Exponential & Log Functions

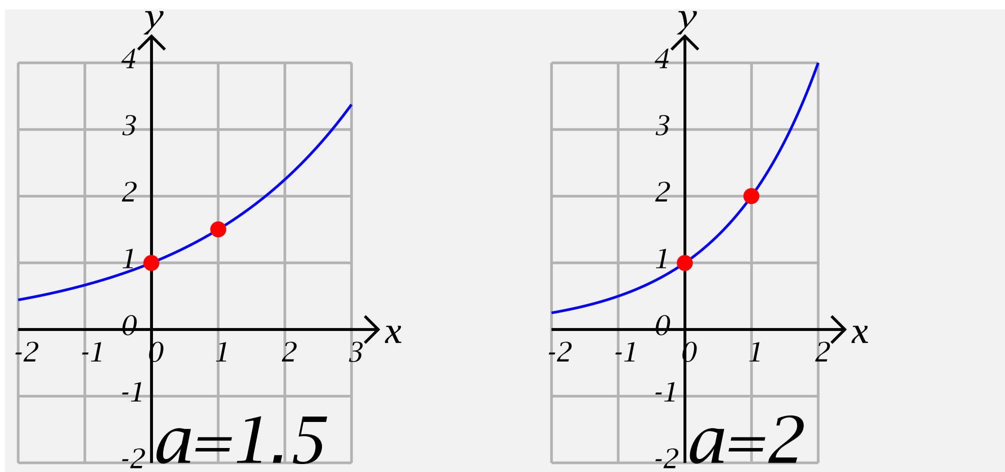
Definition: A function f is said to be an exponential function if its rule is of the form

$$f(x) = ka^x$$

where $a > 0$, $k \neq 0$ are constants.

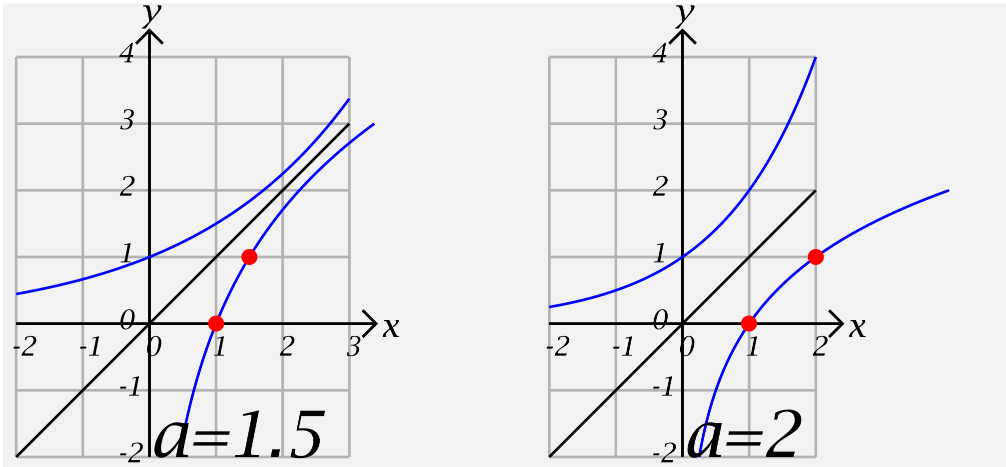
If $a = 1$, then $a^x = 1^x = 1$ and the graph of $y = ka^x$ is the horizontal line $y = k$. It is not 1:1.

If $a \neq 1$, the graph will be a curve, not a straight line, that never crosses the x-axis, and never crosses any other horizontal line more than once, so the function is 1:1. We always have $a^0 = 1$ and $a^1 = a$, so if we look at the graph of an exponential function we can always tell the value of a.



Logarithmic Functions:

$y = \log_a x$ means $a^y = x$. This is called the base a logarithm of x. It is the inverse function of the one-to-one function a^x. Logarithms of negative numbers are not defined.



For example, $\log_a a^2 = 2$.

If $a = 10$, then the logarithm we get is called the common logarithm. Its base is 10.

If $a = e \doteq 2.718281828459045\ldots$, then the logarithm is called the natural logarithm. This is usually written as $\ln x$, and means exactly the same thing as $\log_e x$. On calculators, this function is obtained from the “ln” button, and the common logarithm is obtained from the “log” button.

We always have $\log_a a^x = x$ when the logarithm is defined.

Arithmetic Properties of Logarithmic Functions:

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$\log_a bc = \log_a b + \log_a c$

$\log_a \frac{b}{c} = \log_a b - \log_a c$

$\log_a b^c = c \log_a b$

Since $a^0 = 1$, we have $\log_a 1 = 0$, and since $a^1 = a$, we have $\log_a a = 1$.

Also, $\log_a a^b = b \log_a a = b(1) = b$

$\log_a a^{-1} = (-1) \log_a a = (-1)(1) = -1$

or $\log_a \frac{1}{a} = \log_a 1 - \log_a a = 0 - (1) = -1$

Relation between logs with different bases:

If a, b, and c are positive numbers, then

$\log_a b = \frac{\log_c b}{\log_c a}$

In particular, if $c = e \doteq 2.718281828459045\ldots$, then

$\log_a b = \frac{\log_c b}{\log_c a} = \frac{\log_e b}{\log_e a} = \frac{\ln b}{\ln a}$

Note that this means that the graph of any logarithm function $\log_a x$ is obtained from the graph of $\ln x$ by a vertical scale factor of $\frac{1}{\ln a}$.