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Optimization-3

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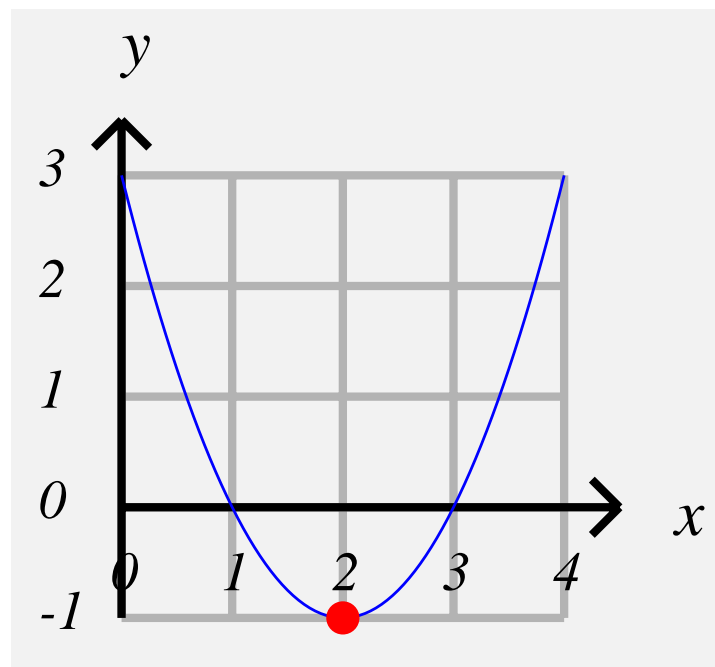
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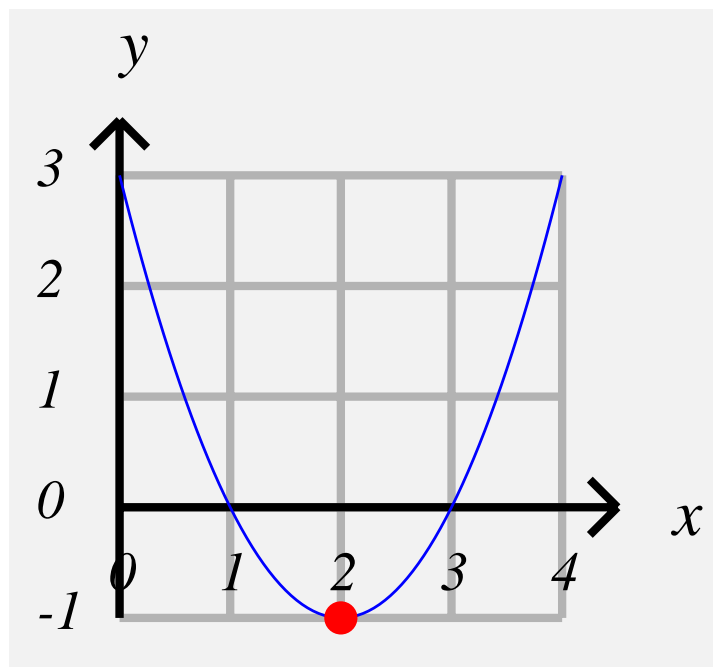
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There are no other possible extrema.



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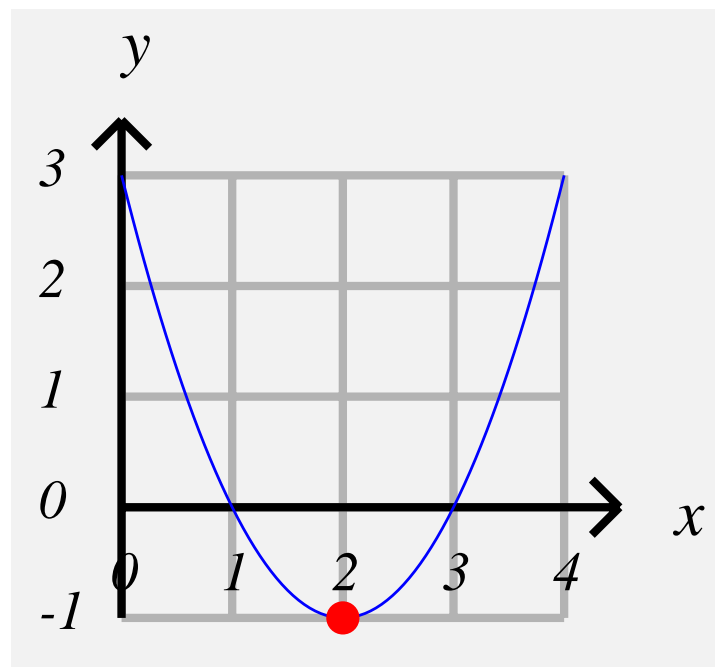
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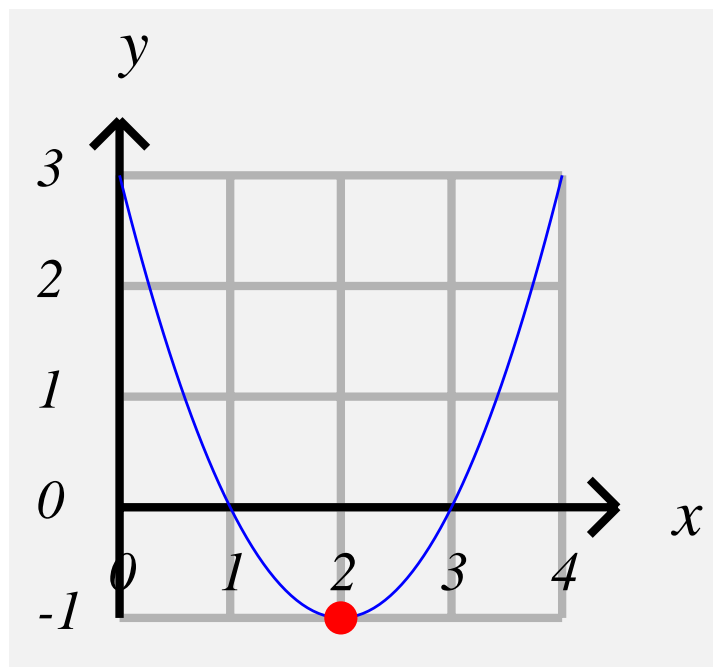
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In our previous example, we conclude that the relative minimum is also the absolute minimum.

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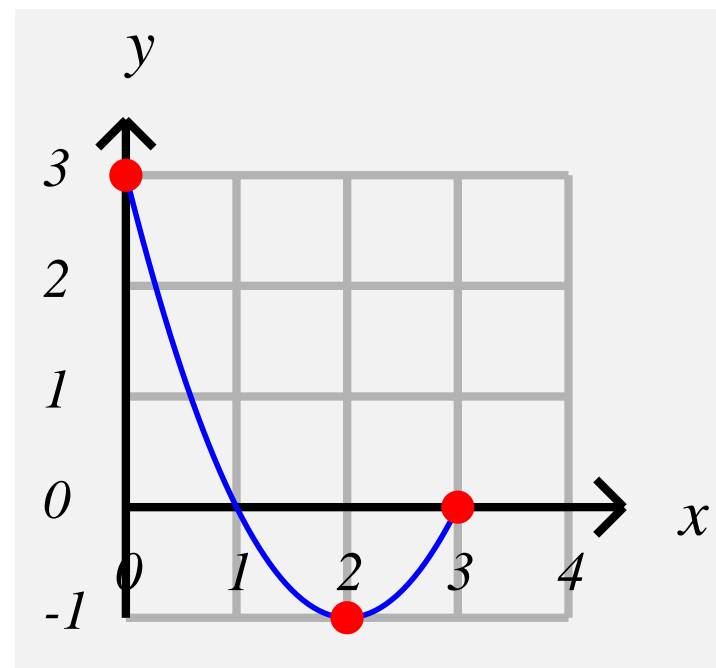
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If we restrict the domain to the interval $[0, 3]$ then the absolute minimum is the same, but the the Extreme Value Theorem tells us that there must be an absolute maximum. This can only occur at the endpoints 0 and 3 of the interval, since there are no other critical numbers.

Optimization-9

We have $f(0) = 3$ and $f(3) = 3^3 - 4(3) + 3 = 0$, so the absolute maximum is 3 at $x = 0$.



Problem 4.7:12. A rectangular storage container with an open top is to have a volume of 10 m^3 . The length of its base is twice the width. Material for the base costs \$10 per m^2 . Material for the sides costs \$6 per m^2 . Find the cost of materials for the cheapest such container.

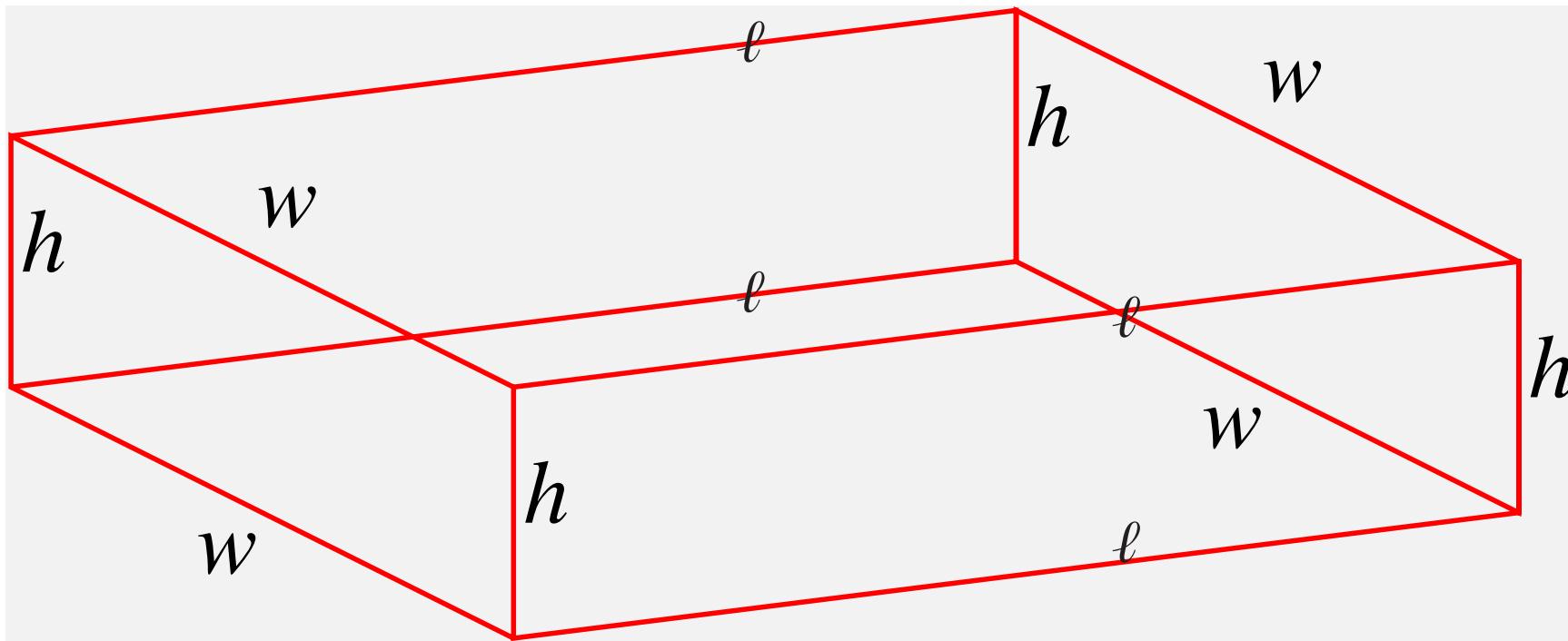
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Solution:

Sketch:



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$$C(w) = C_s + C_b = 180w^{-1} + 20w^2,$$

which is to be minimized.

Optimization-12

Solution of Minimization Problem:

Differentiating with respect to w :

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$$w = \sqrt[3]{\frac{9}{2}}.$$

Optimization-13

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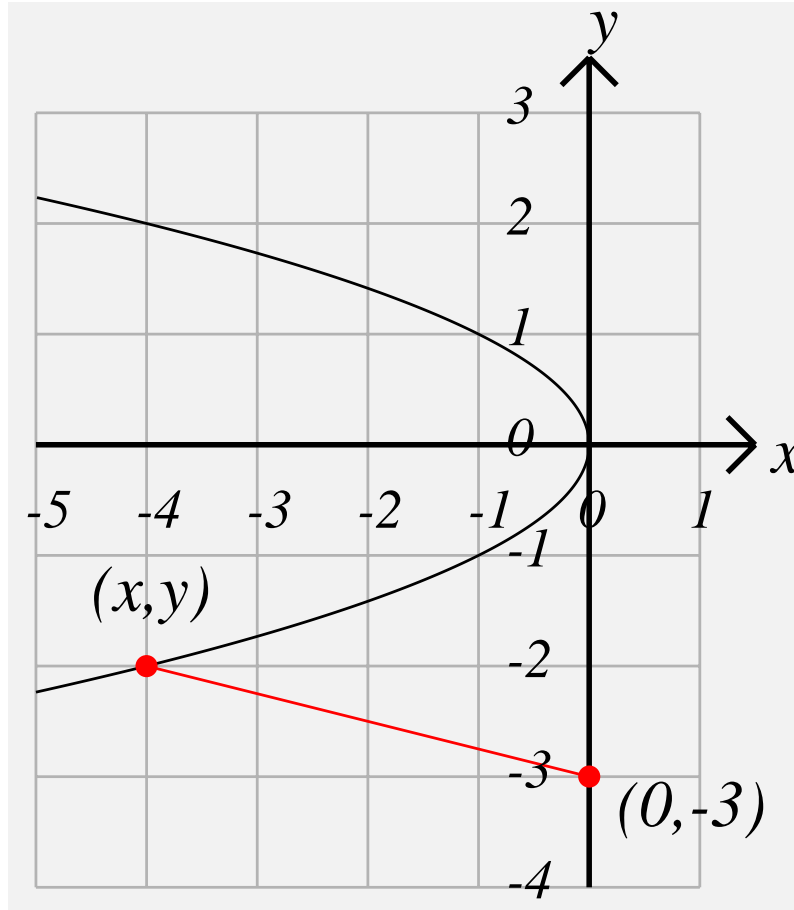
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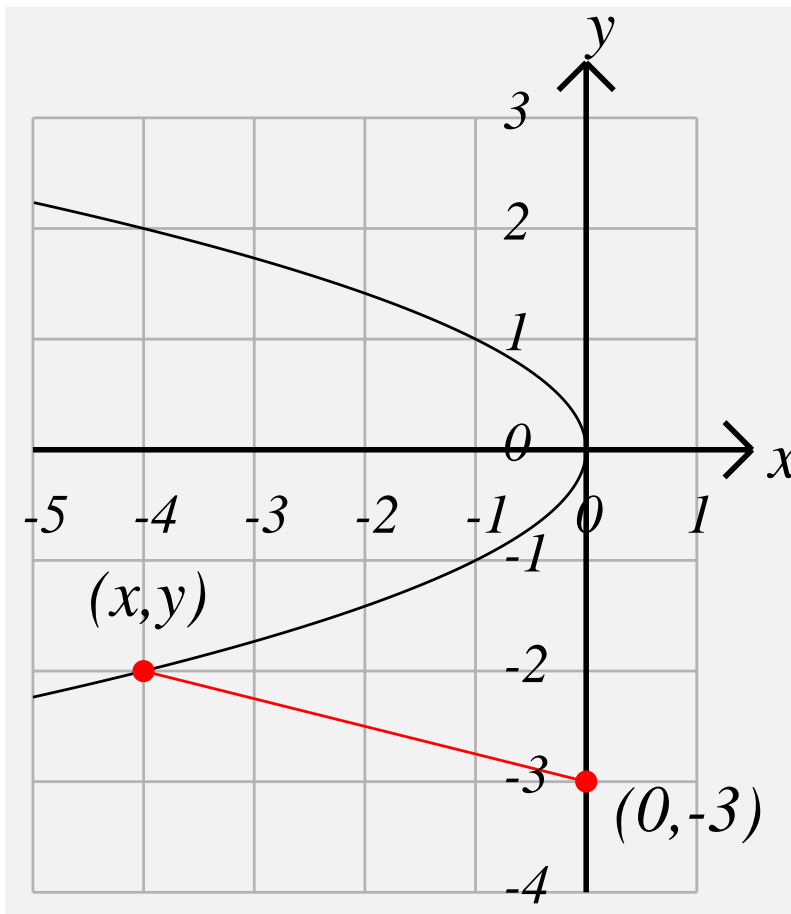
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Problem 4.7:18. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

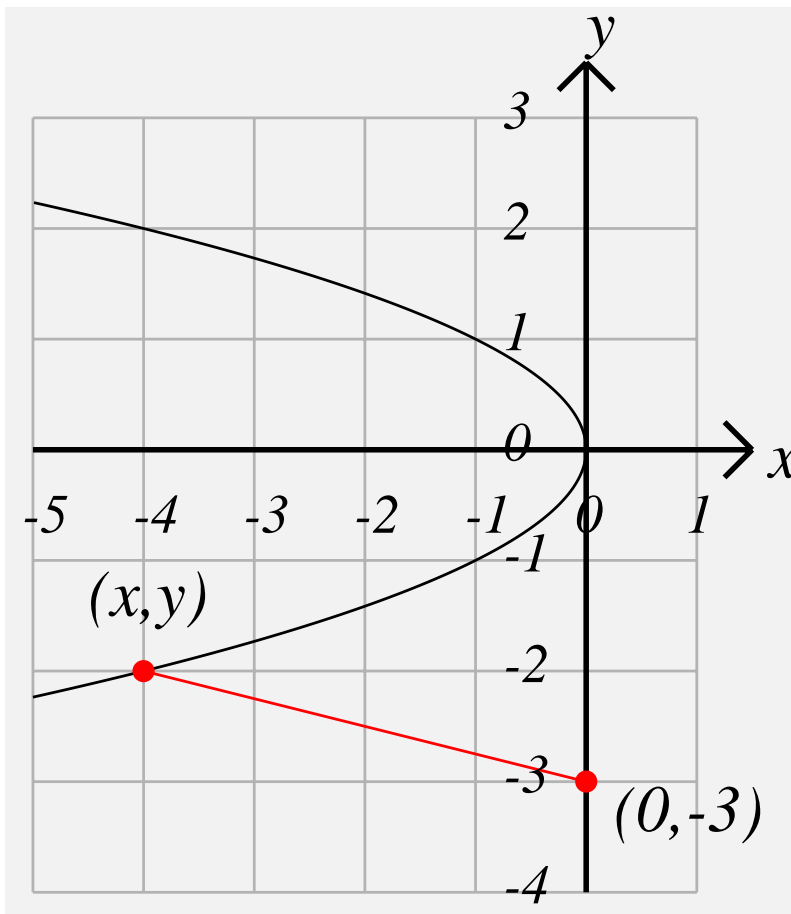


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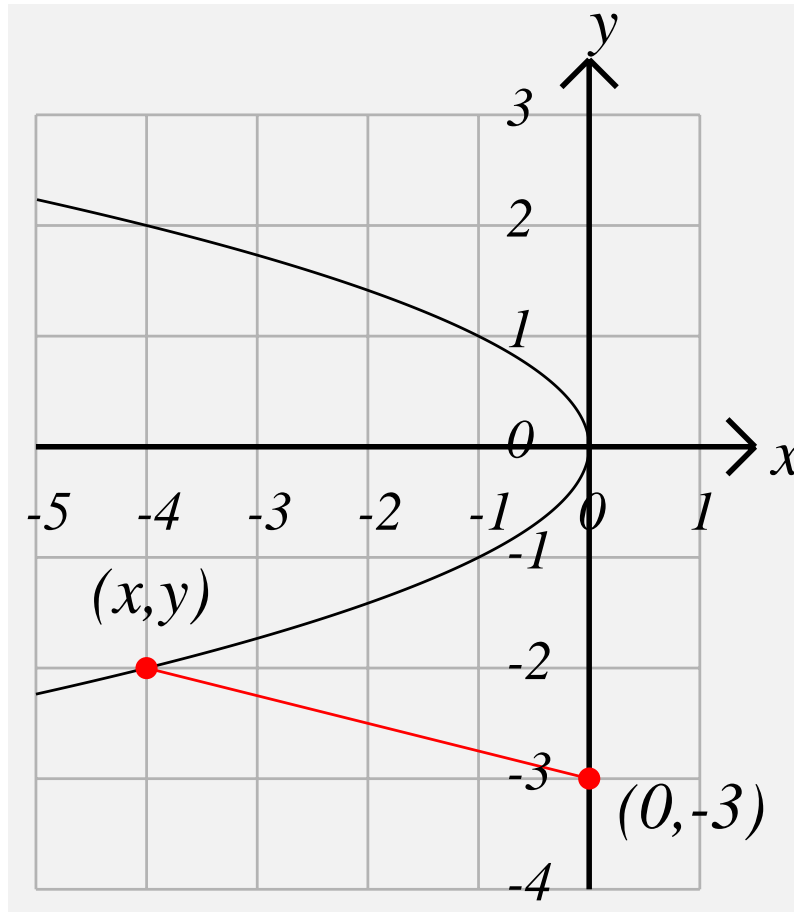
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Solution:

Variables: x , y , and d , the distance to a point

Optimization-15

$(x, y) = (-y^2, y)$ on the curve $x = -y^2$.

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$$\text{Since } 4y^2 - 4y + 6 = 4\left(y^2 - y + \frac{3}{2}\right) = 4\left[\left(y - \frac{1}{2}\right)^2 + \frac{5}{4}\right] > 0,$$

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Since $4y^2 - 4y + 6 = 4(y^2 - y + \frac{3}{2}) = 4[(y - \frac{1}{2})^2 + \frac{5}{4}] > 0$, we

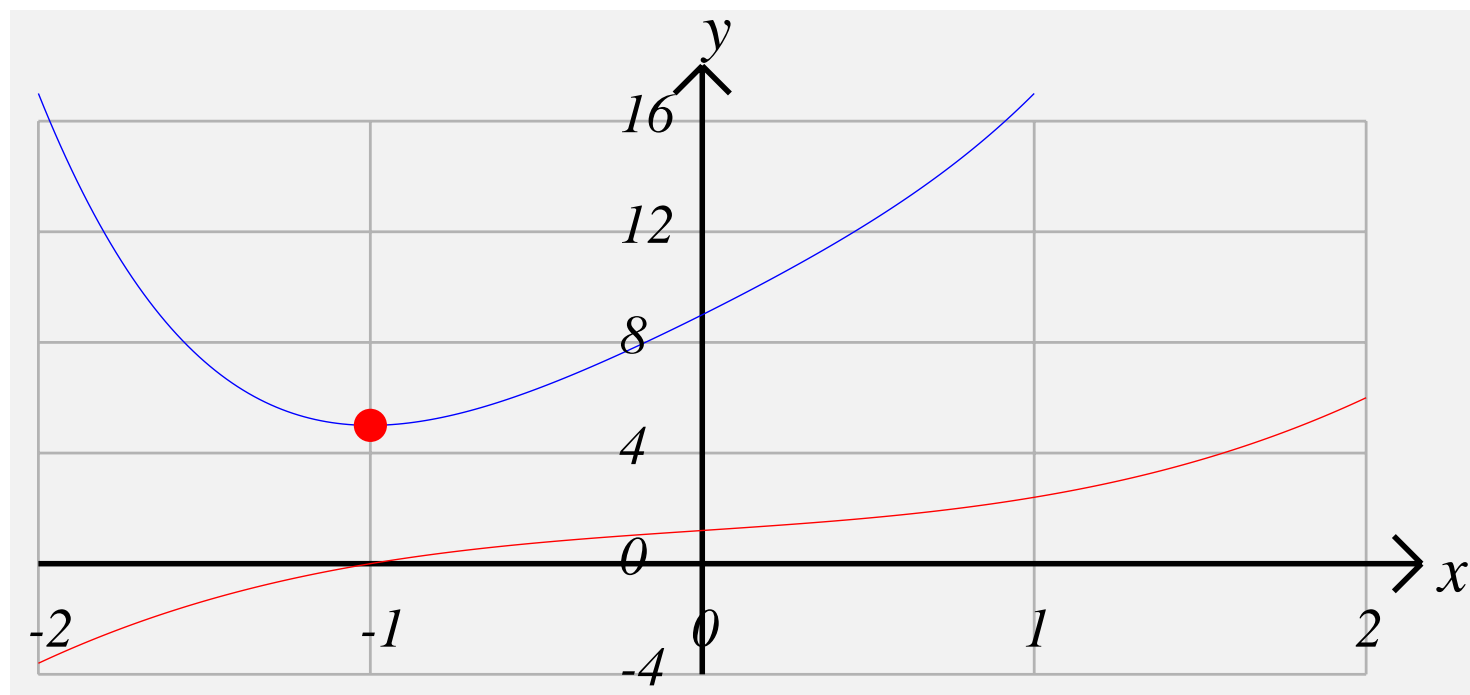
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Optimization-17



The graph of g is blue, and that of g' is red.