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One-to-one functions are functions which do not achieve any value more than once on a specified interval. Any function which is strictly increasing or strictly decreasing on an interval is one-to-one on that interval.

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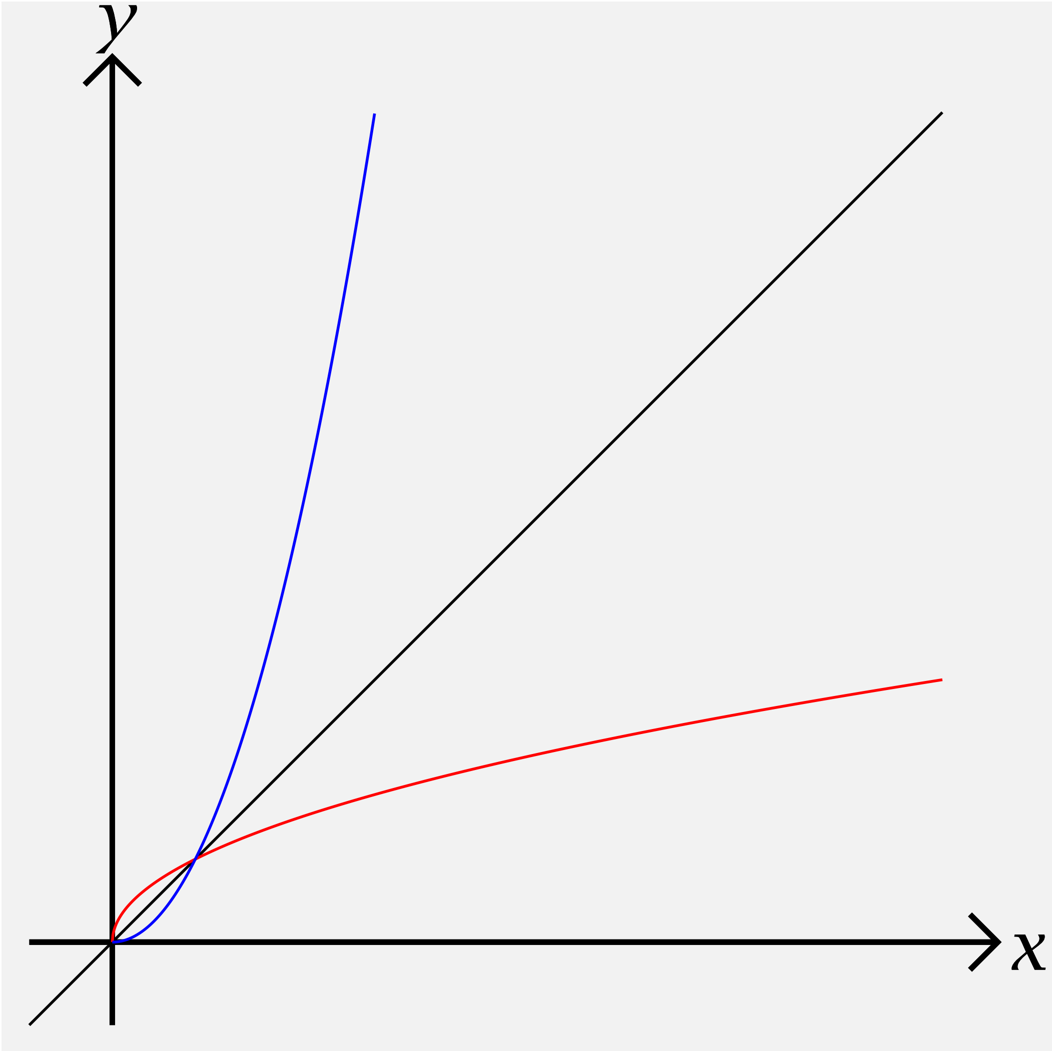

It is best to look at a Java applet .

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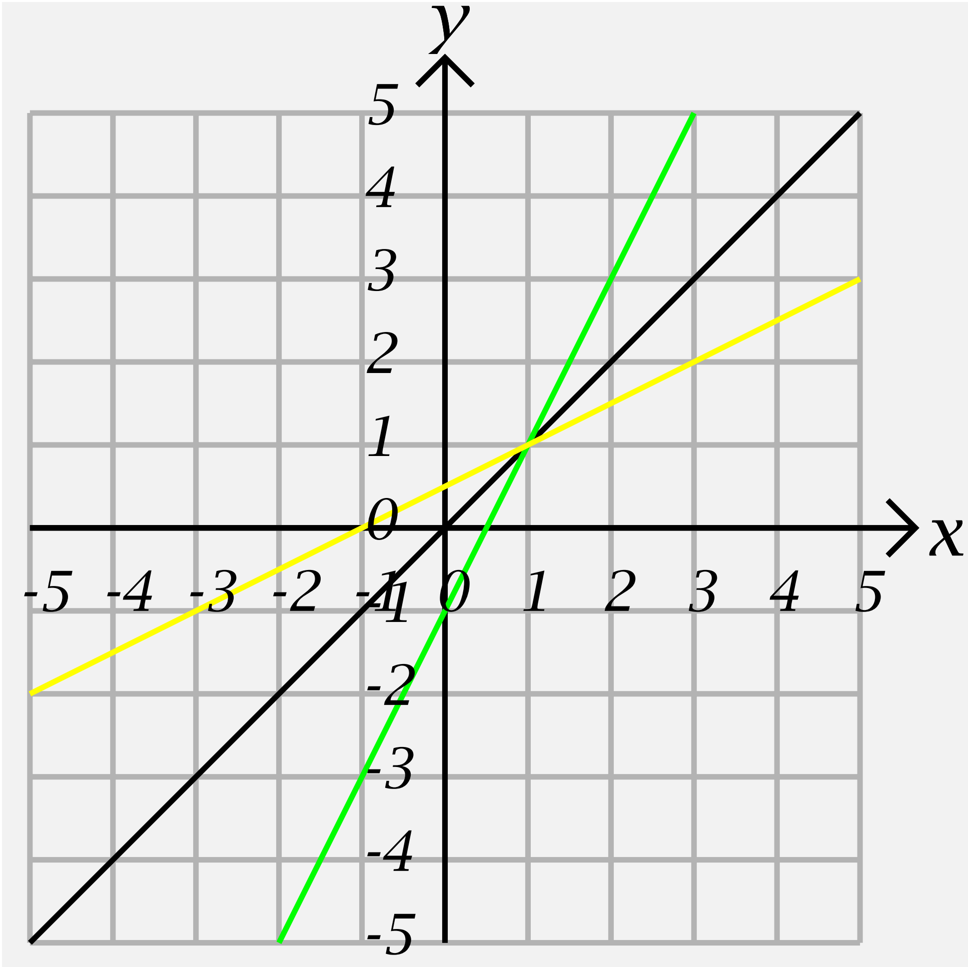

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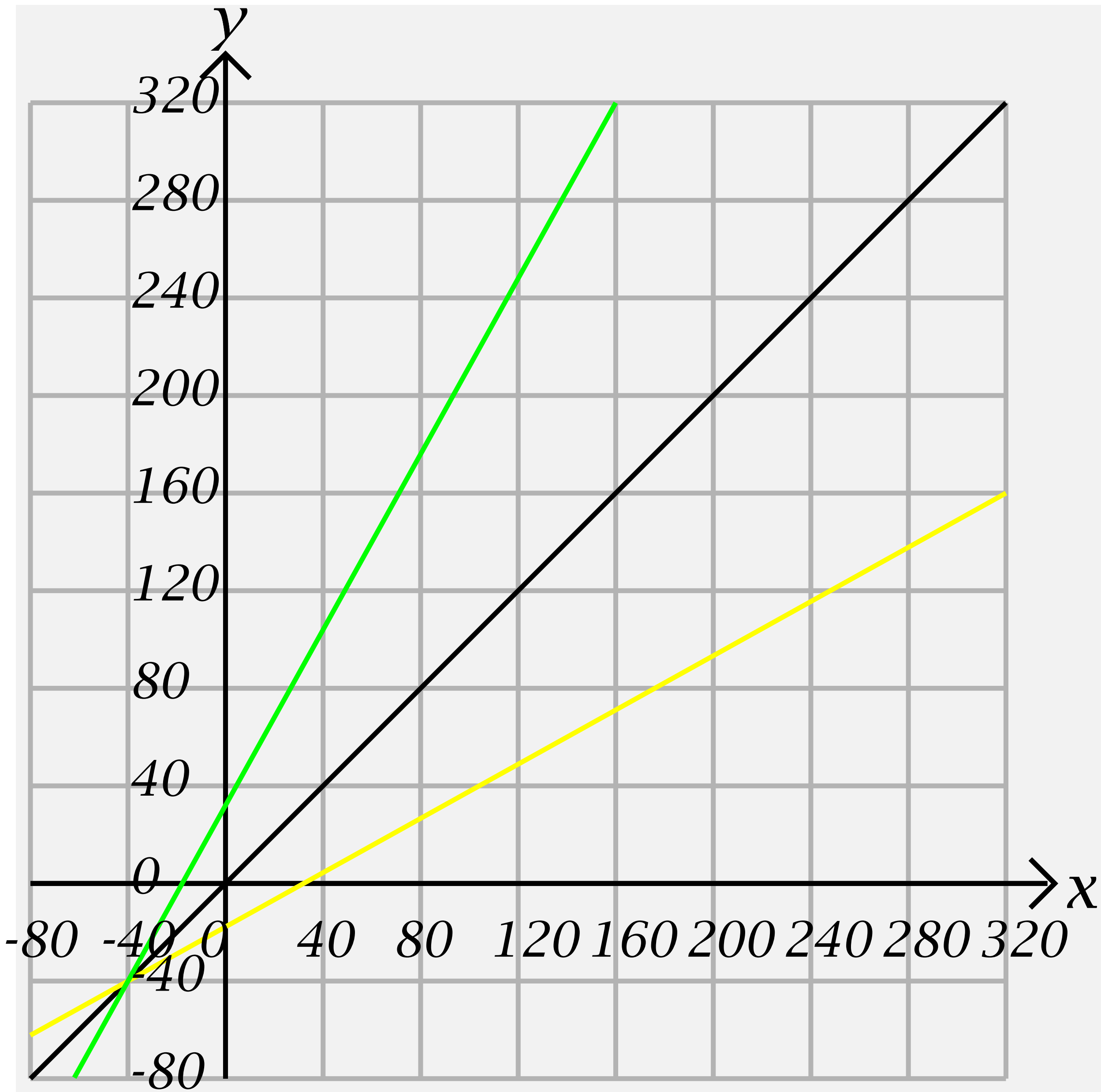
These two formulas are usually written as $C = \frac{5}{9}F - \frac{160}{9}$ and $F = \frac{9}{5}C + 32$, where F and C represent the Fahrenheit and Celsius temperatures respectively.


y
x
320
280
240
200
160
120
80
40
0
-40
-80
-80 -40 0 40 80 120 160 200 240 280 320

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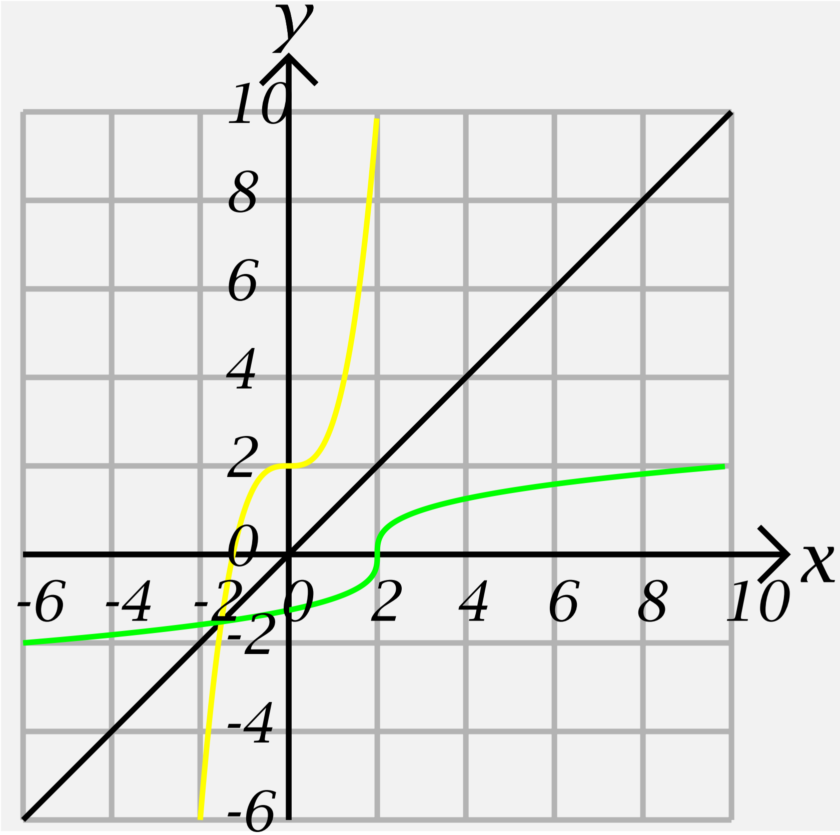
y
x
10
8
6
4
2
0
-2
-4
-6
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0
2
4
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8
10

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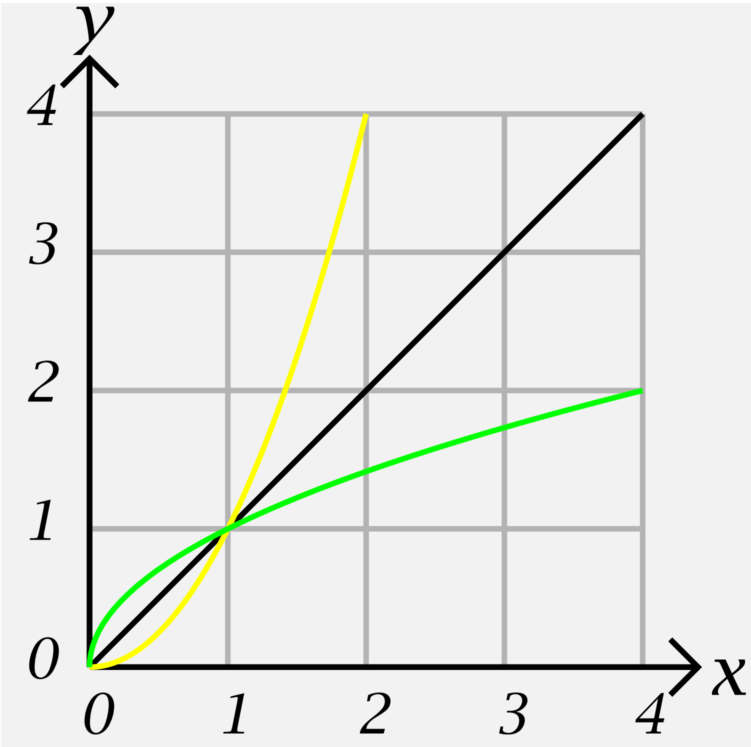

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