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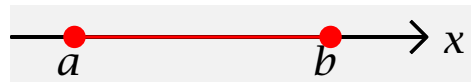
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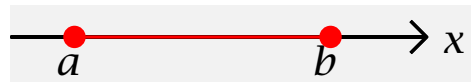
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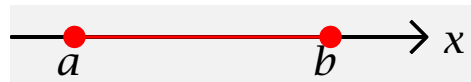
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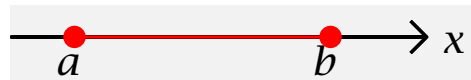
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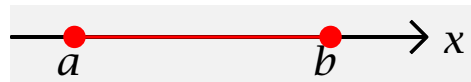
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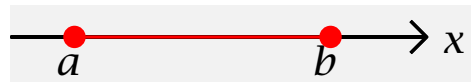
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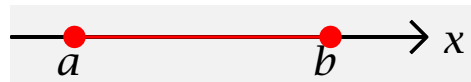
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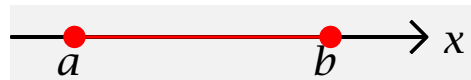
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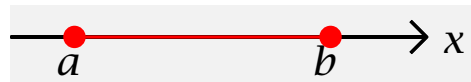
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Finite intervals have a **midpoint** whose numerical value is  $\frac{a+b}{2}$ , the average value of  $a$  and  $b$ . The **length** of a finite interval is  $b - a$ .

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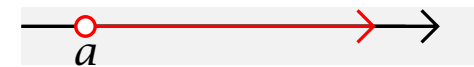
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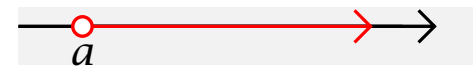
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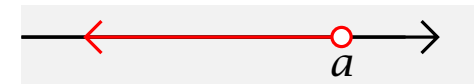
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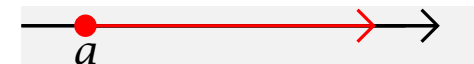


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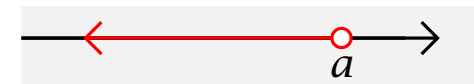
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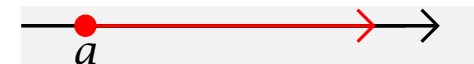
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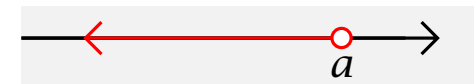
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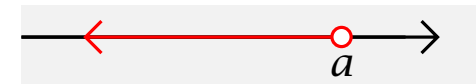
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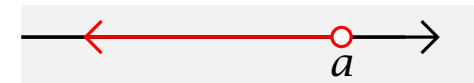
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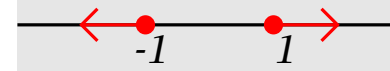
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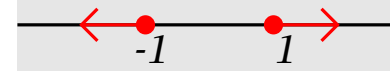
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## Linear Inequalities: An Introduction

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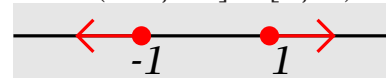
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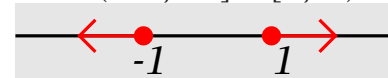
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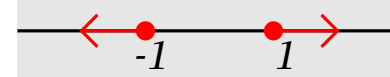
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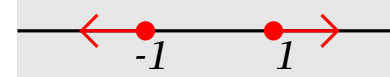
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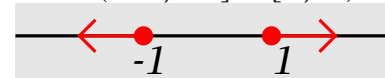
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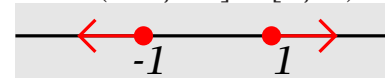
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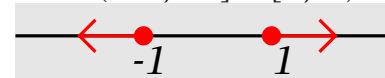
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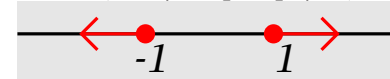
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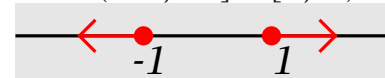
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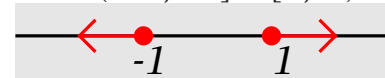
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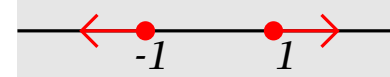
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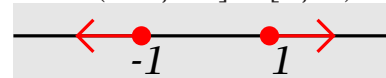
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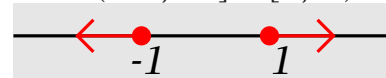
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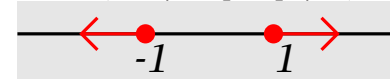
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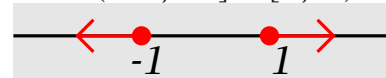
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### Multiplication by a Negative Constant

If  $a \spadesuit b$ , and  $c$  is negative,

### Multiplication by a Positive Constant

If  $a \spadesuit b$ , and  $c$  is positive, then  $ac \spadesuit bc$ , that is:

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Again we can work in at least two different ways:

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## Correctly Chained Inequalities:

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$$a \leq b < c$$

$$a < b \leq c$$

$$a \leq b \leq c$$

$$a > b > c$$

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$$\begin{aligned} 3x - 5 < 7 & \iff \left\| \begin{array}{c} \text{OR} \\ \text{OR} \end{array} \right\| 2x - 1 > 13 & \iff \\ 3x - 5 + 5 < 7 + 5 & \iff \left\| \begin{array}{c} \text{OR} \\ \text{OR} \end{array} \right\| \end{aligned}$$

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### Solution by Equivalent Inequalities

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