UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Quiz #2—Solutions

November 8, 2000 Time:50 minutes Instructor: *Doug MacLean*

CLOSED BOOK — NO CALCULATORS PERMITTED Each question is worth 4%

PART I

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

Evaluate the limits:

(1) $\lim_{x\to 3} \dfrac{-84+19x-x^2}{x-7} = \lim_{x\to 3} \dfrac{(x-7)(12-x)}{x-7} = \lim_{x\to 3} 12-x = 12-3$ =9

(2) $\lim_{h\to 0} \dfrac{f(4+h)-f(4)}{h} = \lim_{h\to 0} \dfrac{4\sqrt{4+h}-4\sqrt{4}}{h} = 4\lim_{h\to 0} \dfrac{\sqrt{4+h}-\sqrt{4}}{h}\dfrac{\sqrt{4+h}+\sqrt{4}}{\sqrt{4+h}+\sqrt{4}} =$

$4\lim_{h\to 0} \dfrac{4+h-4}{h(\sqrt{4+h}+\sqrt{4})} = 4\lim_{h\to 0} \dfrac{h}{h(\sqrt{4+h}+\sqrt{4})} = 4\lim_{h\to 0} \dfrac{1}{\sqrt{4+h}+\sqrt{4}} = 4\dfrac{1}{\sqrt{4+0}+\sqrt{4}} =$

$4\dfrac{1}{2+2}$ =1 if $f(x) = 4\sqrt{x}$ and $x = 4$

Using the limit definition to compute the derivative of $f(x) = x^3 - x^2$, we arrive at $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{[(x+h)^3-(x+h)^2]-[x^3-x^2]}{h} =$

$\lim_{h\to 0} \dfrac{[(x^3+3x^2h+3xh^2+h^3)-(x^2+2xh+h^2)]-[x^3-x^2]}{h} =$

$\lim_{h\to 0} \dfrac{[3x^2h+3xh^2+h^3-2xh-h^2}{h} = \lim_{h\to 0} \dfrac{[3x^2-2x+(ax-b)h+h^2]h}{h} = 3x^2-2x,$

where **(3)** a =3 and **(4)** b =1

Find $f'(1)$ if:

(5) If $f(x) = 5x^6 - 6x^4 - 2x^3 + 5x + 9\ln(\pi^e)$

$f'(x) = 5(6x^5) - 6(4x^3) - 2(3x^2) + 5 + 0$, so $f'(1) = 5(6) - 6(4) - 2(3) + 5$ =5

(6) $f(x) = \frac{1}{256}(5x-1)^4$, $f'(x) = \frac{1}{256}4(5x-1)^3(5) = \frac{5}{64}(5x-1)^3$, so $f'(1) = \frac{5}{64}(5(1)-1)^3 = \frac{5}{64}(4)^3$ =5

(7) $f(x) = 7\sqrt{x^4+3} = 7(x^4+3)^{\frac{1}{2}}$, so $f'(x) = 7\frac{1}{2}(x^4+3)^{-\frac{1}{2}}(4x^3)$, and $f'(1) = \frac{7}{2}(1^4+3)^{-\frac{1}{2}}(4) = \frac{7}{2}(4)^{-\frac{1}{2}}(4)$ =7

(8) $f(x) = 48\frac{x+2}{x+3}$,

$$f'(x) = 48\frac{(x+3)(x+2)' - (x+2)(x+3)'}{(x+3)^2} = 48\frac{(x+3)(1) - (x+2)(1)}{(x+3)^2} = 48\frac{1}{(x+3)^2},$$

so $f'(1) = 48\frac{1}{(1+3)^2} = 48\frac{1}{16}$ =3

(9) $f(x) = \frac{1}{6}(x^2+1)^3$, $f'(x) = \frac{1}{6}3(x^2+1)^2(2x) = x(x^2+1)^2$, so

$f'(1) = 1(1^2+1)^2 = 2^2$ =4

(10) $f(x) = -\frac{3(x+1)^4}{32x^4} = -\frac{3}{32}\left(\frac{x+1}{x}\right)^4$

$$f'(x) = -\frac{3}{32}4\left(\frac{x+1}{x}\right)^3\left(\frac{x+1}{x}\right)' = -\frac{3}{8}\left(\frac{x+1}{x}\right)^3\left(\frac{x(x+1)' - (x+1)x'}{x^2}\right)' =$$

$$-\frac{3}{8}\left(\frac{x+1}{x}\right)^3\left(\frac{x(1) - (x+1)(1)}{x^2}\right)' = -\frac{3}{8}\left(\frac{x+1}{x}\right)^3\left(\frac{-1}{x^2}\right)' = \frac{3}{8}\frac{(x+1)^3}{x^5},$$ so

$f'(1) = \frac{3}{8}\frac{(1+1)^3}{1^5} = \frac{3}{8}\frac{2^3}{1} = \frac{3}{8}8$ =3

(11) $f(x) = \ln\left(2\left(e^{\pi}\right)^3\right) + 3x^2$

$f'(x) = 0 + 6x = 6x$, so $f'(1)$ =6

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(12) The y-intercept of the line tangent to the graph $y = \frac{2}{x}$ at the point $(1, 2)$ is: 4

$y' = -\frac{2}{x^2}$, so the slope of the tangent line is $m = -\frac{2}{1^2} = -2$. The Point-Slope equation is $y - 2 = -2(x-1)$. Setting $x = 0$ in this equation we get that its y-intercept is 4 .

Solve for x:

(13) $\log_e e^x = 4$

$\log_e e^x = x \log_e e = x(1) = x$, so $\boxed{x = 4}$

(14) $\log_5 x^2 + \log_5 x^3 = 5$

$\log_5 x^2 + \log_5 x^3 = \log_5 x^2 x^3 = \log_5 x^5 = 5 \log_5 x$, so $\log_5 x = 1$. Thus $\boxed{x = 5}$

(15) $\log_{16} x^{\frac{7}{2}} - \log_{16} \sqrt{x^3} = \log_{16} 81$

$\log_{16} x^{\frac{7}{2}} - \log_{16} x^{\frac{3}{2}} = \log_{16} \frac{x^{\frac{7}{2}}}{x^{\frac{3}{2}}} = \log_{16} x^{\frac{7}{2} - \frac{3}{2}} = \log_{16} x^2$, so $x^2 = 81$, and thus $\boxed{x = 9}$

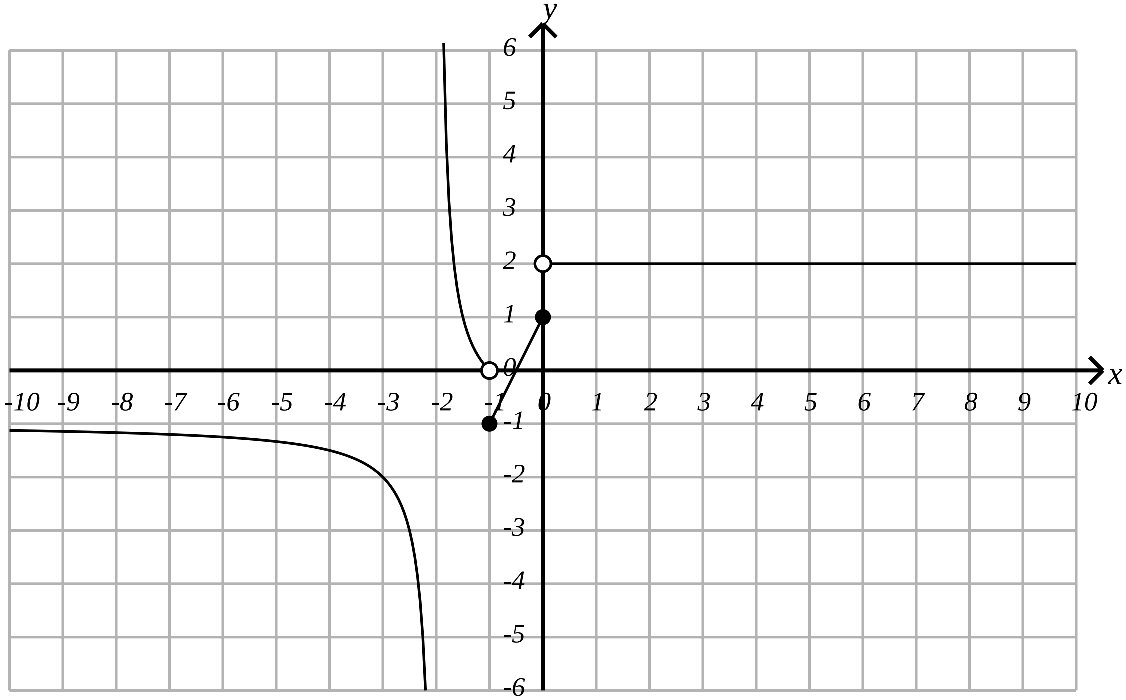
(16) $\log_4 4^{4x} = 16$

$\log_4 4^{4x} = 4x \log_4 4 = 4x(1) = 4x$, so $4x = 16$ and thus $\boxed{x = 4}$

PART II

The possible answers to all questions in Part II are in the **ANSWER SET:**

(A) $-\infty$ (B) -4 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 4 (I) 8 (J) ∞



Part of the graph of $y = f(x) = \begin{cases} \dfrac{x-3}{2-x} & \text{if } x \leq 1 \text{ and } x \neq -2 \\ 2x+1 & \text{if } -1 \leq x \leq 0 \\ 2 & \text{if } 0 < x \end{cases}$ is shown above. Find:

(17) $\lim_{x\to-\infty} f(x)$ $= -1$ **(18)** $\lim_{x\to-2^-} f(x)$ $=-\infty$ **(19)** $\lim_{x\to-2^+} f(x)$ $= \infty$ **(20)** $\lim_{x\to-1^-} f(x)$ $=0$

(21) $\lim_{x\to-1^+} f(x)$ =-1 **(22)** $\lim_{x\to0^-} f(x)$ =1 **(23)** $\lim_{x\to0^+} f(x)$ =2 **(24)** $\lim_{x\to\infty} f(x)$ =2

(25) $f(0)$ =1