UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Quiz #2—Solutions

March 7, 2001 Time:50 minutes Instructor: *Doug MacLean*

CLOSED BOOK — NO CALCULATORS PERMITTED Each question is worth 4%

PART I

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

Evaluate the limits:

(1) $\lim_{x\to 2} \frac{x^3+2x-7}{x-1}$ **Solution:** $\frac{2^3+2(2)-7}{2-1} = \frac{8+7-7}{1} = \boxed{5}$

(2) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ if $f(x) = -\frac{16}{x}$ and $x = 2$

Solution: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-\frac{16}{x+h} - \left(-\frac{16}{x}\right)}{h} = 16 \lim_{h\to 0} \frac{-\frac{1}{x+h}+\frac{1}{x}}{h} =$

$$16 \lim_{h\to 0} \frac{\frac{-x+(x+h)}{x(x+h)}}{h} = 16 \lim_{h\to 0} \frac{\frac{h}{x(x+h)}}{h} = 16 \lim_{h\to 0} \frac{1}{x(x+h)} = \frac{16}{x^2} = \frac{16}{2^2} = \boxed{4}$$

Using the limit definition to compute the derivative of $f(x) = 2x^3 - 3x^2$, we arrive at $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[3x^2 - 2x + (ax - b)h + 2h^2]h}{h} = 3x^2 - 2x$,

where **(3)** $a =$ and **(4)** $b =$

Solution: $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[2(x+h)^3 - 3(x+h)^2] - [2x^3 - 3x^2]}{h} =$

$$\lim_{h\to 0} \frac{[2\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - 3\left(x^2 + 2xh + h^2\right)] - 2x^3 + 3x^2}{h} =$$

$$\lim_{h\to 0} \frac{[2x^3 + 6x^2h + 6xh^2 + 2h^3 - 3x^2 - 6xh - 3h^2] - 2x^3 + 3x^2}{h} =$$

$$\lim_{h\to 0} \frac{6x^2h + 6xh^2 + 2h^3 - 6xh - 3h^2}{h} =$$

$$\lim_{h\to 0} \frac{\left(6x^2 - 6x + (6x - 3)h + 2h^2\right)h}{h} =$$

so **(3)** $a = 6$ and **(4)** $b = 3$

Find $f'(1)$ if:

(5) If $f(x) = 6x^5 - 5x^4 - 2x^3 + 5x + \pi^4$

Solution: $f'(x) = 6\left(5x^{5-1}\right) - 5\left(4x^{4-1}\right) - 2\left(3x^{2-1}\right) + 5(1) + 0 = 30x^4 - 20x^3 - 6x + 5$, so $f'(1) = 30\left(1^4\right) - 20\left(1^3\right) - 6(1) + 5 = 30 - 20 - 6 + 5 =$ **9**

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(6) $f(x) = \frac{1}{324}(4x-1)^5$ **Solution:** $f'(x) = \frac{1}{324}5(4x-1)^{5-1}(4x-1)' =$

$\frac{1}{324}5(4x-1)^4(4) = \frac{20}{324}(4x-1)^4 = \frac{5}{81}(4x-1)^4 =$,

so $f'(1) = \frac{5}{81}(4(1)-1)^4 = \frac{5}{81}(3)^4 =$ **5**

(7) $f(x) = \frac{6}{5}\sqrt{5x^5+4}$ **Solution:** $f(x) = \frac{6}{5}\left(5x^5+4\right)^{\frac{1}{2}}$, so

$$f'(x) = \frac{6}{5}\frac{1}{2}\left(5x^5+4\right)^{\frac{1}{2}-1}\left(5x^5+4\right)' = \frac{3}{5}\left(5x^5+4\right)^{-\frac{1}{2}}\left(5\left(5x^{5-1}\right)+0\right) =$$

$$\frac{3}{5\left(5x^5+4\right)^{\frac{1}{2}}}\left(25x^4\right) = \frac{15x^4}{\left(5x^5+4\right)^{\frac{1}{2}}} =, \text{ so}$$

$$f'(1) = \frac{15(1)^4}{\left(5(1)^5+4\right)^{\frac{1}{2}}} = \frac{15}{(5+4)^{\frac{1}{2}}} = \frac{15}{\sqrt{9}} = \frac{15}{3} = \mathbf{5}$$

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(8) $f(x) = 72\frac{x+4}{x+5}$ **Solution:** $f'(x) = 72\frac{(x+5)(x+4)' - (x+4)(x+5)'}{(x+5)^2} =$ $72\frac{(x+5)(1)-(x+4)(1)}{(x+5)^2} = 72\frac{1}{(x+5)^2}$, so

$$f'(1) = 72\frac{1}{(1+5)^2} = 72\frac{1}{36} = \mathbf{2}$$

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(9) $f(x) = \frac{9}{128}(x^4+1)^4$ **Solution:** $f'(x) = \frac{9}{128}4(x^4+1)^{4-1}\left(x^4+1\right)' =$

$f'(x) = \frac{9}{32}(x^4+1)^3\left(4x^{4-1}+0\right) = \frac{9}{8}(x^4+1)^3x^3$, so $f'(1) = \frac{9}{8}((1)^4+1)^3(1)^3 = \frac{9}{8}(2)^3 =$ $\mathbf{9}$

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(10) $f(x) = -\frac{192(x+2)^5}{405(x+1)^5}$ **Solution:** $f(x) = -\frac{192}{405}\left(\frac{x+1}{x}\right)^5$, so

$$f'(x) = -\frac{192}{405}5\left(\frac{x+2}{x+1}\right)^{5-1}\left(\frac{x+2}{x+1}\right)' = -\frac{192}{81}\left(\frac{x+2}{x+1}\right)^4\left(\frac{(x+1)(x+2)'-(x+2)(x+1)'}{x^2}\right) =$$

$$-\frac{192}{81}\left(\frac{x+2}{x+1}\right)^4\left(\frac{(x+1)(1)-(x+2)(1)}{(x+1)^2}\right) = -\frac{192}{81}\left(\frac{x+2}{x+1}\right)^4\left(\frac{-1}{(x+1)^2}\right) = \frac{192}{81}\left(\frac{x+2}{x+1}\right)^4\left(\frac{1}{(x+1)^2}\right),$$

so

$$f'(1) = \frac{192}{81}\left(\frac{1+2}{1+1}\right)^4\left(\frac{1}{(1+1)^2}\right) = \frac{192}{81}\left(\frac{1+2}{1+1}\right)^4\left(\frac{1}{(1+1)^2}\right) = \frac{192}{81}\left(\frac{3^4}{2^4}\right)\left(\frac{1}{2^2}\right) =$$

$\mathbf{3}$

(11) $f(x) = \dfrac{280}{3}\sqrt[5]{x^3+x^2+x+29}$ **Solution:** $f(x) = \dfrac{280}{3}\left(x^3+x^2+x+29\right)^{\frac{1}{5}}$, so

$$f'(x) = \frac{280}{3}\frac{1}{5}\left(x^3+x^2+x+29\right)^{\frac{1}{5}-1}\left(x^3+x^2+x+29\right)' =$$

$$\frac{56}{3}\left(x^3+x^2+x+29\right)^{-\frac{4}{5}}\left(3x^2+2x+1\right) = \frac{56}{3\left(\sqrt[5]{x^3+x^2+x+29}\right)^4}\left(3x^2+2x+1\right),$$

so $f'(1) = \dfrac{56}{3\left(\sqrt[5]{(1)^3+(1)^2+1+29}\right)^4}\left(3(1)^2+2(1)+1\right) = \dfrac{56}{3\,(\sqrt[5]{32})^4}6 = \dfrac{56}{2^4}2 =$ **7**

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(12) The x-intercept of the line tangent to the graph $y = x^2 + 4$ at the point $(2, 8)$ is:

Solution: Since $y' = 2x$, the slope of the tangent line at $(2, 8)$ is 2(2)=4.

The Point-Slope equation of the tangent line(the line through $(2, 8)$ with slope 4) is thus $y - 8 = 4(x - 2)$. This line intersects the x-axis when $y = 0$, so we must have $0 - 8 = 4(x - 2)$ or $-2 = x - 2$, so $x =$ **0**

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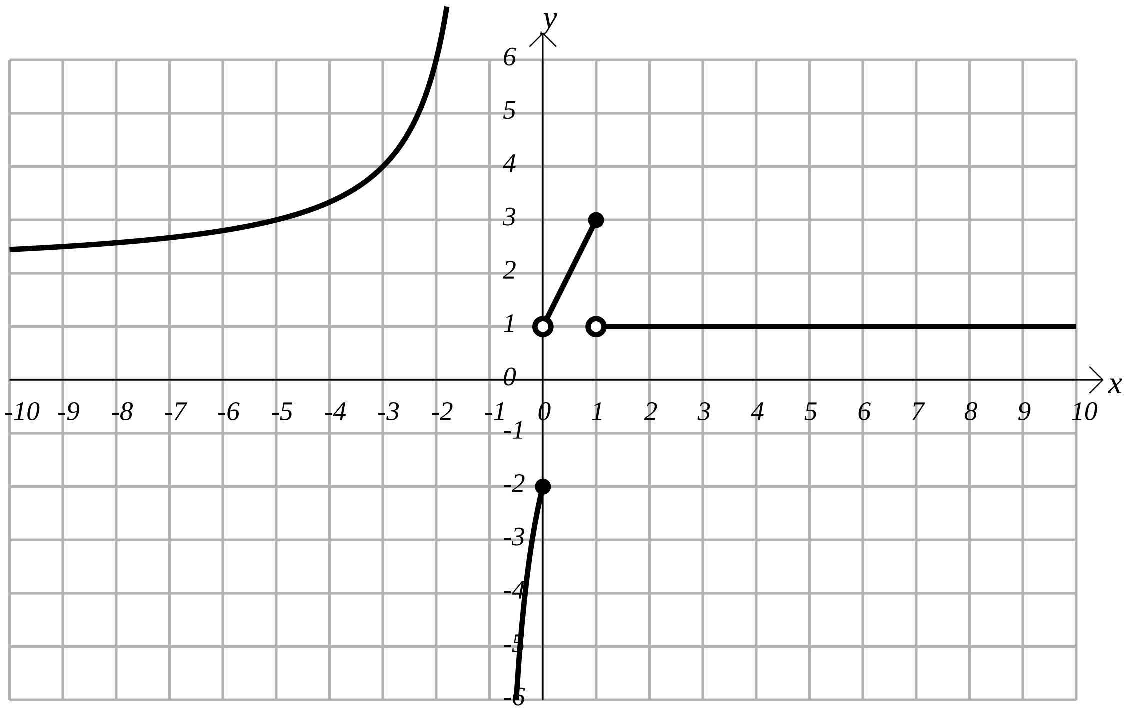
PART II

The possible answers to all questions in Part II are in the **ANSWER SET:**

(A) $-\infty$ (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (J) ∞



Part of the graph of $y = f(x) = \begin{cases} \dfrac{x-3}{x+1} + 1 & \text{if } x \le 0 \text{ and } x \ne -1 \\ 2x+1 & \text{if } 0 < x \le 1 \\ 1 & \text{if } 1 < x \end{cases}$ is shown above.

Find:

(13) $\lim_{x\to-\infty} f(x)$ **2(G)**

(14) $f(-5)$ **3(H)**

(15) $f(-3)$ **4(I)**

(16) $\lim_{x\to-1^-} f(x)$ **∞(J**

(17) $\lim_{x\to-1^+} f(x)$ **$-\infty$(A)**

(18) $\lim_{x\to0^-} f(x)$ **-2(C)**

(19) $f(0)$ **-2(C)**

(20) $\lim_{x\to0^+} f(x)$ **1(F)**

(21) $\lim_{x\to1^-} f(x)$ **3(H)**

(22) $f(1)$ **3(H)**

(23) $\lim_{x\to1^+} f(x)$ **1(F)**

(24) $f(2)$ **1(F)**

(25) $\lim_{x\to\infty} f(x)$ **1(F)**