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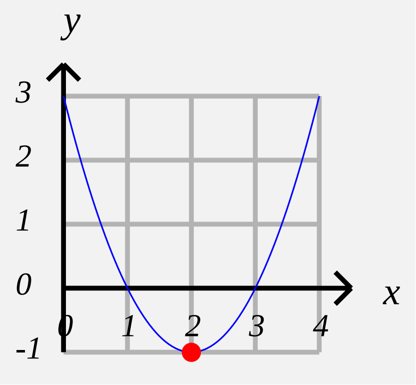

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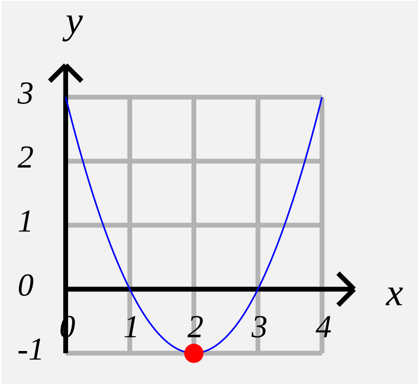

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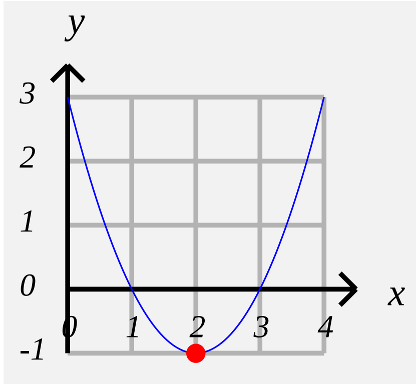

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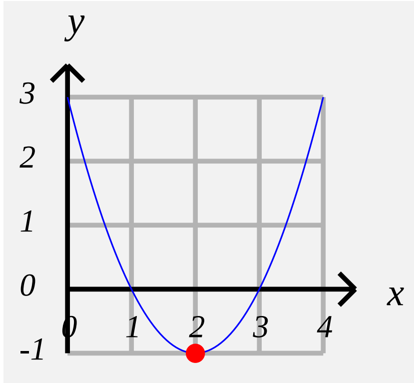

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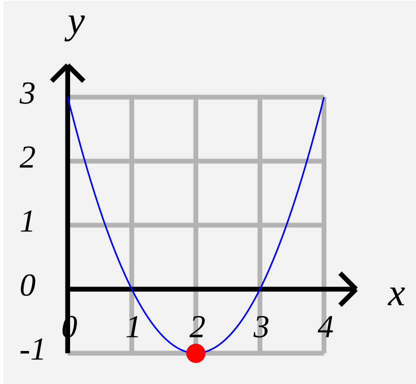

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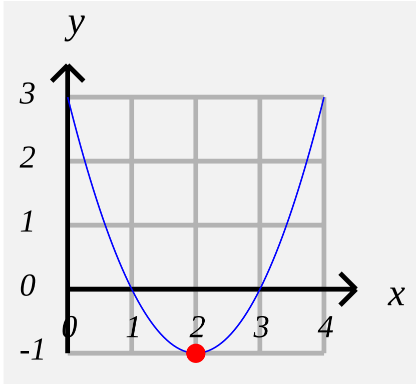

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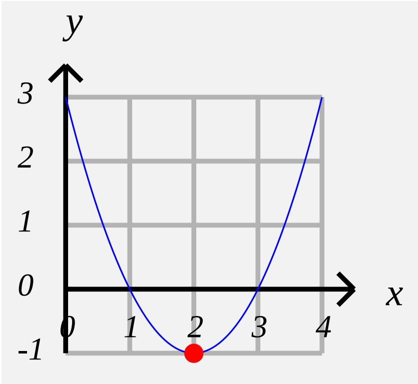

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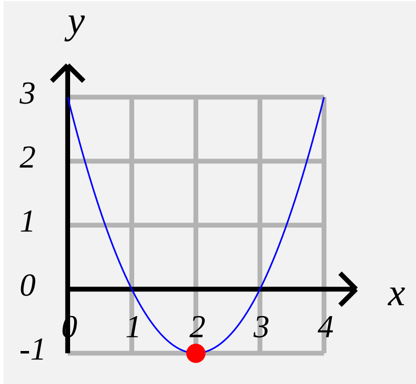

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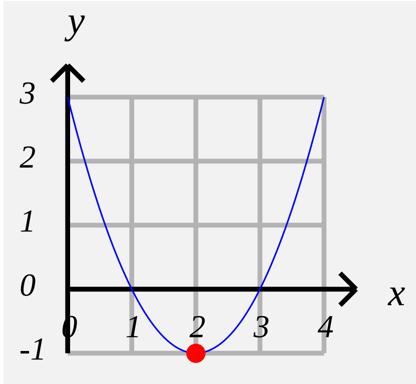

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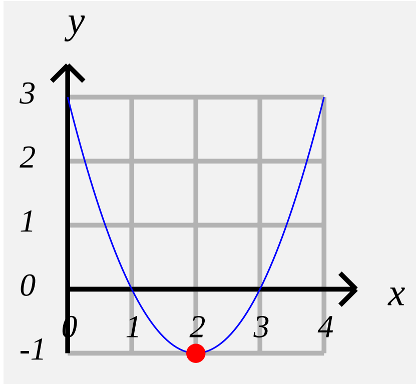

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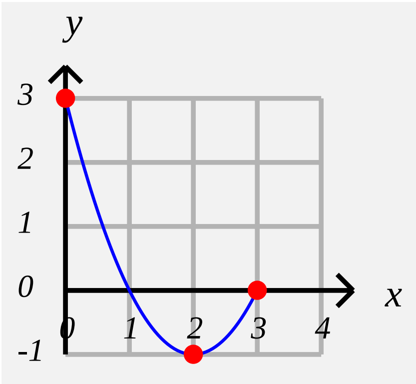
We have $f(0) = 3$ and $f(3) = 3^3 - 4(3) + 3 = 0$, so the absolute maximum is 3 at $x = 0$.



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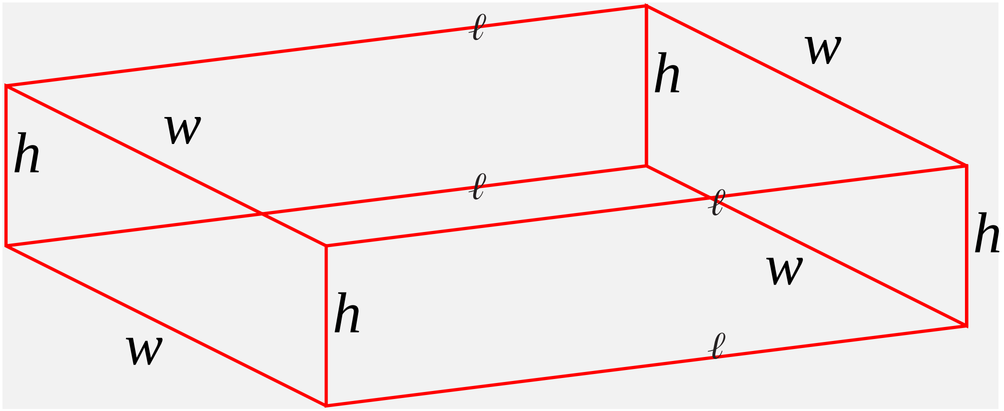

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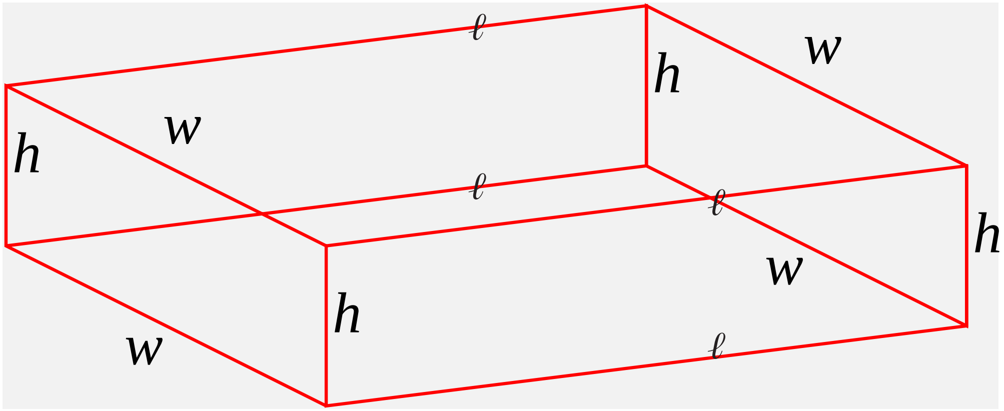

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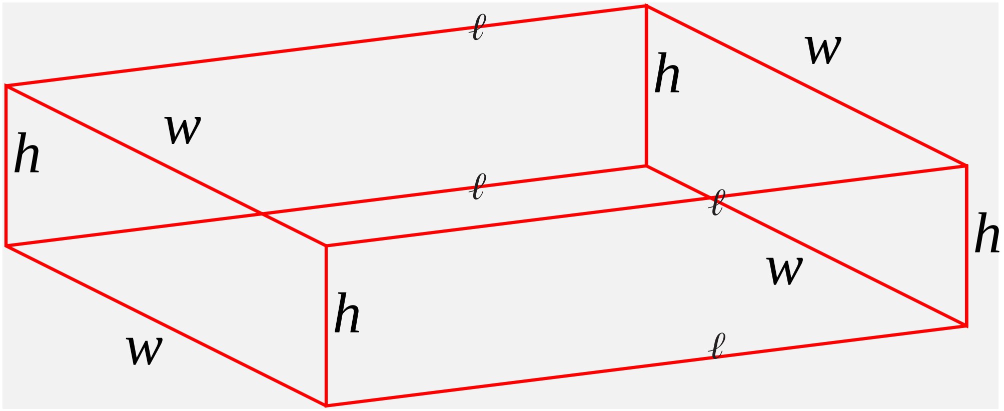

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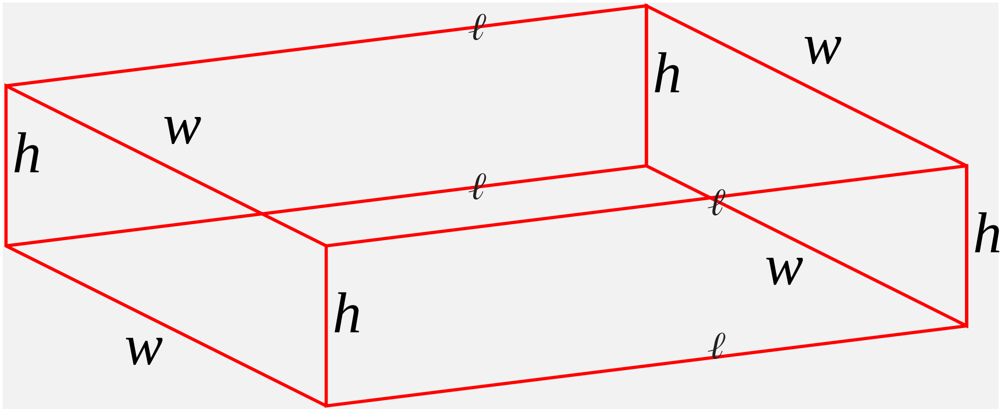

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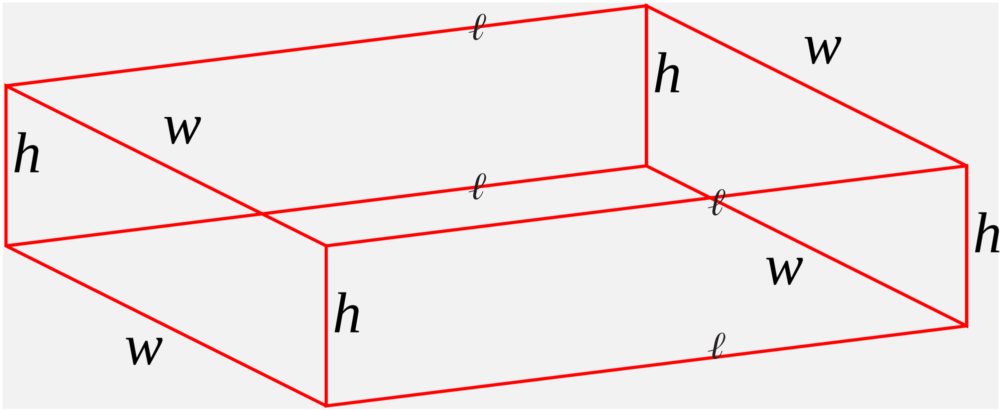

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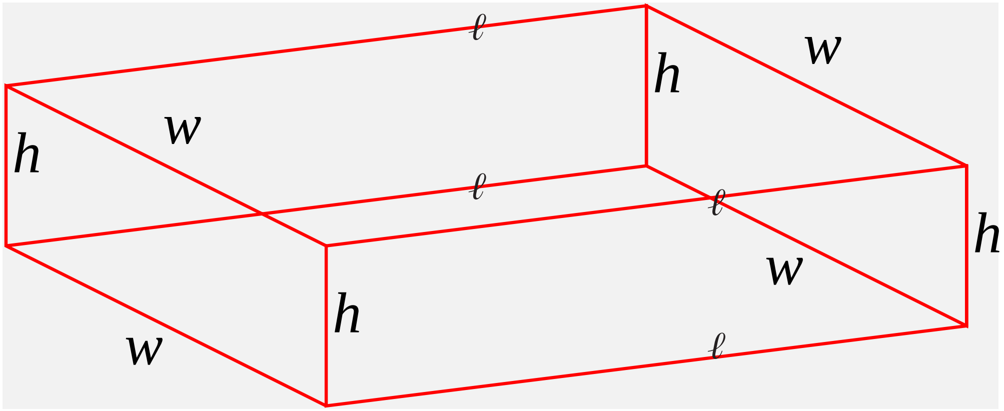

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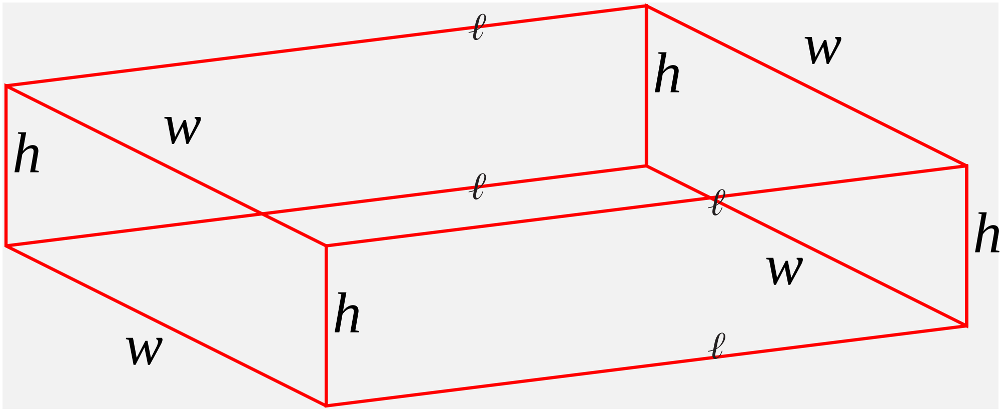

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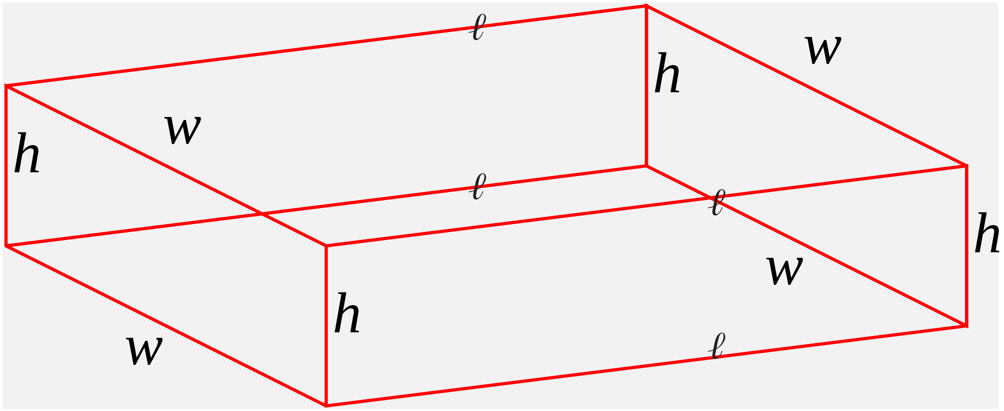

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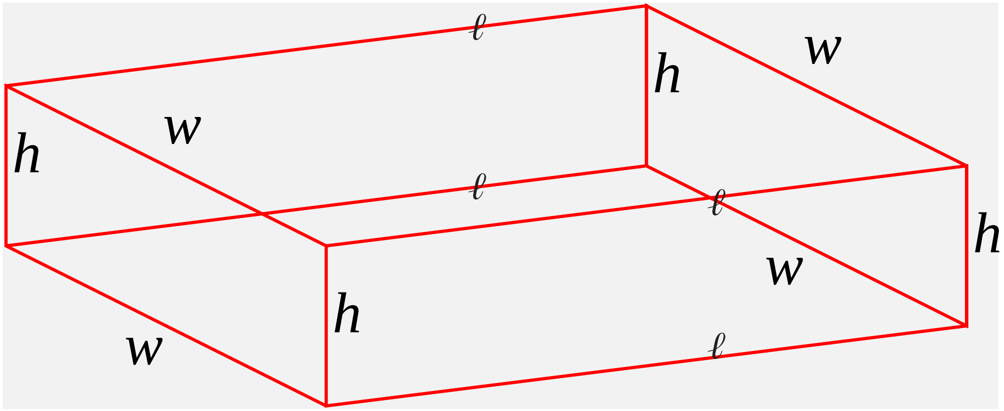

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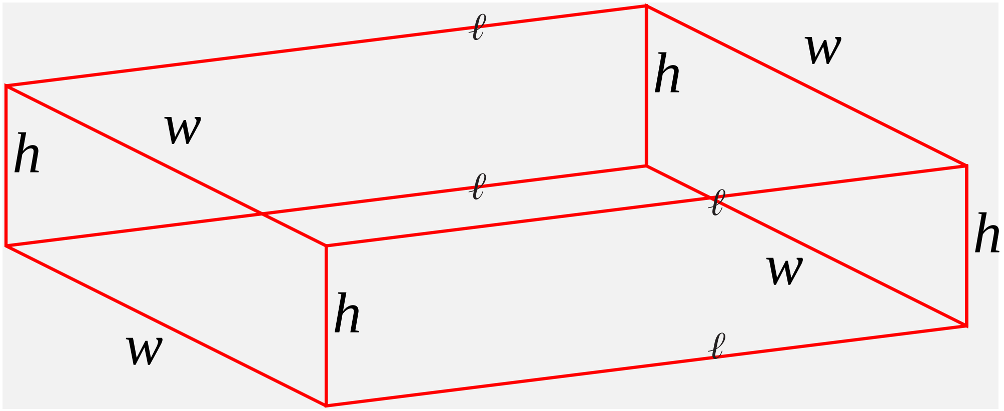

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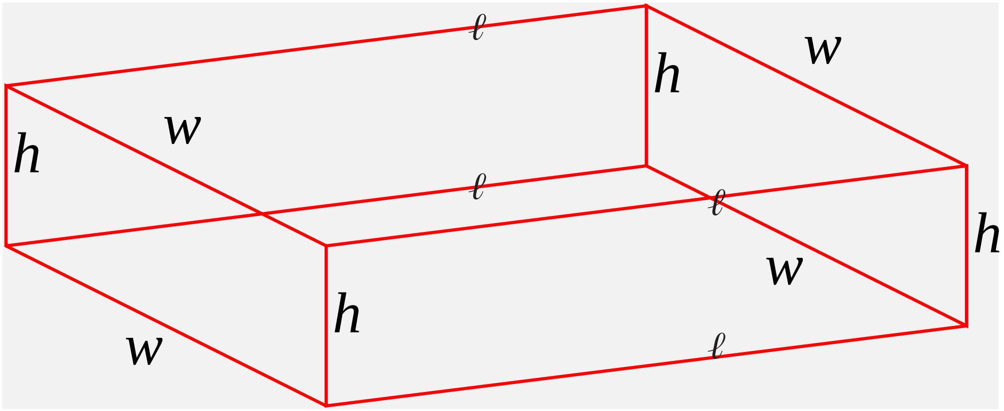

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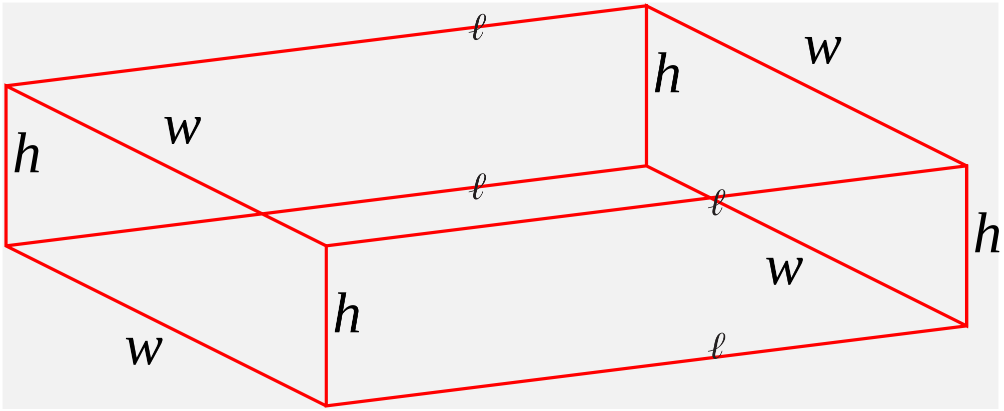

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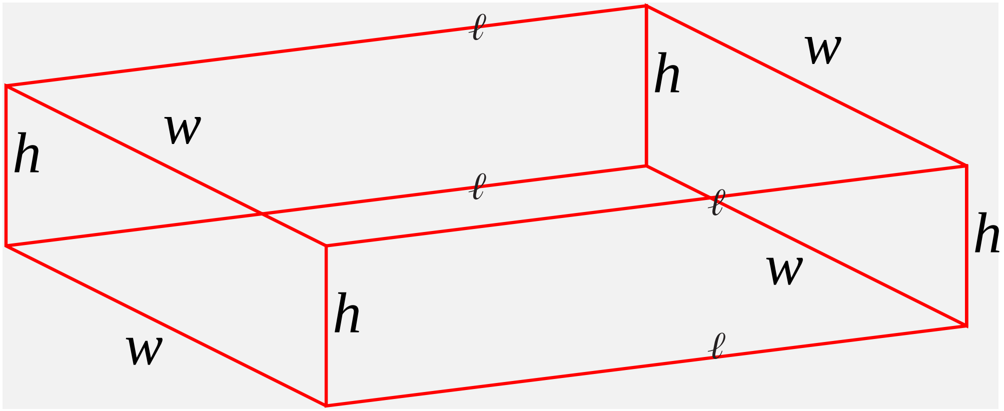

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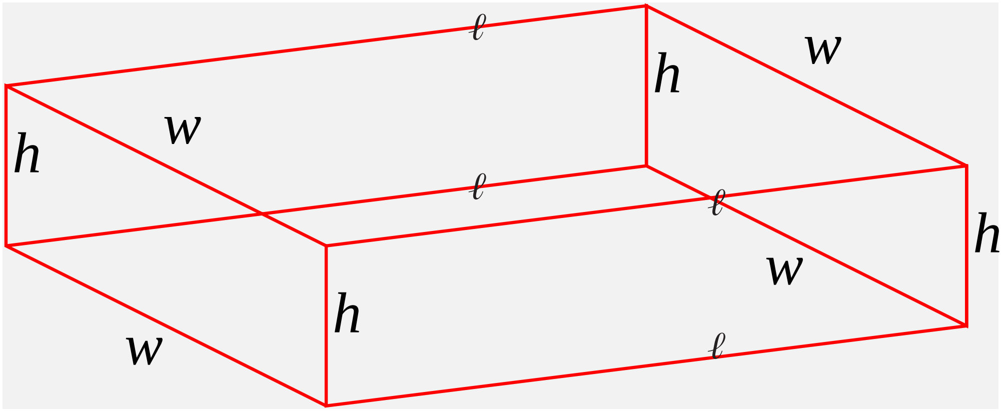

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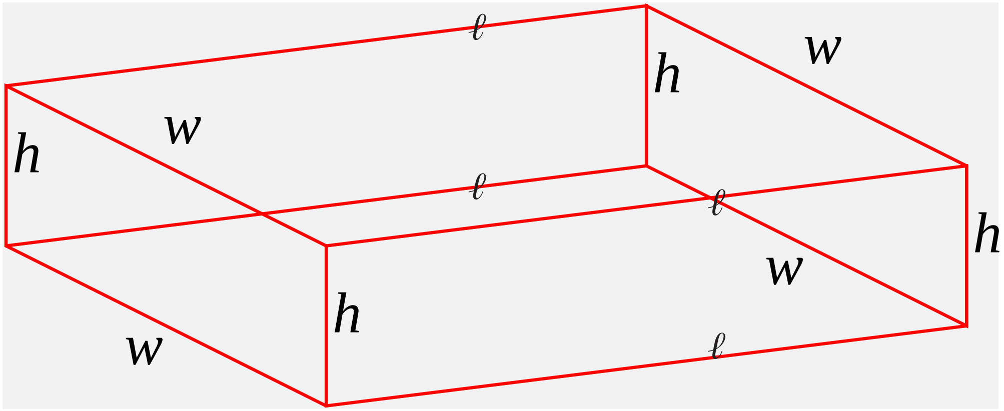

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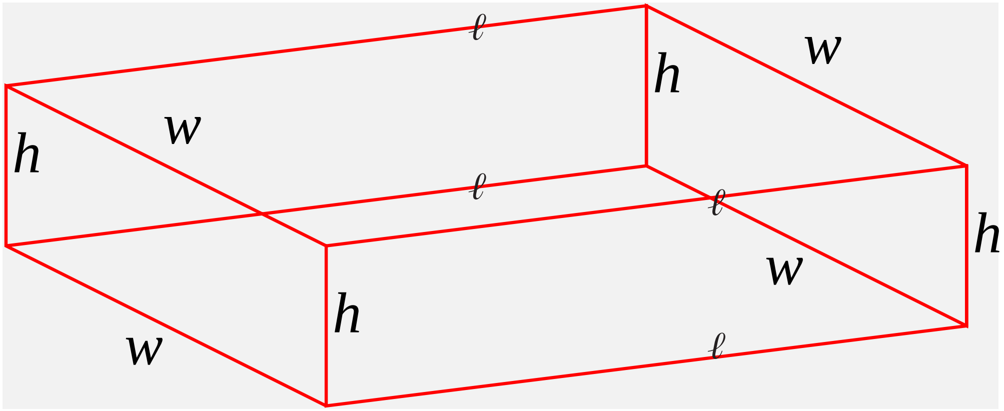

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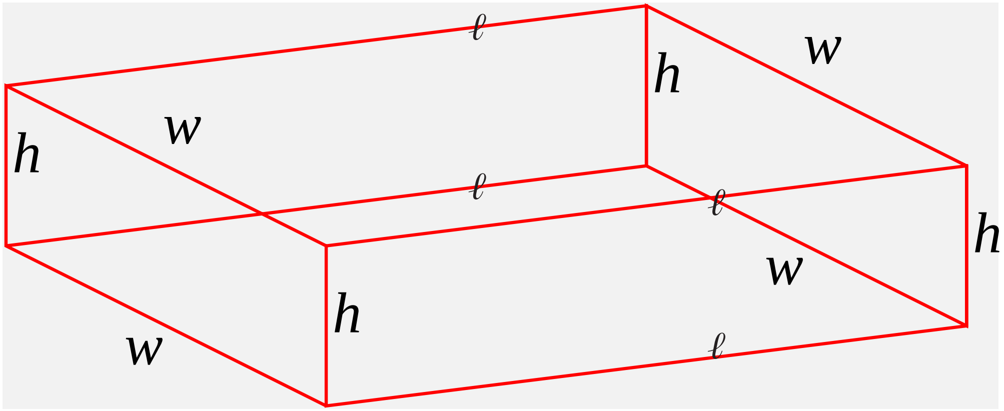

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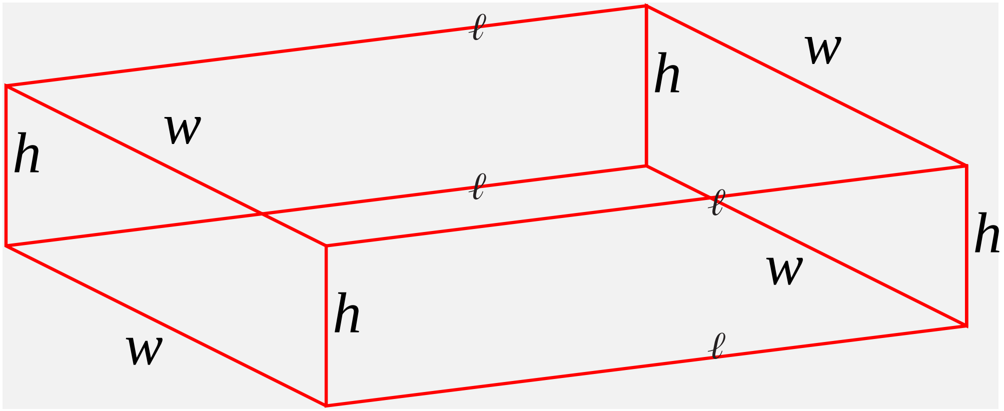

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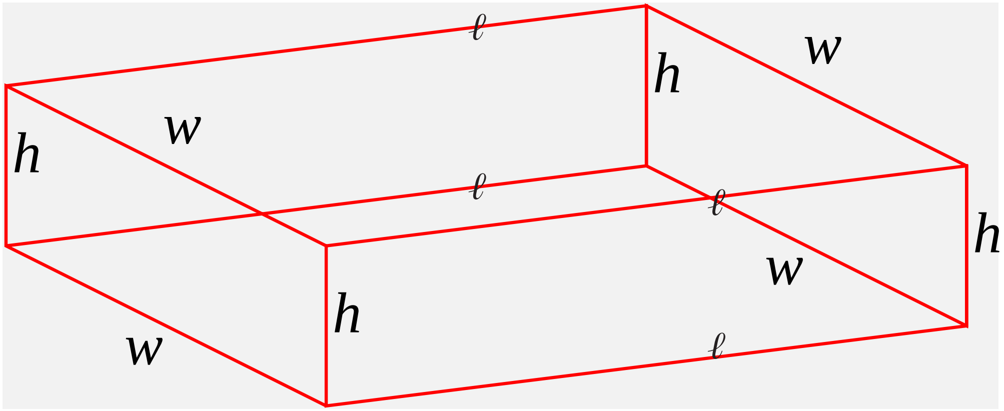

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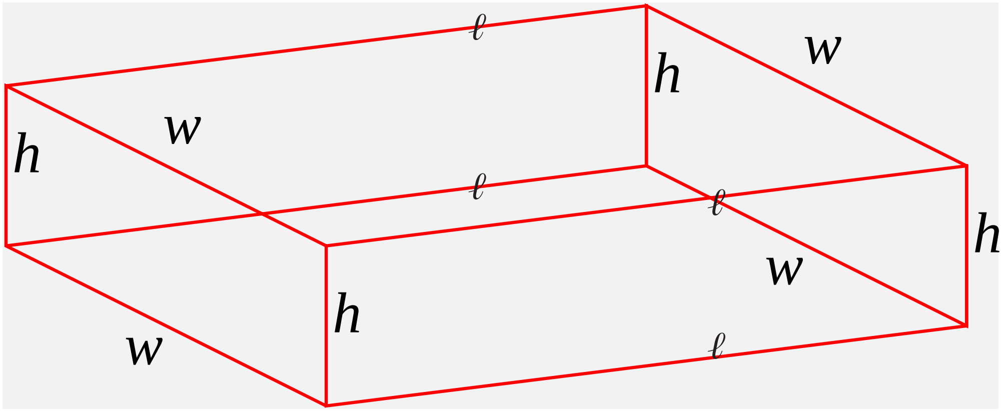

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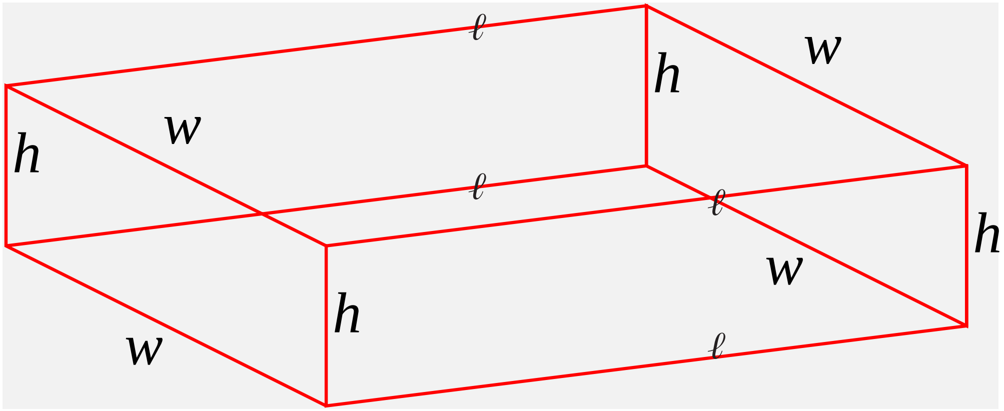

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C_b = cost of base,

C = Total Cost.

Relations:

$\ell = 2w$,

$V = \ell w h = 2w^2 h = 10m^3$,so

$h = 5w^{-2}$

$A_s = wh + \ell h + \ell h + wh = h(2w + 2\ell) = h(6w) = 5w^{-2}6w = 30w^{-1}$

$C_s = 6A_s = 6(30w^{-1}) = 180w^{-1}$

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Problem 4.7:12. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per m^2. Material for the sides costs \$6 per m^2. Find the cost of materials for the cheapest such container.

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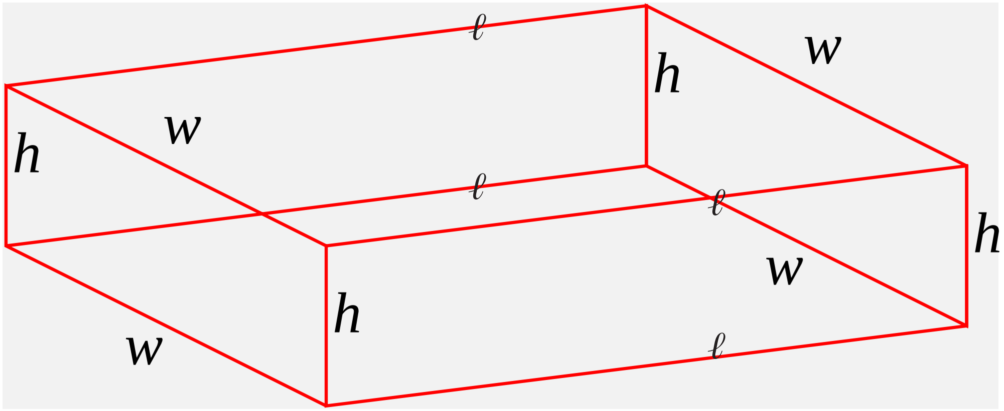

Variables:

ℓ = length,
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h = height,
V = volume,
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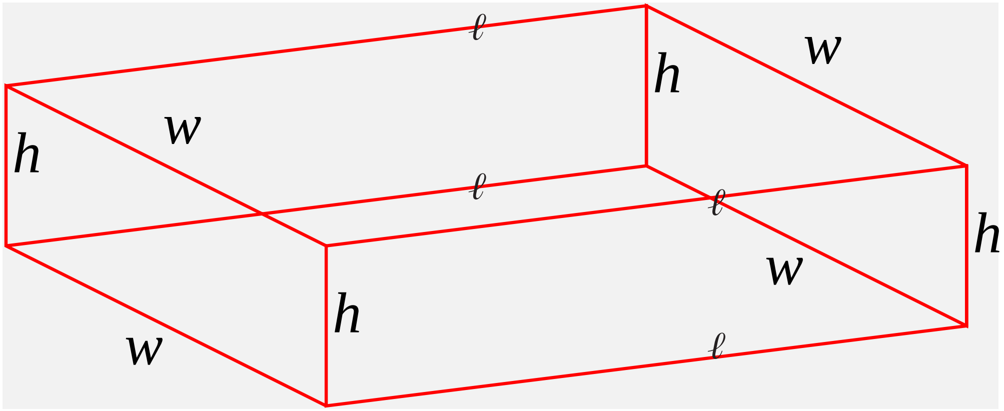

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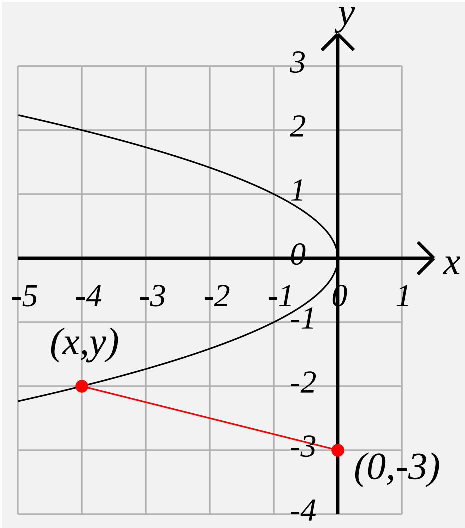
Problem 4.7:18. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.



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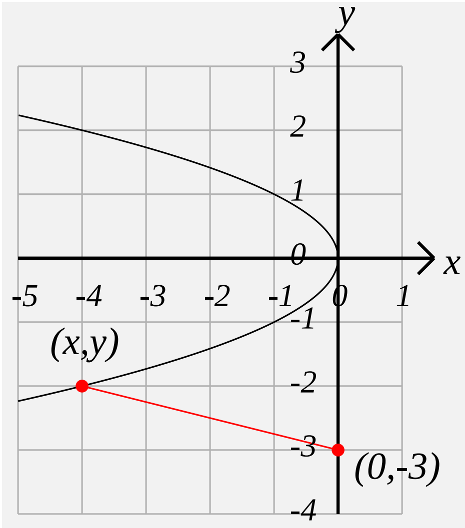

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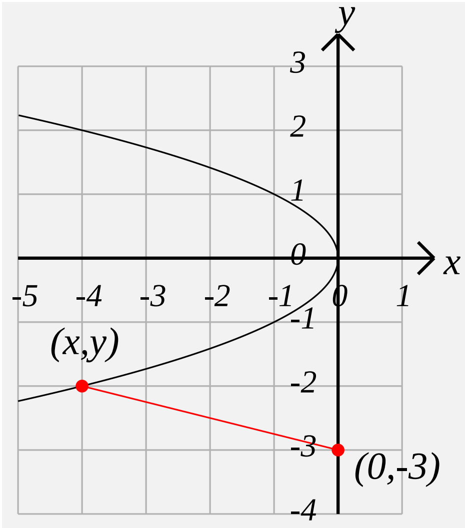

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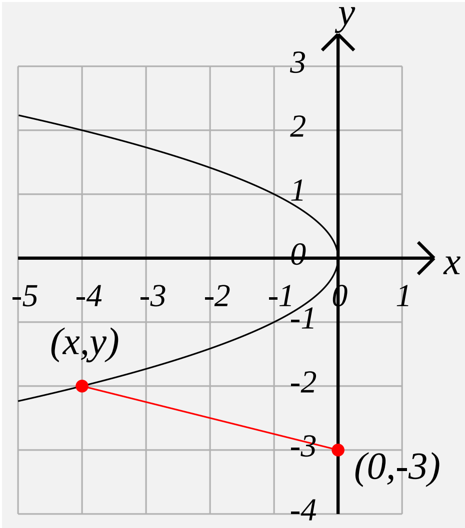

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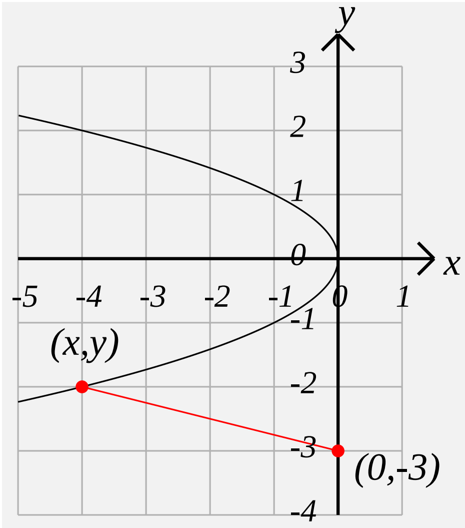

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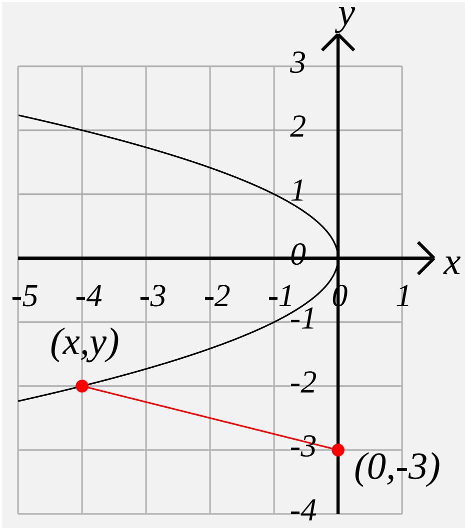

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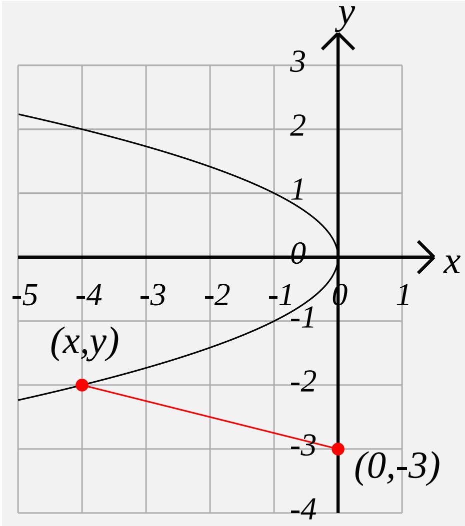

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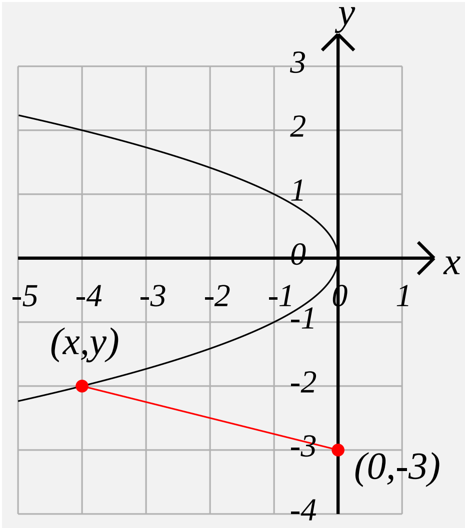

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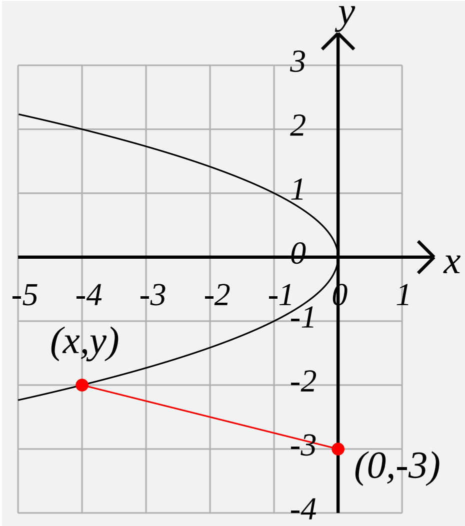

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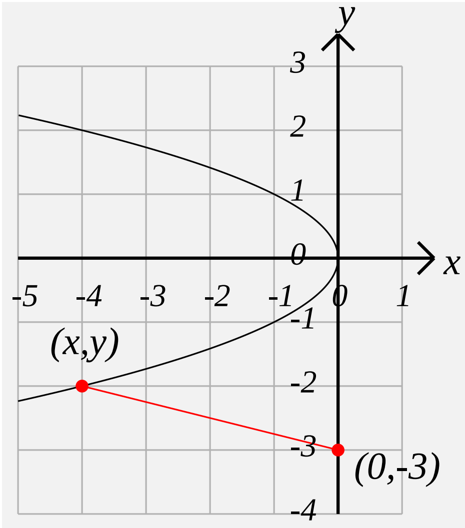

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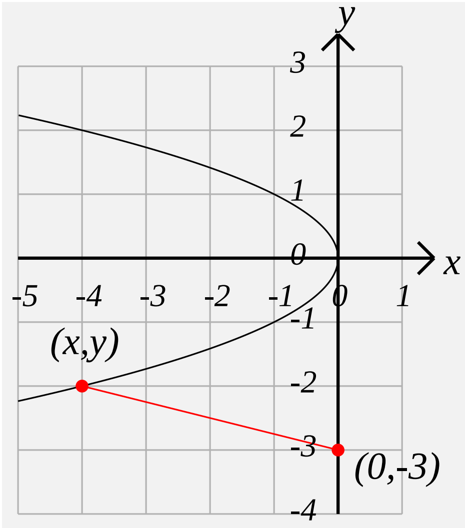

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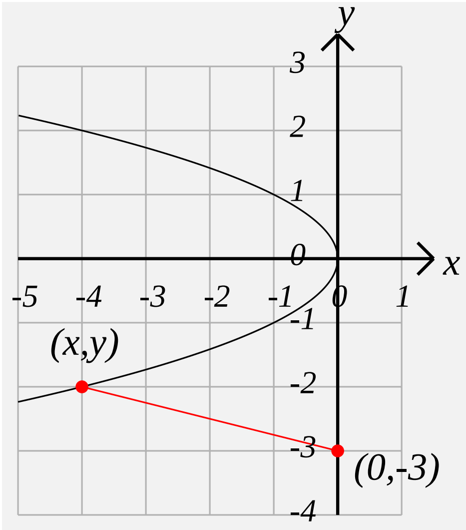

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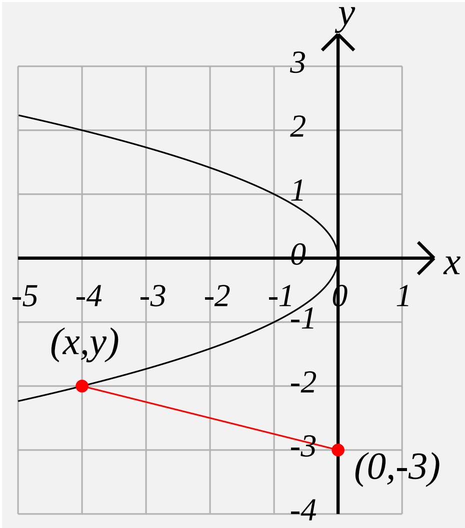

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$g'(y) = 4y^3 + 2y + 6 = (y+1)(4y^2 - 4y + 6) = 0$ iff $y = -1$.

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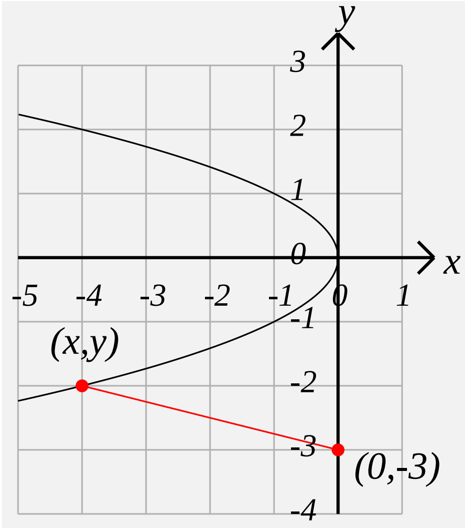

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Since $4y^2 - 4y + 6 = 4(y^2 - y + \frac{3}{2}) = 4[(y - \frac{1}{2})^2 + \frac{5}{4}] > 0$, Since $4y^2 - 4y + 6 = 4(y^2 - y + \frac{3}{2}) = 4[(y - \frac{1}{2})^2 + \frac{5}{4}] > 0$, we have $g(y) < 0$ if $y < -1$ and $g(y) > 0$ if $y > -1$, Since $4y^2 - 4y + 6 = 4(y^2 - y + \frac{3}{2}) = 4[(y - \frac{1}{2})^2 + \frac{5}{4}] > 0$, we have $g(y) < 0$ if $y < -1$ and $g(y) > 0$ if $y > -1$, so by the First Derivative test we have a relative minimum.

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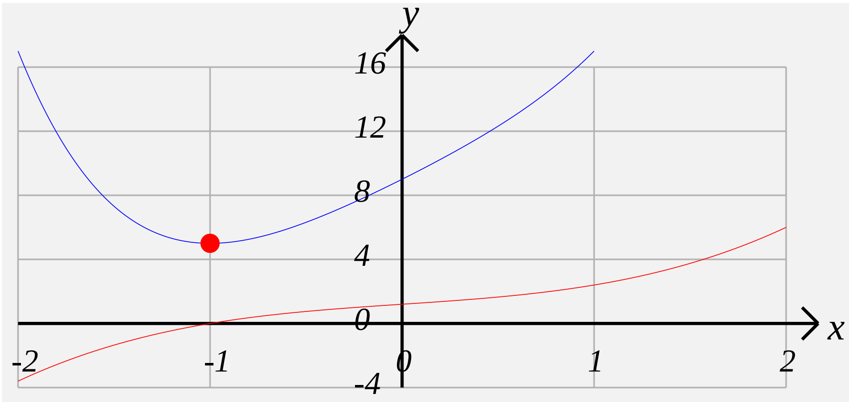

The graph of g is blue, and that of g' is red.