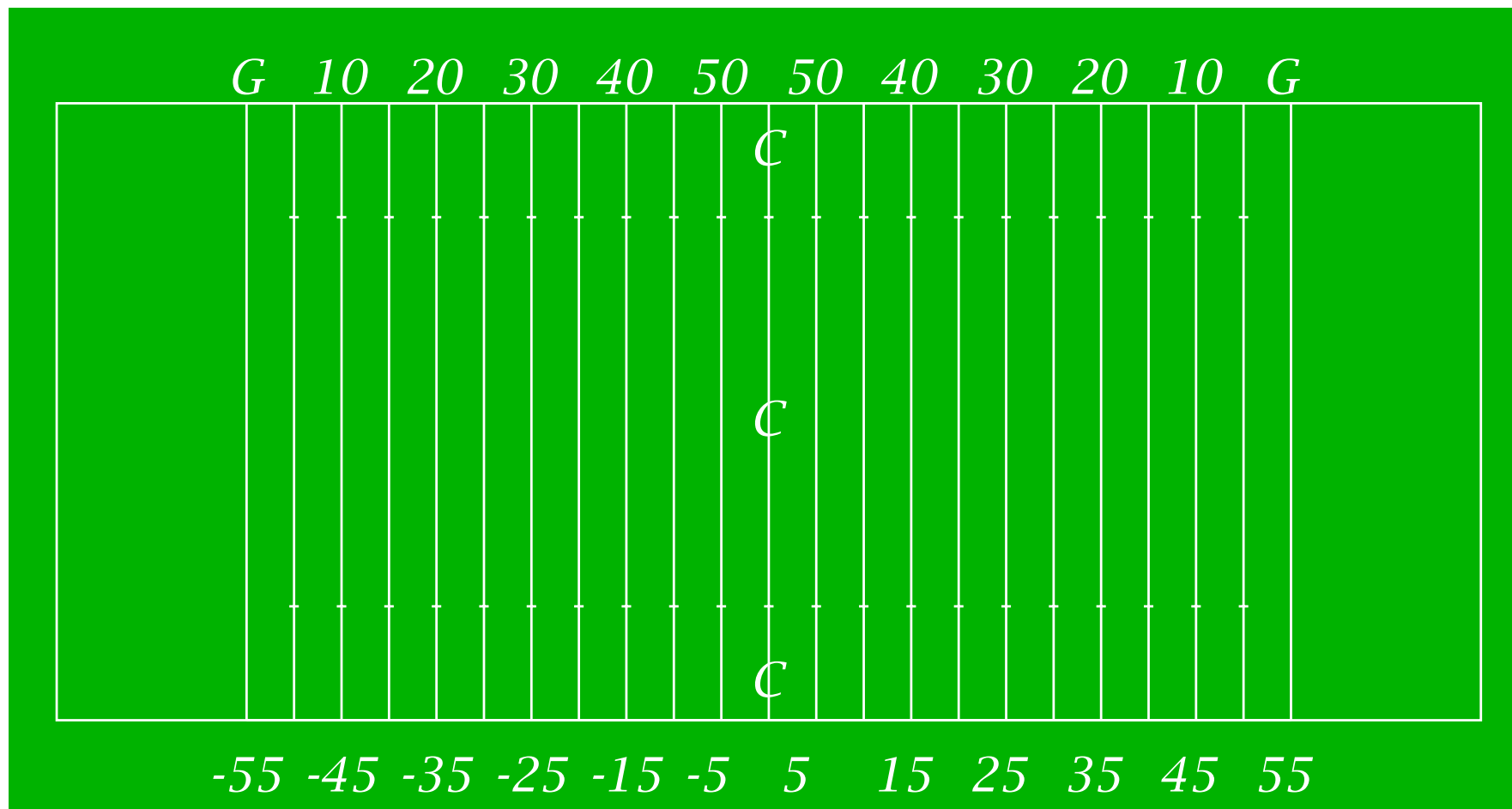
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**Example 9:** Express the condition  $x < 0$  or  $100 < x$  using absolute values and an inequality.

The midpoint of the interval (0,100) is 50 and its length is 100, so our solution is

$$|x - 50| > 50$$

## Absolute Value-22



If a football is kicked from the 35-yard line and lands on the opposing 25 yard line, how far has it been kicked? What does this have to do with absolute values?