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$$x = 2x - 2$$

$$-x = -2,$$

Example 6: Solve $\log_2 x - \log_2(x-1) = 1$

Solution: $\log_2 x - \log_2(x-1) = \log_2 \dfrac{x}{x-1} = 1$, so

$$2^{\log_2 \frac{x}{x-1}} = 2^1 = 2$$

$$\frac{x}{x-1} = 2$$

$$x = 2(x-1)$$

$$x = 2x - 2$$

$$-x = -2, \quad x = 2$$

Problem 1: Solve $\log x - \log 8 = 1$

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$$\frac{x}{8} = 10 \quad x = 80$$

Problem 2:

Solve $\log_5 x + 2\log_5 x = 3$

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Solve $\log_5 x + 2\log_5 x = 3$

Solution: First simplify:

$$3\log_5 x = 3$$

$$\log_5 x = 1$$

$$5^{(\log_5 x)} = 5^1$$

$$x = 5$$

Problem 3:

Solve $\log_6(a+2) - \log_6 \dfrac{a-7}{5} = 1$

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$$6^{\left(\log_6\left(\frac{5(a+2)}{a-7}\right)\right)} = 6^1$$

1.13

$$\frac{5(a+2)}{a-7} = 6$$

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$$5(a+2) = 6(a-7)$$

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$$5a + 10 = 6a - 42$$

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$$a = 52$$