UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics

Mathematics 101.3 Quiz #3 — Solutions

March 28, 2001 Time:50 minutes Instructor: *Doug MacLean*

CLOSED BOOK — NO CALCULATORS PERMITTED Each question is worth 4%

PART I

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

Evaluate

(1) $2\left(e^{\ln 3}\right)$ **Solution:** $2\left(e^{\ln 3}\right) = 2(3) =$ 6

(2) $\log_2 128$ **Solution:** $\log_2 128 = \log_2 2^7 = 7\log_2 2 = 7(1) =$ 7

Find $f'(1)$ if:

(3) $f(x) = \frac{30}{7}\ln(7x+3)$ **Solution:** $f'(x) = \frac{30}{7}\frac{1}{7x+3)}(7x+3)' = \frac{30}{7}\frac{1}{7x+3)}(7) = \frac{30}{7x+3}$, so $f'(1) = \frac{30}{7(1)+3} = \frac{30}{10} =$ 3

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(4) $f(x) = \frac{4}{e^2}e^{\frac{x+3}{2}}$ **Solution:** $f'(x) = \frac{4}{e^2}e^{\frac{x+3}{2}}\left(\frac{x+3}{2}\right)' = \frac{4}{e^2}e^{\frac{x+3}{2}}\left(\frac{1}{2}\right) = \frac{2}{e^2}e^{\frac{x+3}{2}}$, so $f'(1) = \frac{2}{e^2}e^{\frac{1+3}{2}} = \frac{2}{e^2}e^{\frac{4}{2}} = \frac{2}{e^2}e^2 =$ 2

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(5) $f(x) = 150\ln\left(\dfrac{x+4}{x+5}\right)$ **Solution:** $f(x) = 150\left[\ln(x+4) - \ln(x+5)\right]$,

so $f'(x) = 150\left[\dfrac{1}{x+4} - \dfrac{1}{x+5}\right]$, and

$f'(1) = 150\left[\dfrac{1}{1+4} - \dfrac{1}{1+5}\right] = 150\left[\dfrac{1}{5} - \dfrac{1}{6}\right] = 150\left[\dfrac{6-5}{5(6)}\right] = 150\dfrac{1}{30} =$ 5

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(6) $f(x) = \dfrac{8^x}{2\ln 8}$ **Solution:** $f'(x) = \ln 8\dfrac{8^x}{2\ln 8} = \dfrac{8^x}{2}$, so $f'(1) = \dfrac{8^1}{2} = \dfrac{8}{2} =$ 4

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(7) $f(x) = \dfrac{8}{3\ln 7}7^{x^3-1}$ **Solution:** $f'(x) = \dfrac{8}{3\ln 7}(\ln 7)7^{x^3-1}\left(x^3-1\right)' = \dfrac{8}{3}7^{x^3-1}\left(3x^2\right)$,

so $f'(1) = \dfrac{8}{3}7^{(1)^3-1}\left(3(1)^2\right) = \dfrac{8}{3}7^0(3) =$ 8

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(8) $f(x) = 80\ln\left(\dfrac{(x+4)^8}{(x+3)^6}\right)$ **Solution:**

$f(x) = 80\left[8\ln(x+4) - 6\ln(x+3)\right]$, so

$f'(x) = 80\left[8\dfrac{1}{x+4} - 6\dfrac{1}{x+3}\right]$, and

$f'(1) = 80\left[8\dfrac{1}{1+4} - 6\dfrac{1}{1+3}\right] = 80\left[\dfrac{8}{5} - \dfrac{6}{4}\right] = 80\left[\dfrac{8(4)-5(6)}{5(4)}\right] = 80\left[\dfrac{32-30}{20}\right] =$

$80\dfrac{2}{20}80\left[\dfrac{1}{10}\right] =$ 8

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(9) $f(x) = x^6e^{-x+1}$ **Solution:** $f'(x) = \left(x^6\right)'e^{-x+1} + x^6\left(e^{-x+1}\right)' =$

$6x^5e^{-x+1} + x^6e^{-x+1}(-x+1)' = 6x^5e^{-x+1} + x^6e^{-x+1}(-1) = x^5e^{-x+1}(6-x)$, so $f'(1) =$

$(1)^5e^{-1+1}(6-1) = e^0(5) =$ 5

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(10) $f(x)=\dfrac{x^6 e}{e^x}$ **Solution:** $f(x)=e\dfrac{x^6}{e^x}$, so

$f'(x)=e\dfrac{e^x\left(x^6\right)'-x^6\left(e^x\right)'}{\left(e^x\right)^2}=e\dfrac{e^x\left(6x^5\right)-x^6e^x}{\left(e^x\right)^2}=e\dfrac{6x^5-x^6}{e^x}$, so $f'(1)=e\dfrac{6(1)^5-(1)^6}{e^1}=$

$e\dfrac{6-1}{e}=$ 5

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(11) $f(x)=-6\dfrac{e^{x-1}}{5x^6}$ **Solution:**

$f(x)=-\dfrac{6}{5}\dfrac{e^{x-1}}{x^6}$, so

$f'(x)=-\dfrac{6}{5}\dfrac{x^6\left(e^{x-1}\right)'-e^{x-1}\left(x^6\right)'}{\left(x^6\right)^2}=-\dfrac{6}{5}\dfrac{x^6e^{x-1}(x-1)'-e^{x-1}\left(6x^5\right)}{x^{12}}=$

$-\dfrac{6}{5}\dfrac{x^6e^{x-1}(1)'-6x^5e^{x-1}}{x^{12}}=-\dfrac{6}{5}e^{x-1}\dfrac{x^6-6x^5}{x^{12}}=-\dfrac{6}{5}e^{x-1}\dfrac{x-6}{x^7}$, so

$f'(1)=-\dfrac{6}{5}e^{1-1}\dfrac{1-6}{1^7}=-\dfrac{6}{5}e^0\dfrac{-5}{1}=$ 6

The line tangent to the graph of $y = 7 - e^{2(x-1)}$ at the point $(1, 6)$ has its

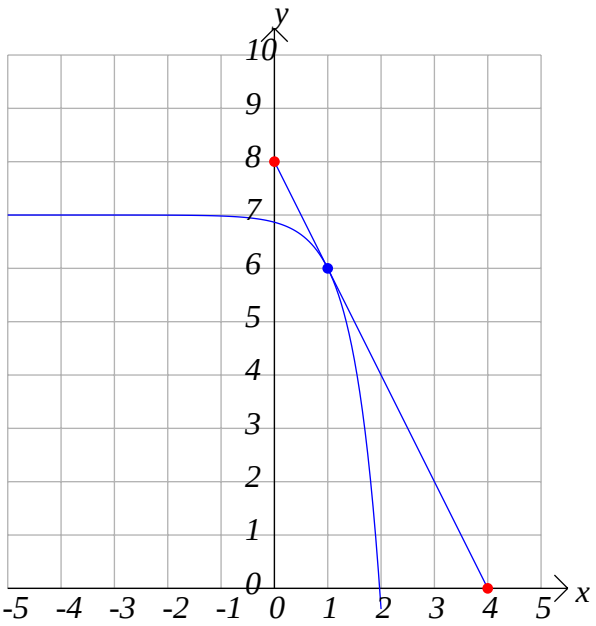
x-intercept = **(12)** and its y-intercept =**(13)**

Solution: $y' = -e^{2(x-1)}(2(x-1))' = -e^{2(x-1)}(2) = -2e^{2(x-1)}$,

and when $x = 1$, we have $y' = -2e^{2(1-1)} = -2$. The equation of the tangent line is therefore

$y - 6 = -2(x-1)$. Setting $y = 0$, we get $0 - 6 = -2(x-1)$, so the x-intercept is 4 .

Setting $x = -$, we get $y - 6 = -2(0-1) = 2$, so the y-intercept is 8



Solve for x:

(14) $\log_x 16 = 2$ **Solution:** We must have $x^2 = 16$, so $x =$ 4

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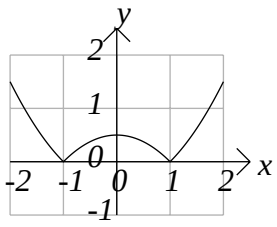
(15) $\log_{(x^2)} 16 = 2$ **Solution:** We must have $\left(x^2\right)^2 = 16$, so $x =$ 2

PART II

The possible answers to all questions in Part II are the letters A to J

Part of the graph of $y = f(x) = \left|\dfrac{x^2 - 1}{2}\right|$ is shown to the right.



Parts of the graphs of

(16) $y = f(x+1)$, B

(17) $y = f(x)+1$, E

(18) $y = f(x-1)$, D

(19) $y = f(x)-1$, C

(20) $y = 2f(x)$, J

(21) $y = -f(x)$, A

(22) $y = f(x)/2$, F

(23) $y = f(2x)$, I

(24) $y = -f(x)+1$, H and

(25) $y = f(x/2)$, G are shown below. Match them.