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The quantity $\frac{f'(x)}{f(x)}$ is called the **relative rate of change** of the function f , and is very important in practical applications.

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