UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Quiz #1— Solutions

January 31, 2001 Time:50 minutes Instructor: *Doug M*

CLOSED BOOK — NO CALCULATORS PERMITTED
Each question is worth 4%
The possible answers to all questions are the digits in the **ANSWER SET:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7

If $\frac{8}{9} - \frac{7}{8}$ is written in its simplest form as $\frac{a}{10b+c}$, where a, b, and c are digits, then

Solution: $\frac{8}{9} - \frac{7}{8} = \frac{8(8) - 7(9)}{9(8)} = \frac{1}{72} = \frac{1}{10(7)+2}$

(1) $a =$ 1 **(2)** $b =$ 7 **(3)** $c =$

$x^2 + 10x + 32$ is to be written in the form $(x+a)^2 + b$ by completing squares. We must have:

Solution: $x^2 + 10x + 32 = x^2 + 2\frac{10}{2}x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + 32 =$

$x^2 + 2(5)x + 5^2 - 5^2 + 32 = (x+5)^2 + 7$

(4) $a =$ 5 **(5)** $b =$ 7

$9x^2 + 126x + 405$ is to be written in the form $a[(x+b)^2 - c]$ by completing squares. We must have:

Solution: $9x^2 + 126x + 405 = 9[x^2 + 14x + 45] = 9\left[x^2 + 2\frac{14}{2}x + 7^2 - 7^2 + 45\right] = 9\left[(x+7)^2 - 49\right.$

(6) $a =$ 9 **(7)** $b =$ 7 **(8)** $c =$

If $a = \left[(x-1)^{\frac{4}{36}}\right] - (x-1)^{\frac{1}{3}} + 6$, then

Solution: $a = \left[(x-1)^{\frac{1}{3}}\right] - (x-1)^{\frac{1}{3}} + 6 = 6$

(9) $a =$ 6

If $x = 9$ and $h = 1$, then

(10) $\left(\frac{x+h}{x-h}\right)^{-\frac{2}{3}} \cdot \frac{x+h}{\sqrt[3]{x^2-h^2}} \cdot \frac{\sqrt[3]{(x-h)(x^2+xh+h^2)}}{(x^3-h^3)^{\frac{1}{3}}} =$

Solution: $\left(\frac{x-h}{x+h}\right)^{\frac{2}{3}} \cdot \frac{x+h}{\sqrt[3]{(x-h)(x+h)}} \cdot \frac{\sqrt[3]{x^3-h^3}}{(x^3-h^3)^{\frac{1}{3}}} =$

$$\frac{(x-h)^{\frac{2}{3}}}{(x+h)^{\frac{2}{3}}} \cdot \frac{x+h}{(x-h)^{\frac{1}{3}}(x+h)^{\frac{1}{3}}} = (x-h)^{\frac{1}{3}} = (9-1)^{\frac{1}{3}} = 8^{\frac{1}{3}} =$$ 2

If $a = 9$, then **(11)** $\frac{\left(\frac{4a^3}{9}\right)^{\frac{1}{2}} - \frac{2}{3}\sqrt{a}}{a-1} =$

Solution: $\frac{\frac{2a^{\frac{3}{2}}}{3} - \frac{2}{3}a^{\frac{1}{2}}}{a-1} = \frac{2}{3}a^{\frac{1}{2}}\frac{a-1}{a-1} = \frac{2}{3}9^{\frac{1}{2}} = \frac{2}{3}3 =$ 2

The roots of $5x^2 + 7x + 2 = 0$ in their simplest form are $-A$ and $-\frac{B}{C}$. We must have:

Solution: Using the Quadratic Formula, the roots are:

$$\frac{-7 \pm \sqrt{7^2 - 4(5)2}}{2(5)} = \frac{-7 \pm \sqrt{49-40}}{10} = \frac{-7 \pm 3}{10} = \frac{-10}{10}, \frac{-4}{10} = -1, -\frac{2}{5}.$$

(12) $A =$ 1 **(13)** $B =$ 2 **(14)** $C =$

The polynomial $p(x) = 6x^4 + x^3 - 25x^2 - 4x + 4$ can be factored in the form $(x-a)(x+b)(cx+1)(dx-1)$, where a, b, c, and d are digits. Their values are:

Solution: The only integer values of the roots are $\pm 1, \pm 2$. We have $p(1) = -18$, $p(-1) = -12$, $p(2) =$ p
is a factor of $p(x)$.

After long division, we have $\dfrac{p(x)}{(x-2)(x+2)} = 6x^2 + x - 1 = (3x-1)(2x+1)$

(15) $a =$ 2 **(16)** $b =$ 2 **(17)** $c =$ 2 **(18**

$\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ can be simplified to the expression $a - \sqrt{10b + c}$, where a, b, and c are digits. Their values are:

Solution: $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{(\sqrt{7}-\sqrt{5})^2}{\sqrt{7}^2-\sqrt{5}^2} = \dfrac{\sqrt{7}^2 - 2\sqrt{7}\sqrt{5} + \sqrt{5}^2}{7-5} =$

$\dfrac{7 - 2\sqrt{35} + 5}{2} = \dfrac{12 - 2\sqrt{35}}{2} = 6 - \sqrt{35} =$ $6 - \sqrt{10(3) + 5}$

(19) $a =$ 6 **(20)** $b =$ 3 **(21)** $c =$

The inequality $10 - 7x < 4$ has solution of the form $\left(\dfrac{a}{b}, \infty\right)$, where a and b are positive digits.
The values of a and b are:

Solution: $10 - 7x < 4 \iff 6 < 7x \iff \dfrac{6}{7} < x$

(22) $a =$ 6 **(23)** $b =$ 7

If we solve the inequalities $-2 \leq \dfrac{x-5}{2} \leq 2$, the solution is an interval $[a, b]$, where a and b are positive digits

Solution: $-2 \le \frac{x-5}{2} \le 2 \iff -4 \le x-5 \le 4 \iff 1 \le x \le 9$

(24) $a =$ 1 **(25)** $b =$ 9