UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Quiz #3

November 24, 2000 Time:50 minutes Instructor: *Doug MacLean*

PART I

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

Using the limit definition to compute the derivative of $f(x) = 3x^3 - 2x^2$, we arrive at $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[9x^2 - 4x + (ax - b)h + 3h^2]h}{h} = 9x^2 - 4x$,
where **(1)** $a =$ and **(2)** $b =$

Solution: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[3(x+h)^3 - 2(x+h)^2] - [3x^3 - 2x^2]}{h} =$

$\lim_{h \to 0} \frac{[3(x^3 + 3x^2h + 3xh^2 + h^3) - 2(x^2 + 2xh + h^2)] - [3x^3 - 2x^2]}{h} =$

$\lim_{h \to 0} \frac{[9x^2h + 9xh^2 + 3h^3 - 4xh - 2h^2}{h} = \lim_{h \to 0} \frac{[9x^2 - 4x + (9x - 2)h + h^2]h}{h} = 3x^2 - 2x$,
so **(1)** $a = 9$ and **(2)** $b = 2$

Find $f'(1)$ if:

(3) $f(x) = \frac{32}{5} \ln(5x + 3)$

Solution: $f'(x) = \frac{32}{5} \frac{(5x+3)'}{5x+3} = \frac{32}{5} \frac{5}{5x+3} = \frac{32}{5x+3}$,

so $f'(1) = \frac{32}{5(1) + 3} = \frac{32}{8} = 4$

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(4) $f(x) = \frac{18}{e} e^{\frac{x+2}{3}}$

Solution: $f'(x) = \frac{18}{e} e^{\frac{x+2}{3}} \left(\frac{x+2}{3}\right)' = \frac{18}{e} e^{\frac{x+2}{3}} \left(\frac{1}{3}\right) = \frac{6}{e} e^{\frac{x+2}{3}}$, so

$f'(1) = \frac{6}{e} e^{\frac{1+2}{3}} = \boxed{6}$

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(5) $f(x) = 100 \ln\left(\frac{x+3}{x+4}\right)$

Solution: $f(x) = 100\,(\ln(x+3) - \ln(x+4))$, so

$f'(x) = 100\left(\frac{(x+3)'}{x+3} - \frac{(x+4)'}{x+4}\right) = 100\left(\frac{1}{x+3} - \frac{1}{x+4}\right)$, and thus

$f'(1) = 100\left(\frac{1}{1+3} - \frac{1}{1+4}\right) = 100\left(\frac{1}{4} - \frac{1}{5}\right) = 100\left(\frac{5}{20} - \frac{4}{20}\right) = 100\left(\frac{1}{20}\right) = \boxed{5}$

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(6) $f(x) = \frac{9^x}{\ln 9}$

Solution: $f'(x) = \frac{1}{\ln 9}(9^x)' = \frac{1}{\ln 9}(\ln 9)9^x = 9^x$ so $f'(1) = 9^1 = \boxed{9}$

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(7) $f(x) = \frac{5}{3\ln 5} 5^{x^3-1}$

Solution: $f'(x) = \frac{5}{3\ln 5}(5^{x^3-1})' = \frac{5}{3\ln 5}(\ln 5)5^{x^3-1}(x^3-1)' = \frac{5}{3}5^{x^3-1}3x^2 = 5x^2 5^{x^3-1}$, so

$f'(1) = 5(1)^2 5^{1^3-1} = \boxed{5}$

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(8) $f(x) = 36\ln\left(\dfrac{(x+3)^7}{(x+2)^5}\right)$

Solution: $f(x) = 36\left(7\ln(x+3) - 5\ln(x+2)\right)$, so

$$f'(x) = 36\left(7\frac{(x+3)'}{x+3} - 5\frac{(x+2)'}{x+2}\right) = 36\left(7\frac{1}{x+3} - 5\frac{1}{x+2}\right) = 36\left(\frac{7}{x+3} - \frac{5}{x+2}\right) =$$

$$36\left(\frac{7(x+2)}{(x+2)(x+3)} - \frac{5(x+3)}{(x+2)(x+3)}\right) = 36\left(\frac{7x+14}{(x+2)(x+3)} - \frac{5x+15}{(x+2)(x+3)}\right) = 36\frac{2x-1}{(x+2)(x+3)},$$
and thus

$$f'(1) = 36\frac{2(1)-1}{(1+2)(1+3)} = \frac{36}{(3)(4)} = \boxed{3}$$

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(9) $f(x) = 2x^5e^{-x+1}$

Solution: $f'(x) = 2\left(x^5e^{-x+1}\right)' = 2\left[\left(x^5\right)'e^{-x+1} + x^5\left(e^{-x+1}\right)'\right] =$

$$2\left[5x^4e^{-x+1} + x^5e^{-x+1}(-x+1)'\right] = 2\left[5x^4e^{-x+1} + x^5e^{-x+1}(-1)\right] =$$

$$2e^{-x+1}\left[5x^4 - x^5\right] = 2x^4e^{-x+1}[5-x], \text{ so}$$

$$f'(1) = 2(1)^4e^{-1+1}[5-1] = 2e^04 = \boxed{8}$$

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(10) $f(x) = \dfrac{x^5e}{e^x}$

Solution: Use logarithmic differentiation:

$$\ln(f(x)) = \ln(x^5e) - \ln(e^x) = \ln x^5 + \ln e - x = 5\ln x + 1 - x.$$

Differentiating, we get

$$\frac{f'(x)}{f(x)} = 5\frac{1}{x} + 0 - 1 = \frac{5}{x} - 1, \text{ so}$$

$f'(x) = f(x)\left(\dfrac{5}{x} - 1\right)$. Setting $x = 1$, we get:

$$f'(1) = f(1)\left(\frac{5}{1} - 1\right) = \frac{(1)^5e}{e^1}4 = \boxed{4}.$$

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(11) $f(x) = -2\frac{e^{x-1}}{x^5}$

Solution: Using the Quotient Rule, we get

$$f(x) = -2\frac{x^5\left(e^{x-1}\right)' - e^{x-1}\left(x^5\right)'}{\left(x^5\right)^2} = -2\frac{x^5e^{x-1}(x-1)' - e^{x-1}5x^4}{x^{10}} =$$

$$-2\frac{x^5e^{x-1} - e^{x-1}5x^4}{x^{10}} = -2e^{x-1}\frac{x^5 - 5x^4}{x^{10}} = -2e^{x-1}\frac{x-5}{x^6}, \text{ so}$$

$$f'(1) = -2e^{1-1}\frac{1-5}{1^6} = \boxed{8}$$

The line tangent to the graph of $y = 5 - e^{2(x-1)}$ at the point $(1, 4)$ has its

x-intercept = **(12)** and its y-intercept =**(13)**

Solution: $y' = 0 - \left(e^{2(x-1)}\right)' = -e^{2(x-1)}\left(2(x-1)\right)' = -e^{2(x-1)}2 = -2e^{2(x-1)}$, so when $x = 1$, $y' = -2$.

The Point-Slope Form of the line with slope -2 passing through the point $(1, 4)$ is

$y - 4 = -2(x-1)$

Setting $y = 0$ in this equation, we get **(12)** $x = 3$, and Setting $x = 0$ in this equation we get **(13)** $y = 6$

Solve for x:

(14) $\log_x 16 = 2$

Solution: $\log_x 16 = \frac{\ln 16}{\ln x} = \frac{\ln 2^4}{\ln x} = \frac{4\ln 2}{\ln x}$, so we have

$\frac{4\ln 2}{\ln x} = 2$, or $4\ln 2 = 2\ln x$, so $\ln x = 2\ln 2 = \ln 4$. Therefore $x = 4$.

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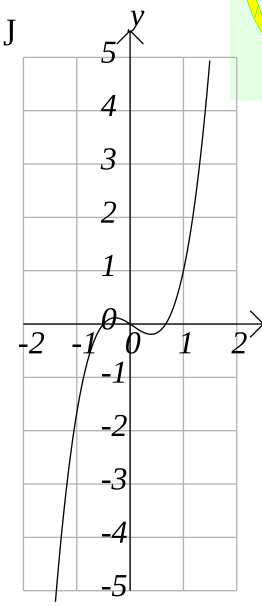
(15) $\log_{(x^2)} 16 = 2$

Solution: $\log_{(x^2)} 16 = \dfrac{\ln 16}{\ln x^2} = \dfrac{4\ln 2}{2\ln x} = 2\dfrac{\ln 2}{\ln x}$, so $2\dfrac{\ln 2}{\ln x} = 2$. Thus $\ln 2 = \ln x$, and $x = 2$.

PART II

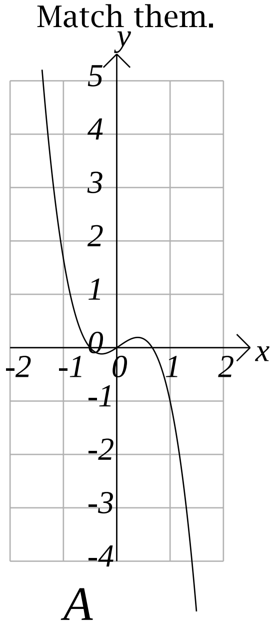
The possible answers to all questions in Part II are the letters A to J



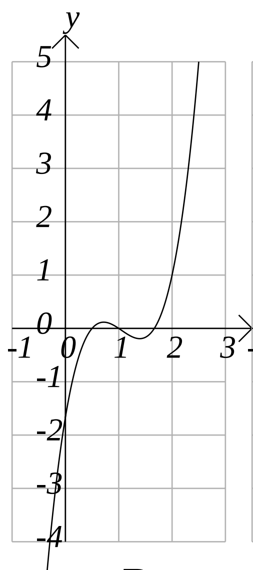
Part of the graph of
$y = f(x) = x(2x+1)(3x-2)/3$
is shown to the right.
Parts of the graphs of

(16) $y = f(x+1)$, D
(17) $y = f(x) + 1$, E
(18) $y = f(x-1)$, B
(19) $y = f(x) - 1$, C
(20) $y = 2f(x)$, J
(21) $y = -f(x)$, A
(22) $y = f(x)/2$, H
(23) $y = f(2x)$, I
(24) $y = -f(x) + 1$, F and
(25) $y = f(x/2)$, G

are shown below.
Match them.



A



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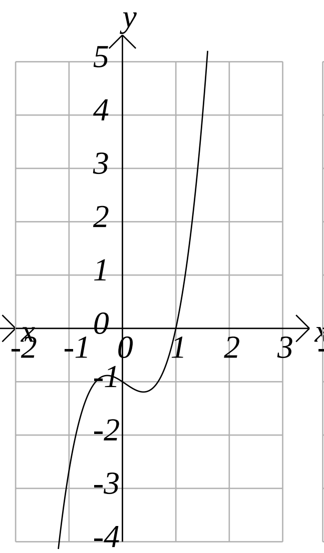

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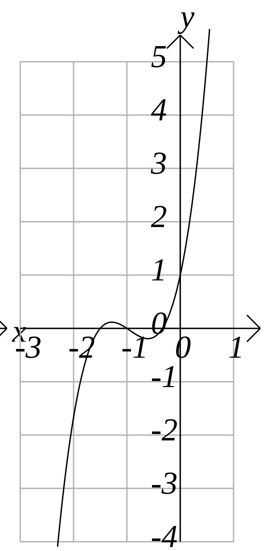

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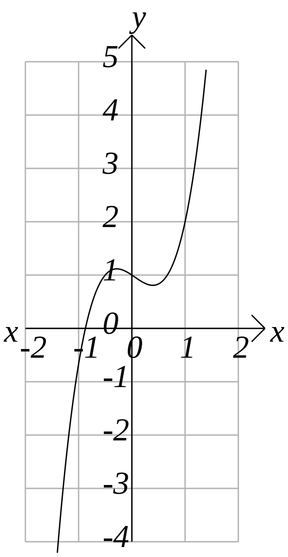

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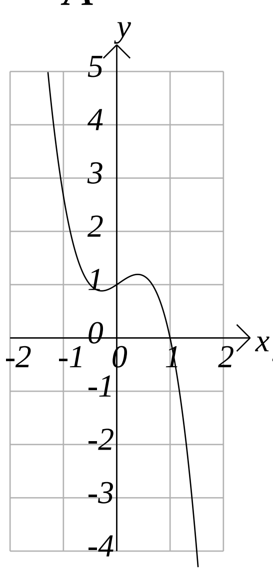

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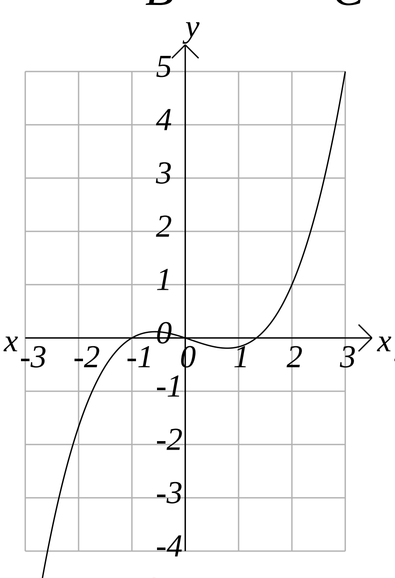

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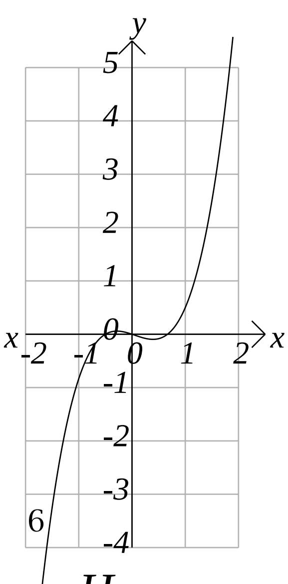

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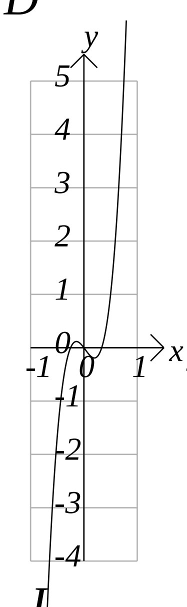
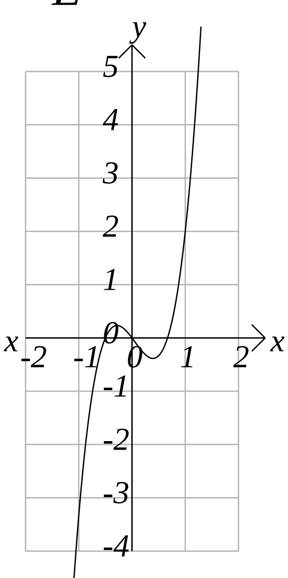


I



J