UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Quiz #1—Solutions

October 4, 2000 Time:50 minutes Instructor: *Doug MacLean*

CLOSED BOOK — NO CALCULATORS PERMITTED
Each question is worth 4%
The possible answers to all questions are the digits in the **ANSWER SET:**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

If $\frac{5}{7} - \frac{3}{5}$ is written in its simplest form as $\frac{a}{10b+c}$, where a, b, and c are digits, then

Solution:

$$\frac{5}{7} - \frac{3}{5} = \frac{25-21}{35} = \frac{4}{35} = \frac{4}{3 \cdot 10 + 5}$$

(1) $a =$ 4 **(2)** $b =$ 3 **(3)** $c =$ 5

$x^2 + 16x + 70$ is to be written in the form $(x+a)^2 + b$ by completing squares. We must have:

Solution: $x^2 + 16x + 70 = x^2 + 2\frac{16}{2}x + 70 = x^2 + 2(8)x + (8^2 - 8^2) + 70 = (x^2 + 2(8)x + 8^2) - 64 + 70 = (x+8)^2 + 6$

(4) $a =$ 8 **(5)** $b =$ 6

$5x^2 + 30x + 40$ is to be written in the form $a[(x+b)^2 - c]$ by completing squares. We must have:

Solution: $5x^2 + 30x + 40 = 5[x^2 + 6x + 8] = 5[x^2 + 2(3)x + 3^2 - 3^2 + 8] = 5[(x+3)^2 - 9 + 8] = 5[(x+3)^2 - 1]$

(6) $a =$ 5 **(7)** $b =$ 3 **(8)** $c =$ 1

If $a = \left[(x+1)^{\frac{1}{6}}\right]^3 - (x+1)^{\frac{1}{2}}$, then

Solution: **(9)** $a = (x+1)^{\frac{3}{6}} - (x+1)^{\frac{1}{2}} = \boxed{0}$

If $x = 9$ and $h = 1$, then **Solution:**

(10) $\left(\frac{x+h}{x-h}\right)^{-\frac{2}{3}} \cdot \frac{x+h}{\sqrt[3]{x^2-h^2}} = \left(\frac{x-h}{x+h}\right)^{\frac{2}{3}} \cdot \frac{x+h}{\sqrt[3]{(x-h)(x+h)}} = \frac{(x-h)^{\frac{2}{3}}}{(x+h)^{\frac{2}{3}}} \cdot \frac{x+h}{(x-h)^{\frac{1}{3}}(x+h)^{\frac{1}{3}}} =$

$(x-h)^{\frac{1}{3}} = (9-1)^{\frac{1}{3}} = (8)^{\frac{1}{3}} = \boxed{2}$

If $a = 4$, then **Solution:** **(11)**

$$3\frac{\left(\frac{4a^3}{9}\right)^{\frac{1}{2}} - \frac{2}{3}\sqrt{a}}{a-1} = 3\frac{\left(\frac{4(4)^3}{9}\right)^{\frac{1}{2}} - \frac{2}{3}\sqrt{4}}{4-1} = 3\frac{\left(\frac{4^4}{3^2}\right)^{\frac{1}{2}} - \frac{2}{3}2}{3} = \frac{4^2}{3} - \frac{2}{3}2 = \frac{16-4}{3} = \boxed{4}$$

The roots of $4x^2 - 2x - 1 = 0$ in their simplest form are $\frac{A \pm \sqrt{B}}{C}$. We must have:

Solution: $\frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-1)}}{2(4)} = \frac{2 \pm \sqrt{20}}{8} = \frac{2 \pm 2\sqrt{5}}{8} = \frac{1 \pm \sqrt{5}}{4}$

(12) $A = \boxed{1}$ **(13)** $B = \boxed{5}$ **(14)** $C = \boxed{4}$

The polynomial $p(x) = 6x^4 + x^3 - 7x^2 - x + 1$ can be factored in the form $(x-a)(x+b)(cx+1)(dx-1)$, where a, b, c, and d are digits. Their values are:

Solution: The only possible rational roots are $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$. Testing the integer values first, we get $p(1) = p(-1) = 0$, so $p(x)$ is divisible by $x+1$ and $x-1$. Long division gives $p(x) = (x-1)(x+1)(6x^2+x-1)$. Using the quadratic formula, we find that the roots of $6x^2 + x - 1$ are

$\frac{-1 \pm \sqrt{1^2 - 4(6)(-1)}}{2(6)} = \frac{-1 \pm \sqrt{25}}{12} = \frac{-1 \pm 5}{12} = \frac{-1-5}{12}, \frac{-1+5}{12} = -\frac{1}{2}, \frac{1}{3}$. Thus

(15) $a = \boxed{1}$ **(16)** $b = \boxed{1}$ **(17)** $c = \boxed{2}$ **(18)** $d = \boxed{3}$

$\frac{2\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ can be simplified to the expression $a - b\sqrt{c}$, where a, b, and c are digits. Their values are:

Solution: $\left(\frac{2\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \frac{(2\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{3-2} =$

$(2\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2}) = (2\sqrt{3}-\sqrt{2})\sqrt{3}-(2\sqrt{3}-\sqrt{2})\sqrt{2} = 2\sqrt{3}\sqrt{3}-\sqrt{2}\sqrt{3}-2\sqrt{3}\sqrt{2}+\sqrt{2}\sqrt{2} =$

$2(3) - \sqrt{6} - 2\sqrt{6} + 2 = 8 - 3\sqrt{6}$

(19) $a =$ 8 **(20)** $b =$ 3 **(21)** $c =$ 6

The inequality $8 - 5x < 2$ has solution of the form $\left(\frac{a}{b}, \infty\right)$, where a and b are positive digits.
The values of a and b are:

Solution: $8 - 5x < 2 \iff 6 < 5x \iff \frac{6}{5} < x$

(22) $a =$ 6 **(23)** $b =$ 5

If we solve the inequalities $-2 \le \frac{3-x}{2} \le 2$, the solution is an interval:
$[-a, b]$, where a and b are positive digits.
The values of a and b are:

Solution: $-2 \le \frac{3-x}{2} \le 2 \iff -4 \le 3 - x \le 4 \iff -7 \le -x \le 1 \iff -1 \le x \le 7$

(24) $a =$ 1 **(25)** $b =$ 7
